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Finite Time Synergetic Control for Quadrotor UAV with Disturbance Compensation

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ABSTRACT

In this paper, a finite time tracking control scheme is proposed for a quadrotor unmanned aerial vehicle by constructing a terminal synergetic manifold. The whole control system is divided into two loops: the inner-loop for the attitude control and the outer-loop for the position control. Through integrating the terminal sliding mode techniques and synergetic control theory, the finite time synergetic controllers are designed to achieve both the position and attitude tracking control performance. Moreover, a disturbance observer is employed in the inner-loop to compensate for effect of external disturbances. Simulation results are given to validate the effectiveness and satisfactory performance of the proposed scheme.

1. Introduction

As a rotor-type aircraft, quadrotor unmanned aerial vehicle (UAV) has many advantages, such as small size, simple structure, low cost, rapid maneuverability and flexible operability. Quadrotor UAV is a kind of nonlinear system with underactuation, strong coupling and static instability (Zhao et al., 2015). Generally, the control of rotors is directly influencing the quadrotor's attitude, and the position is indirectly controlled by the change of the attitude.

In order to solve the flight control problems of quadrotor UAV, numerous schemes have been put forward so far. In (Bouabdallah et al., 2004), PID control and LQR control methods are proposed, but the anti-disturbance performance of the above linear control methods is not satisfactory. In (Xu et al., 2015), a nonlinear control method of quadrotor UAV is designed based on singular perturbation with taking into account the robustness and tracking accuracy of the system. In (Liao et al., 2015), a finite time flight control based on fast terminal sliding mode is proposed, and the finite time stability and anti-disturbance performance are guaranteed, however, the nature of the discontinuous switching characteristic determines that the chattering phenomenon would exist, which may affect its practical application.

In (Kolesnikov et al, 2000), a synergetic control scheme is proposed and it can lead to the closed-loop system reduction without any chattering problem. The controller is easy to be implemented, and provides a good dynamic performance and stability characteristics. Therefore, the synergetic control scheme has been widely used in power systems (Zhao et al., 2013; Santi et al., 2004). The synergetic control design is based on a particular choice of the macro-variable, which results in the determination of a control law to force the system to track a reference signal. Hadjer Abderrezek et al. developed a nonlinear macro-variable in the terminal synergetic control for the buck DC/DC converter and a fast finite time convergence was achieved (Abderrezek et al., 2016). Chi-Hua Liu et al. presented a new evolution constraint of macro-variable in the finite time synergetic control scheme for robot manipulators (Liu et al., 2012).

Moreover, external disturbances exist in most control systems and may cause a system crash. Then, the disturbance suppression is an important issue in the field of control. In (Isidori et al., 1992), a measurement feedback is used for disturbance attenuation in affine nonlinear systems. In (Chen et al., 2004), several control techniques based on disturbance observers are used to introduce general frameworks for nonlinear systems subjected to disturbances. The disturbance attenuation and suppression problem of a class of multi-input and multi-output (MIMO) nonlinear systems with exogenous-system-generated disturbances is discussed in (Guo et al., 2005). A trajectory tracking controller of airship horizontal model based on active disturbance rejection is proposed in (Zhu et al.,

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2014). The existing work shows that the observer-based active disturbance suppression method is popular due to its effectiveness and ease of use.

Motivated by the aforementioned discussion, this paper develops a nonsingular terminal synergetic control scheme for quadrotor UAV with external disturbances. The developed terminal synergetic manifold contains terminal synergetic macro-variable and nonsingular terminal constraint, such that the proposed synergetic controller can achieve finite time convergence of macro variables without any singularity problems. Besides, disturbance observer is employed to compensate for the system uncertainty and external diturbances.

The rest of this paper is organized as follows. Section 2 presents the dynamic model of the quadrotor UAV. Section 3 describes the control design process. Section 4 provides the simulation results, and the conclusion is given in Section 5.

2. Dynamic Model

For the convenience of an intuitive understanding of the quadrotor UAV configuration, the schematic configuration, the illustrations of reference frames, and the force from each of the four rotors are shown in Fig. 1. There are two reference frames defined in the model. A fixed inertia frame {E} represented by $OX_EY_EZ_E$ is established to locate the absolute position of the center of the mass of the quadrotor vehicle. Body frame {B}, which is represented by $OX_BY_BZ_B$, is a frame attached to the center of mass of the vehicle, shifting and/or rotating with the quadrotor vehicle. The Euler angles of the vehicle with respect to reference frame {E} is denoted by $\{\psi,\theta,\phi\}$, which represent the yaw angle (ψ, rotation around OZ_E), the pitch angle (θ, rotation around OY_E) and the roll angle (ϕ, rotation around OX_E), respectively. The four rotors rotate in different speeds to generate the lift force F_i and balance the yaw torque.

