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The Mechanical Meaning of Bernstein Basis Function

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ABSTRACT

Bernstein basis has a specific mechanics meaning in geometric modeling. In this paper, we combine with the load and bending moment of mechanics to study the mechanical meaning of the Bernstein basis. In mechanics we have found the model of Bernstein basis, and give its particular mechanical meaning. In the triangular domain of binary Bernstein basis we used triangulation to study it. Eventually we find the mechanical meaning of the Bernstein basis.

1. Introduction

Bedform dynamics is of great importance for fluvial and coastal research. Bernstein base function has an important role in geometric modeling. With the rapid development of automobile，in the 1960s, aided design of car styling urgent to find a practical and feasible modeling tools 1972 French Renault engineers Bézier advance a shape of the curve Bézier curves when he explore automotive design. And later, Forrest Research find Bézier curve is a Bernstein polynomial form of vector-valued (see [1-3]).

With the rapid development of CAGD people have done a lot of researches about Bernstein basis (see [4-6]). But Geometric modeling indispensable in the mechanics, when we study the shape of the object shell, we should be clear what kind of external force we impose will be the object shape what we want (see [7-9]). This requires us to fully understand the Bézier curve (surface), Bézier curve (surface) construction are based on the Bernstein basis. This paper focuses on the mechanical meaning of the Bernstein basis. We have not enough research on the mechanical meaning of the Bernstein basis, but on the mechanical meaning of the spline there are a number of studies, such as WANG Ren-hong, CHANG Jin-cai *A kind of bivariate spline and pure bending of thin plate*. Bernstein basis has some connection with the spline function, this study provides some reference(see [10-12]) .

The paper is organized as follows. The second part describes the generalization of Bernstein basis. Including the definition of a Bernstein basis, the area coordinates of the dual Bernstein basis in triangular domain. The three parts of the article describes the mechanical significance of the Bernstein basis, which came with the introduction of single-span statically indeterminate beam. The fourth section describes the mechanical meaning of the dual Bernstein basis functions in triangular domain(see [13-15]).

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2 The definition of the Bernstein basis

2.1 The definition of one variable Bernstein basis

2.1 Derivation of Navier-Stokes equations in vertical velocity formulation. Let $f(t)$ be a function on the interval [0, 1],

$$B_n(f;t)=\sum_{i=0}^{n} f(\frac{i}{n})B_i^n(t), 0\le t\le 1$$

It called the n-th Bernstein polynomial. Of which

$$B_i^n(t)=\binom{n}{i}t^i(1-t)^{n-i} \qquad i=0,1,\cdots,n; 0\le t\le 1$$

The combination coefficients $\binom{n}{i}$ are defined as

$$\binom{n}{i} = \begin{cases} \dfrac{n!}{i!(n-i)!} & 0 \le i \le n \\ 0 & \text{else} \end{cases}$$

$\{B_i^n(t)\}_{i=0}^n$ are linearly independent and constitute a set of n degree polynomials space basement P_n, which known as the Bernstein basis. Each function $B_i^n(t)(i = 0,1,\cdots,n)$ is called the Bernstein basis. The following figure shows Bernstein basis when n = 3

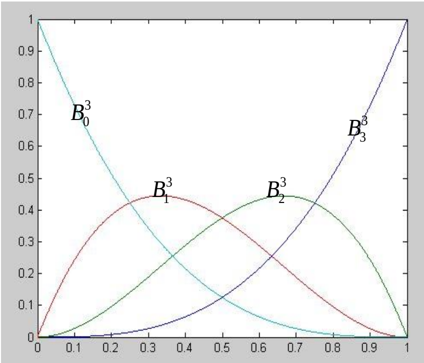

Fig2.1 Bernstein basis when n=3

2.2. **Bernstein basis on the triangular domain**

(1) Area coordinates in the plane

If all the number does not exceed n degrees in the binary polynomial space

$$\mathrm{P}_n = span\{x^i y^j, i + j \le n, i, j = 0,1,\cdots,n\}$$

The basis functions $x^i y^j (i + j \le n)$ of the P_n corresponding to the triangular structure,

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & x & & y & & \\ & x^2 & & xy & & y^2 & \\ x^3 & & x^2y & & xy^2 & & y^3 \\ \vdots & & \vdots & & \vdots & & \vdots \end{array}$$

So it can be called a triangular polynomial space, in order transformed a binary power plot into dual Bernstein basis. The following text give the concept of barycentric coordinates.

Given two points v_1, v_2 in space and determine a straight line. The relative position of any point may be sole determined by v_1 and v_2. Take this line for the axis, and set up x_1, x_2 and x as the coordinates of v_1, v_2 and v, respectively $\overrightarrow{v_1v_2}, \overrightarrow{v_1v}$ and $\overrightarrow{v\,v_2}$ to a distance of line segments is

$$\left|\overrightarrow{v_1v_2}\right| = x_2 - x_1 = \begin{vmatrix} 1 & x_1 \\ 1 & x_2 \end{vmatrix}$$

$$\left|\overrightarrow{v_1v}\right| = x - x_1 = \begin{vmatrix} 1 & x_1 \\ 1 & x \end{vmatrix}$$

$$\left|\overrightarrow{vv_2}\right| = x_2 - x = \begin{vmatrix} 1 & x \\ 1 & x_2 \end{vmatrix}$$

If

$$\tau_1 = \frac{\left|\overrightarrow{vv_2}\right|}{\left|\overrightarrow{v_1v_2}\right|} = \frac{x_2 - x}{x_2 - x_1}$$

$$\tau_2 = \frac{\left|\overrightarrow{v_1v}\right|}{\left|\overrightarrow{v_1v_2}\right|} = \frac{x - x_1}{x_2 - x_1}$$

(τ_1, τ_2) Known as the one-dimensional center of gravity coordinates, and $\tau_1 + \tau_2 = 1$.