The rotation matrix for transforming coordination from {B} to {E} is given as

$$R_B^E=\begin{bmatrix} c\theta\cdot c\psi & s\phi\cdot s\theta\cdot c\psi-c\phi\cdot s\psi & c\phi\cdot s\theta\cdot c\psi+s\phi\cdot s\psi \\ c\theta\cdot s\psi & s\phi\cdot s\theta\cdot s\psi+c\phi\cdot c\psi & c\phi\cdot s\theta\cdot s\psi-s\phi\cdot c\psi \\ -s\theta & s\phi\cdot c\theta & c\phi\cdot c\theta \end{bmatrix} \quad (1)$$

where $c\theta$ and $s\theta$ denote $cos\theta$ and $sin\theta$, respectively, and similarly for ϕ and ψ.

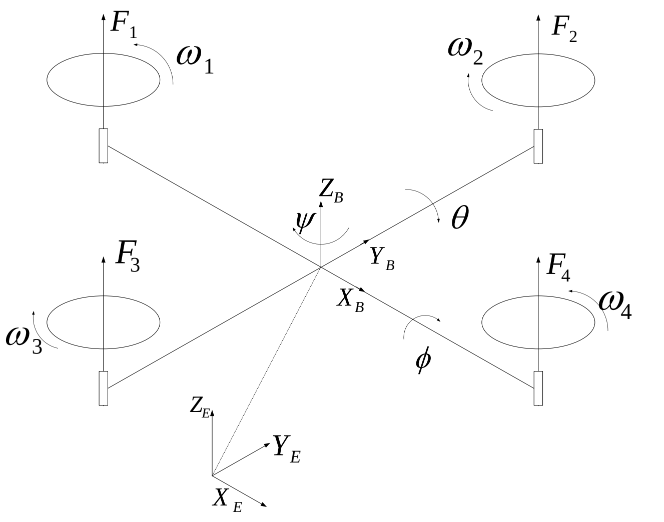

Fig.1. Quadrotor configuration frame scheme

At present, the Newton-Euler formula is used to modelling the quadrotor UAV in most research works (Mellinger et al., 2012; Elsamanty et al., 2013; Islam et al., 2014). Generally, the air resistance and the gyro effect are ignored in the kinematic and dynamic analysis. Prior to modeling the quadrotor UAV, the following reasonable and necessary assumptions are provided.

Assumption 2.1 (Bouabdallah et al., 2007) The quadrotor structure is rigid and strictly symmetrical with respect to the body coordinate system.

Assumption 2.2 (Bouabdallah et al., 2007) The center of gravity of the quadrotor coincides with the origin of the body coordinate system.

Under these assumptions, the quadrotor UAV can be described as a rigid body. So all the external forces acting on the rigid body can be regarded as acting on the center of mass. The acceleration of the center of mass governed by the Newton equation is

$$\begin{bmatrix}\tau_x\\ \tau_y\\ \tau_z\end{bmatrix}=\begin{bmatrix}I_x&0&0\\0&I_z&0\\0&0&I_z\end{bmatrix}\begin{bmatrix}\dot p\\ \dot q\\ \dot r\end{bmatrix}+\begin{bmatrix}p\\ q\\ r\end{bmatrix}\times\begin{bmatrix}I_x&0&0\\0&I_z&0\\0&0&I_z\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix} \quad (3)$$

where $\{\tau_x,\tau_y,\tau_z\}$ represents the moment applied to the vehicle's center of mass; $\{I_x,I_y,I_z\}$ represents the moment of inertia of the quadrotor vehicle; $\{p,q,r\}$ denotes the angular velocity with respect to frame {B}. All the components represent the ones around OX_B, OY_B, OZ_B, respectively.

Taking into account that the quadrotor is generally in a low speed flight or hover state, the change of attitude angle is small, reference (Liao et al., 2015) notices that $\dot\phi\approx p,\dot\theta\approx q,\dot\psi\approx r,\ddot\phi\approx\dot p,\ddot\theta\approx\dot q,\ddot\psi\approx\dot r$. The system parameters and states in equations (2) and (3) are not accurately obtained due to the influence of measurement noise, power supply changes and external disturbances. Therefore, combining (1) to (3), the mathematical model can be expressed as

$$\begin{cases}\ddot x=U_x+d_x\\ \ddot y=U_y+d_y\\ \ddot z=U_z+d_z\\ \ddot\phi=a_1\dot\theta\dot\psi+b_1\tau_x+d_\phi\\ \ddot\theta=a_2\dot\phi\dot\psi+b_2\tau_y+d_\theta\\ \ddot\psi=a_3\dot\phi\dot\theta+b_3\tau_z+d_\psi\end{cases} \quad (4)$$

Where $U_x=\frac{U_F}{m}(cos\phi sin\theta cos\psi+sin\phi sin\psi)$,$U_y=\frac{U_F}{m}(cos\phi sin\theta sin\psi-sin\phi cos\psi)$,$U_z=-g+\frac{U_F}{m}(cos\phi cos\theta)$; $a_1=\frac{I_y-I_z}{I_x}$, $a_2=\frac{I_z-I_x}{I_y}$, $a_3=\frac{I_x-I_y}{I_z}$, $b_1=\frac{1}{I_x}$, $b_2=\frac{1}{I_y}$, $b_3=\frac{1}{I_z}$;

$\{d_x,d_y,d_z,d_\phi,d_\theta,d_\psi\}$ represent external disturbance and model uncertainty.

From (4), we can see that the former three terms are the position related equations, the latter three terms are the attitude related equations. Therefore, the control problem could be divided into double loops to realize the flight control of the quadrotor UAV. But it should be noticed that the position term $\{U_x,U_y,U_z\}$ contains the attitude terms like that $cos\phi$, $sin\theta$, $cos\psi$ and so on, which results in the coupling problem between the position and attitude and increases the design difficulty of position and attitude controllers.

3. Control Design

In this section, we design the finite time synergetic control strategy for the quadrotor vehicle. To obtain a clear understanding of the control strategy, the overall control structure diagram is given in Fig. 2, which is mainly divided into outer-loop position controller and inner-loop attitude controller.

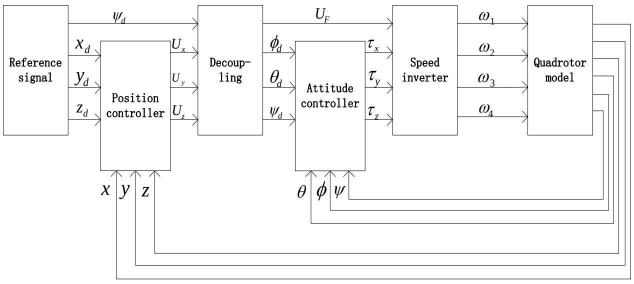

Fig. 2. Control system structure

3.1 Preliminaries

As shown in Fig. 2, $\{U_x, U_y, U_z\}$ is regarded as the outputs of position controller and $\{\tau_x, \tau_y, \tau_z\}$ are regarded as the outputs of attitude controller. Notice that the reference signals include $\{x_d, y_d, z_d, \psi_d\}$. The control structure mainly includes three parts: position-attitude decoupling, rotational speed inverter, and the position & attitude controllers.

A. Position-attitude decoupling

According to equation (4), the position and attitude relationship can be decoupled, the results are as follows:

$$\begin{cases} U_F = m\sqrt{{U_x}^2 + {U_y}^2 + (U_z+g)^2} \\ \phi_d = sin^{-1}\left[\frac{m}{U_F}\left(U_x \sin\psi_d - U_y \cos\psi_d\right)\right] \\ \theta_d = tan^{-1}\left[\frac{1}{U_z+g}\left(U_x \cos\psi_d + U_y \sin\psi_d\right)\right] \end{cases} \tag{5}$$

After decoupling calculation, the attitude controller inputs, that is $\{\phi_d, \theta_d\}$, can be obtained from the outputs of position controller $\{U_x, U_y, U_z\}$. Therefore, the position controller and attitude controller are connected in this way.

B. Rotational speed inverter

The quadrotor UAV is usually driven by DC motors, so the relationship between the control torque and the rotational speed of the rotor are stated as follows (Wei et al., 2015):

$$\begin{bmatrix} U_F \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} k_F & k_F & k_F & k_F \\ 0 & k_F L & 0 & -k_F L \\ -k_F L & 0 & k_F L & 0 \\ k_M & -k_M & k_M & -k_M \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \tag{6}$$

Then, the speed inverter is shown in the following:

$$\begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} = \frac{1}{4k_F k_M L} \begin{bmatrix} k_M L & 0 & -2k_M & k_F L \\ k_M L & 2k_M & 0 & -k_F L \\ k_M L & 0 & 2k_M & k_F L \\ k_M L & -2k_M & 0 & -k_F L \end{bmatrix} \begin{bmatrix} U_F \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \tag{7}$$

where L denotes the distance from the center of mass to the rotation axis of the rotor; $\omega_i (i = 1, 2, 3, 4)$ represent the rotational speed of the rotor; k_F represents the lift coefficient; k_M represents the torque coefficient.

The outputs of attitude controller $\{\tau_x, \tau_y, \tau_z\}$, which represent the overall torque acting on the rigid body, are transformed into the every rotor's desired speed. When the quadrotor is in the hover state, the lift force U_F is equal with the gravity of quadrotor mg.

C. Position & attitude controllers

Considering that the position and attitude equations belong to the second order MIMO nonlinear system, the attitude equations become more complicated, so in order to facilitate the design of the controllers, system (4) can be expressed as

$$\begin{cases} \ddot{X} = f(X) + B(X)U + D \\ Y = X \end{cases} \tag{8}$$

where $\ddot{X} = [\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\phi}, \ddot{\theta}, \ddot{\psi}]^T$, $f(X) = [0,0,0, \mathrm{a}_1\dot{\theta}\dot{\psi}, \mathrm{a}_2\dot{\phi}\dot{\psi}, \mathrm{a}_3\dot{\phi}\dot{\theta}]^T$, $B(X) = diag\{1,1,1, b_1, b_2, b_3\}$, $U = [U_x, U_y, U_z, \tau_x, \tau_y, \tau_z]^T$, $D = [d_x, d_y, d_z, d_\phi, d_\theta, d_\psi]^T$.

The control objective of this paper is to design a finite time synergetic controller U for the system (8), such that the system output Y can track the desired reference signal Y_d and all signals in the closed-loop system are bounded.

3.2 Disturbance observer design

Disturbance observer (DO) is easy to use and can be designed independently from the controller. In this section, a disturbance observer is employed to compensate for the external disturbance and model uncertainty D. The design of the disturbance observer is generally divided into two steps (Chen et al., 2011): (i) design the disturbance observer to estimate the external disturbance; (ii) design the attitude controller to compensate for the negative effect caused by using the estimation of the observer. Before the disturbance observer design, it is necessary to make a reasonable assumption firstly.

*Assumption 3.*1 For all system states X, there exists $\delta > 0$ such that $|\dot{D}_i| \leq \delta$.

To design a disturbance observer, an auxiliary design variable is introduced as follows:

$$z = D - k_d x_2 \tag{9}$$

where $k_d > 0$; $x_2 = \dot{X}$.

Considering (8) and (9), the time derivative of z can be written as

$$\begin{aligned} \dot{z} &= \dot{D} - k_d \dot{x}_2 \\ &= \dot{D} - k_d(f + BU + D) \\ &= \dot{D} - k_d(f + BU + z + k_d x_2) \end{aligned} \tag{10}$$

The estimate of the auxiliary variable z is given by

$$\dot{\hat{z}} = -k_d(f + BU + \hat{z} + k_d x_2) \tag{11}$$

where $\hat{z}$ is the estimate value of z.

Considering (9), we can obtain the estimate of disturbance D

$$\widehat{D} = \hat{z} + k_d x_2 \tag{12}$$

Then, we define the estimate error

$$\tilde{z} = z - \hat{z} = D - \widehat{D} = \widetilde{D} \tag{13}$$

Differentiating (13), and considering (10) and (11) yields

$$\dot{\tilde{z}} = \dot{z} - \dot{\hat{z}} = \dot{D} - k_d\tilde{z} = \dot{D} - k\widetilde{D} = \dot{\widetilde{D}} \tag{14}$$

For the developed disturbance observer (11) and (12), considering the convergent stability of estimate error $\widetilde{D}$, the Lyapunov function can be expressed as

$$V_0 = 0.5\widetilde{D}^2 = 0.5\tilde{z}^2 \tag{15}$$

Considering (15), the derivative of V_0 is

$$\dot{V}_0 = \widetilde{D}\dot{\widetilde{D}} = \widetilde{D}\dot{D} - k_d\widetilde{D}^2 = \tilde{z}\dot{D} - k_d\tilde{z}^2 \tag{16}$$

Due to the basic inequality $2ab \le a^2 + b^2$, we can assure that $\tilde{z}\dot{D} \le 0.5(\tilde{z}^2 + \dot{D}^2)$. Then, (16) can be rewritten as

$$\begin{aligned} \dot{V}_0 &\le 0.5(\tilde{z}^2 + \dot{D}^2) - k_d\tilde{z}^2 \\ &\le (0.5 - k_d)\tilde{z}^2 + 0.5\dot{D}^2 \\ &\le -(k_d - 0.5)\tilde{z}^2 + 0.5\delta^2 \\ &\le -(2k_d - 1)V_0 + 0.5\delta^2 \end{aligned} \tag{17}$$

According to the *lemma 1* in the literature (Yu et al., 2002) and (17), when $k_d > 0.5$, the estimate error $\widetilde{D}$ is bounded by $\epsilon = 0.5\delta^2/(2k_d - 1)$.

Thus, we can conclude that the approximation error $\widetilde{D}$ of the developed disturbance observer is uniformly asymptotically convergent if the parameter k_d is chosen appropriately.

3.3 Terminal synergetic manifold

In this subsection, we provide a new terminal synergetic manifold for quadrotor system (8) by combining the terminal attractor technique.

Firstly, define system tracking error as

$$e = X - X_d \tag{18}$$

where X_d denotes the desired signal. Then, the first order derivative and the second order derivative of (18) are expressed as

$$\dot{e} = \dot{X} - \dot{X}_d \tag{19}$$

$$\ddot{e} = \ddot{X} - \ddot{X}_d \tag{20}$$

Next, define a macro-variable σ to construct the following manifold

$$M = \{\varepsilon : \sigma = s(\varepsilon) = 0, s(\varepsilon) \in R\} \tag{21}$$

where $\varepsilon = \dot{e}$; $s(\varepsilon) = \dot{e} + \alpha e + \beta e^{\gamma}$, $\alpha, \beta > 0$, $0 < \gamma < 1$.

The aim of the synergetic controller is to drive the variable σ to converge from any initial state and remain on the manifold M in finite time. Then, the system tracking error e moves along the manifold to the equilibrium point in finite time. The dynamic process of the variable σ converging to the manifold M can be constrained by the following nonsingular form:

$$\tau\dot{\sigma}^{p/r} + \sigma = 0 \tag{22}$$

where $\tau > 0$; p, r are positive odd integers and satisfy $p > r$.

3.4 Finite time synergetic controller design

In this section, we provide the finite time synergetic controller design based on the proposed terminal synergetic manifold given in Sect. 3.3. The detailed design procedure is given as follows.

Substituting (21) into (22), we have

$$\tau(s_\varepsilon\dot{\varepsilon})^{p/r} + s = 0 \tag{23}$$

where $s_\varepsilon = \frac{\partial s}{\partial \varepsilon} = \frac{\partial s}{\partial \dot{e}} = 1$ .Then consider the system (8) and tracking error (20), and we can obtain

$$\tau(f(x) + D + B(x)U - \ddot{x}_d)^{\frac{p}{r}} + s = 0 \tag{24}$$

So the control law can be designed as

$$\begin{aligned} U &= B(x)^{-1}(\ddot{X}_d - f(x) - \widehat{D}) - B(x)^{-1}\left[(\tau^{-1}s)^{\frac{r}{p}} + k_n s\right] \\ &= U_{eq} + U_{ftsc} \end{aligned} \tag{25}$$

where $U_{eq} = B(x)^{-1}(\ddot{x}_d - f(x) - \widehat{D})$; $U_{ftsc} = -B(x)^{-1}[(\tau^{-1}s)^{r/p} + k_n s]$, $k_n > 0$; $\widehat{D}$ is the estimate of disturbance D, which is designed as (12).

Remark 3.1 According to the control law (25), notice that there is no direct differentiation of s but the partial differential s respect to ε. That means there is no differential term of βe^{γ} in the control law, and thus the singularity is avoided from the underlying cause. In addition, there is no sign function such that there will be no chattering phenomenon.

Lemma 3.1 (Yu et al., 2005) An extended Lyapunov description finite-time stability can be given with the following form

$$\dot{V}(x) + \alpha V(x) + \beta V^{\gamma}(x) \le 0 \tag{26}$$

where $\alpha, \beta > 0$; $0 < \gamma < 1$, and the settling time can be given by

$$t_r = \frac{1}{\alpha(1-\gamma)}\ln\frac{\alpha V^{1-\gamma}(x_0) + \beta}{\beta} \tag{27}$$

Theorem 1 Consider a class of nonlinear systems (8). The variable σ will converge to zero in finite time with the convergence rate depending on the selected parameters τ, p and r if the control law is designed as (25).

Proof

First, choose the following Lyapunov function:

$$V_1 = 0.5\sigma^2 \tag{28}$$

Differentiating (28) with respect to time and using (25) yields

$$\begin{aligned}\dot{V}_1 &= \sigma\dot{\sigma} = ss_\varepsilon\dot{\varepsilon} = ss_\varepsilon(\ddot{X} - \ddot{X}_d)\\ &= s(f(x) + \mathrm{D} + B(x)U - \ddot{x}_d)\\ &= -s(\tau^{-1}s)^{r/p} + \tilde{D}s - k_n s^2\\ &< -\tau^{-\frac{r}{p}}(s^2)^{\frac{r+p}{2p}} + \epsilon(s^2)^{\frac{r+p}{2p}} - k_n s^2\\ &< -(\tau^{-r/p} - \epsilon)(2)^{r/p}(V_1)^{(r+p)/2p} - 2k_n V_1\\ &< -\alpha_1 V_1 - \alpha_2 {V_1}^{\alpha_3}\end{aligned} \tag{29}$$

where $\alpha_1 = 2k_n$, $\alpha_2 = (\tau^{-r/p} - \epsilon)(2)^{r/p}$ and $\alpha_3 = (r+p)/2p$.

Then, (29) can be rewritten as

$$\dot{V}_1(x) + \alpha_1 V_1(x) + \alpha_2 {V_1}^{\alpha_3}(x) < 0 \tag{30}$$

Since $p > r$, that is $0.5 < \alpha_3 = (r+p)/2p < 1$, satisfies the condition of ***Lemma 3.1***, the settling time of the period $\sigma \to 0$ is

$$\mathrm{T}_{r1} = \frac{1}{\alpha_1(1-\alpha_3)}\ln\frac{\alpha_1 V_2^{1-\alpha_3}(x_0)+\alpha_2}{\alpha_2} \tag{31}$$

Theorem 2 Consider a class of nonlinear systems (8). When the variable $\sigma = 0$ is achieved, the tracking error e will converge to the equilibrium point within a finite time.

Proof

When the variable $\sigma = 0$ is achieved, the system tracking error e remains on the manifold M and the manifold behaves like a terminal attractor. On the manifold $s(\varepsilon) = 0$, we can obtain

$$\varepsilon = \dot{e} = -(\alpha e + \beta e^\gamma) \tag{32}$$

Constructing the following Lyapunov candidate

$$V_2 = 0.5e^2 \tag{33}$$

and differentiating V_2 along (32) yields:

$$\begin{aligned}\dot{V}_2 &= -e(\alpha e + \beta e^\gamma)\\ &= -\alpha e^2 - \beta e^{\gamma+1}\\ &= -2\alpha V_2 - \beta 2^{\frac{\gamma+1}{2}} V_2^{\frac{\gamma+1}{2}}\\ &< -\beta_1 V_2 - \beta_2 {V_2}^{\beta_3}\end{aligned} \tag{34}$$

where $\beta_1 = 2\alpha$, $\beta_2 = \beta 2^{\frac{\gamma+1}{2}}$ and $\beta_3 = \frac{\gamma+1}{2}$.

Then, (34) can be rewritten as

$$\dot{V}_2(x) + \beta_1 V_2(x) + \beta_2 {V_2}^{\beta_3}(x) < 0 \tag{35}$$

Since $0 < \gamma < 1$, that is $0.5 < \beta_3 = \frac{\gamma+1}{2} < 1$, satisfies the condition of ***Lemma 3.1***, the settling time of the period $e \to 0$ is

$$\mathrm{T}_{r2} = \frac{1}{\beta_1(1-\beta_3)}\ln\frac{\beta_1 V_2^{1-\beta_3}(x_0)+\beta_2}{\beta_2} \tag{36}$$

In conclusion, the proposed control law (25) can drive the variable σ and the tracking error e to zero within a finite convergence time $T_r = T_{r1} + T_{r2}$.

4. Simulation

In this section, simulations are conducted to verify the feasibility of the proposed scheme. Table 1 gives the system model parameters (Liao et al., 2015), while the initial state of the system are set to zero; the position reference signals are given as $x_d = y_d = z_d = 2m$; yaw angle reference signal is given $\psi_d = 0.5rad$. The parameters of the position controller are set to $\alpha_1 = 1.5$, $\beta_1 = 0.1$, $\gamma_1 = 0.8$, $p_1 = 7$, $r_1 = 5$, $\tau_1 = 0.2$, $k_{n1} = 1$, while the parameters of the attitude controller are set to $\alpha_2 = 2$, $\beta_2 = 0.1$, $\gamma_2 = 0.8$, $p_2 = 7$, $r_2 = 5$, $\tau_2 = 0.5$, $k_{n2} = 1$. The simulation results of the two cases are shown as follows.

Table 1 model parameters

Parameters	Value
m	$0.625kg$
L	$0.1275m$
k_F	$2.103 \times 10^{-6}N/(rad \cdot s^{-2})$
k_M	$2.091 \times 10^{-8}N/(rad \cdot s^{-2})$
I_x	$2.3 \times 10^{-3}kg \cdot m^2$
I_y	$2.4 \times 10^{-3}kg \cdot m^2$
I_z	$2.6 \times 10^{-3}kg \cdot m^2$

Case I: control performance without external disturbance

In this case, we consider the situation where there exists no external disturbance, and simulations are conducted with two different control schemes. In order to verify the effectiveness and superiority of the proposed controller, the terminal sliding mode control (TSMC) in (Mellinger et al., 2012) is employed for the comparison with the proposed finite time synergetic control (FTSC). In the TSMC scheme, the sliding mode is $s = \dot{e} + \alpha e + \beta e^{q/p}$; the control law is $\mathrm{U} = -\mathrm{B}^{-1}[f + \dot{e}(\alpha + \beta e^{q/p-1}) - \ddot{x}_d + k_1 s + k_2 sign(s)]$. The system parameters and the initial values are all set the same as Table 1. Compared simulations results are shown in Figs.3-6, respectively, and we can see that FTSC has a faster trajectory tracking performance due to the quick response of attitude. Fig.7 shows the trajectory of the quadrotor in 3-dimensional space.

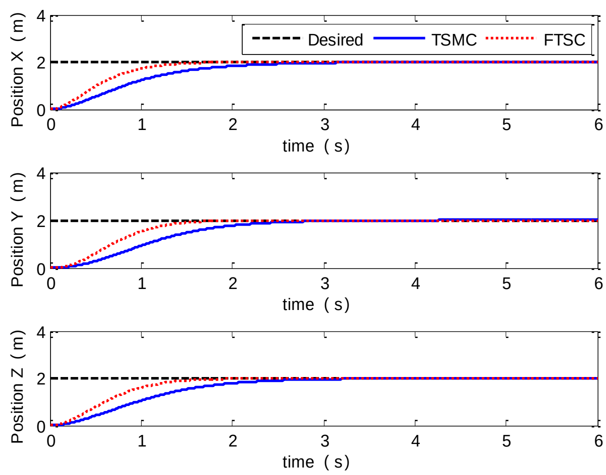

Fig.3 Position tracking performance

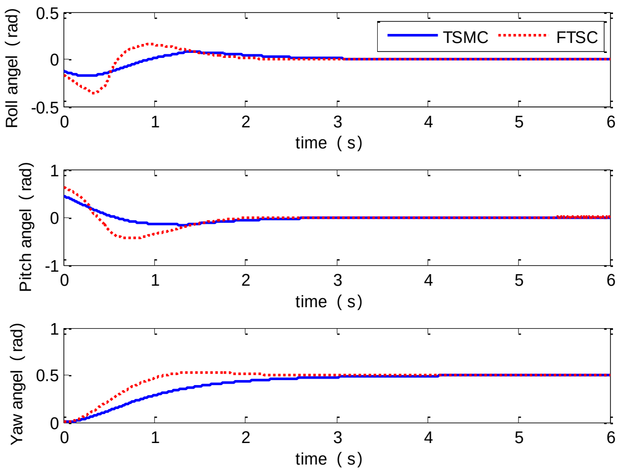

Fig.4 Attitude convergence performance

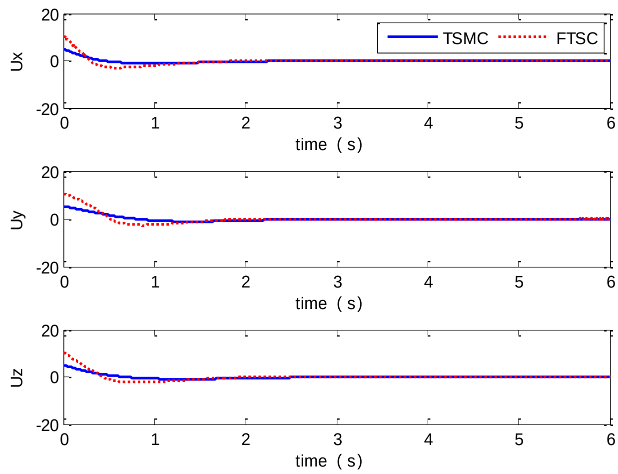

Fig.5 Position controller outputs

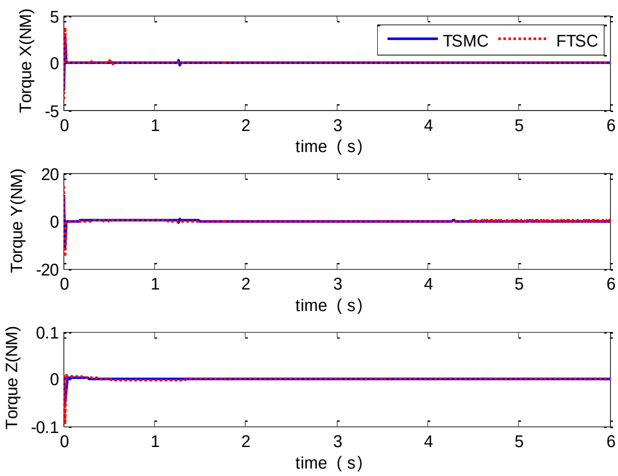

Fig.6 Attitude controller outputs

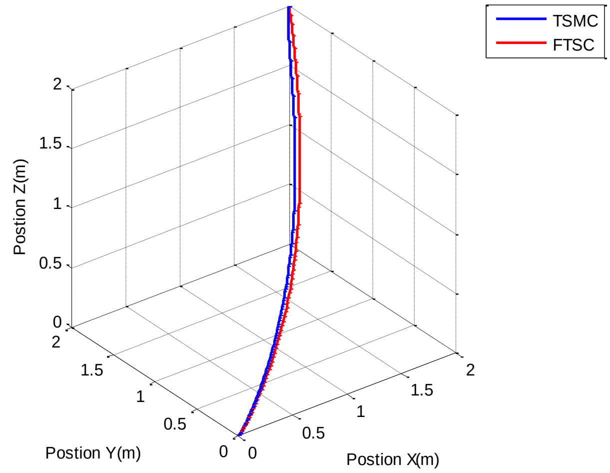

Fig.7 3-Dimentional trajectory

Case II：control performance with external disturbance

In this case, we consider the other situation where external disturbances exist. The disturbances are described as $\{2,4\sin(5t),3\sin(4t)\}$ and added to the outer-loop channel after 3 seconds. Simulations are conducted for the FTSC without DO and the FTSC with DO. The disturbance observer parameter is designed to be $k = 50$. Simulations results are shown in Figs.8-12, respectively, and we can see that the controller with DO has the better capability to stabilize the overall nonlinear quadrotor system and provides a better trajectory tracking performance.

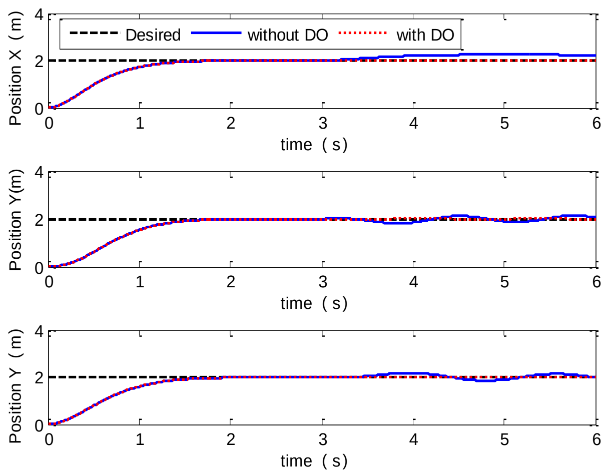

Fig.8 Position tracking performance

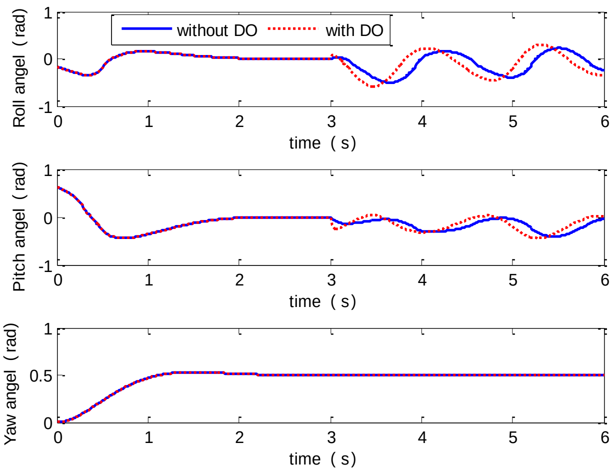

Fig.9 Attitude convergence performance

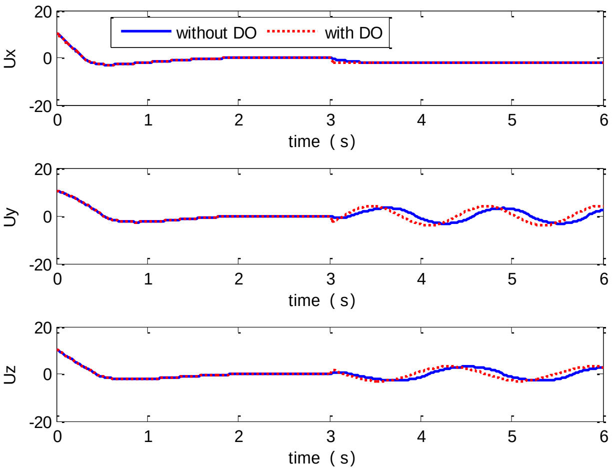

Fig.10 Position controller outputs

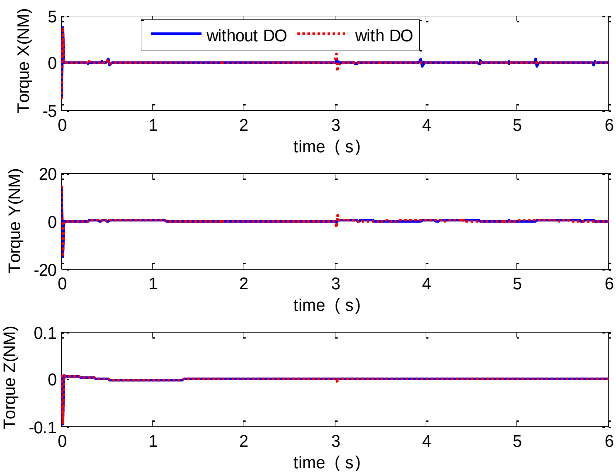

Fig 11 Attitude controller outputs

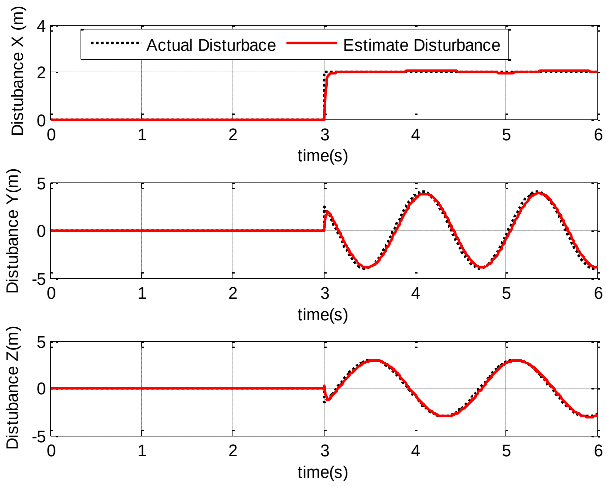

Fig.12 Disturbance estimation

5. Conclusion

In this paper, a finite time synergetic control scheme is proposed for the flight of quadrotor UAV with disturbance. Based on the terminal sliding mode techniques and synergetic control theory, both the position and attitude tracking control performance are guaranteed simultaneously. Besides, the effect of external disturbances is considered and compensated by employing a disturbance observer. Finally, some simulation examples are given to show the effectiveness of the proposed scheme.

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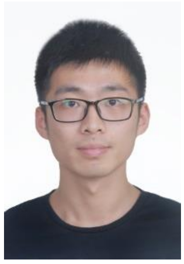

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