So the Bernstein basis is expressed as

$$B_i^n(t) = \frac{n!}{i!(n-1)!} t^i (1-t)^{n-i} = \frac{n!}{\lambda_1!\lambda_2!} \tau_1^{\lambda_1} \tau_2^{\lambda_2}$$

Among them, $\tau_1 = 1 - t$, $\tau_2 = t$, $\lambda_1 = n - i$, $\lambda_1, \lambda_2 = 0,1,\cdots,n$. (τ_1, τ_2) is in the interval $[0,1]$, the point t is in the barycentric coordinates of 0 and 1 will be extended to two-dimensional space available to the center of gravity of the two-dimensional space coordinates, also known as area coordinates.

As figure 3.4 shown, given any triangle T, the three vertices v_1, v_2, v_3 in counterclockwise order, taking any point in T, Denoted their Cartesian coordinates are as follows:

$v_1 = (x_1, y_1)$, $v_2 = (x_2, y_2)$, $v_3 = (x_3, y_3)$, $x = (x, y)$

The area of triangles $\Delta v_1v_2v_3$, Δxv_2v_3, Δxv_3v_1 and Δxv_1v_2 were A, A_1, A_2, A_3.

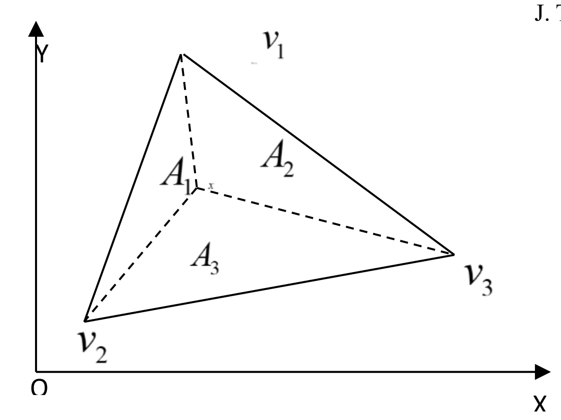

Fig2.2 Area coordinate diagram

$$A=\frac{1}{2}\begin{vmatrix}1 & x_1 & y_1\\ 1 & x_2 & y_2\\ 1 & x_3 & y_3\end{vmatrix}, A_1=\frac{1}{2}\begin{vmatrix}1 & x & y\\ 1 & x_2 & y_2\\ 1 & x_3 & y_3\end{vmatrix}$$
$$A_2=\frac{1}{2}\begin{vmatrix}1 & x & y\\ 1 & x_3 & y_3\\ 1 & x_1 & y_1\end{vmatrix}, A_3=\frac{1}{2}\begin{vmatrix}1 & x & y\\ 1 & x_1 & y_1\\ 1 & x_2 & y_2\end{vmatrix} \quad \text{(2-1)}$$

The area coordinates (τ_1, τ_2, τ_3) of point x defined as

$$\begin{aligned}\tau_1 &= \frac{A_1}{A} = \frac{1}{2A}(a_1 + b_1 x + c_1 y),\\ \tau_2 &= \frac{A_2}{A} = \frac{1}{2A}(a_2 + b_2 x + c_2 y),\\ \tau_3 &= \frac{A_3}{A} = \frac{1}{2A}(a_3 + b_3 x + c_3 y).\end{aligned} \quad \text{(2-2)}$$

Among them, a_i, b_i, c_i ($i = 1, 2, 3.$) are coefficients and have relationship with x_1, x_2, x_3 and y_1, y_2, y_3.

It is easy to verify $\tau_1 + \tau_2 + \tau_3 = 1$.

（2）Triangulations

Put the trilateral of the triangle T n equal portions, connecting each side of the bisector, received $(n+1)(n+2)/2$ points of intersection, called the domain points or nodes, each domain points denoted as $\xi_{\lambda_1,\lambda_2,\lambda_3}$. Correspond to the area coordinates $\left(\frac{\lambda_1}{n}, \frac{\lambda_2}{n}, \frac{\lambda_3}{n}\right)$, in which $\lambda_1, \lambda_2, \lambda_3$ is non-negative integer and $\lambda_1 + \lambda_2 + \lambda_3 = n$. These nodes form set of Larange interpolation nodes on the triangle, If given some values, we can uniquely identify a binary n polynomial. The figure 2.5 shows domain distribution of the triangles when n=3.

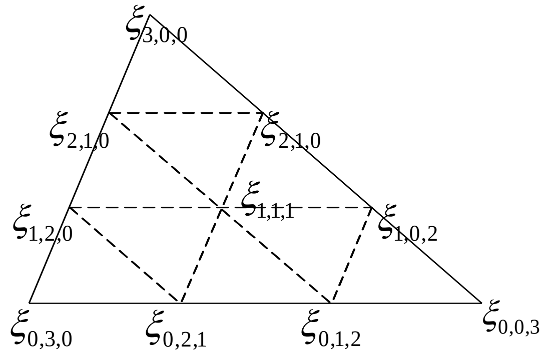

Fig2.3Domain distribution of the triangles when n=3

The introduction of mark is $\tau = (\tau_1, \tau_2, \tau_3)$, $\lambda = (\lambda_1, \lambda_2, \lambda_3)$, $|\lambda| = \lambda_1 + \lambda_2 + \lambda_3$, $\lambda! := \lambda_1!\lambda_2!\lambda_3!$, $\tau^\lambda := \tau_1^{\lambda_1}\tau_2^{\lambda_2}\tau_3^{\lambda_3}$. The promotion of one-dimensional center of gravity coordinates of the Bernstein basis. We can get $(n+1)(n+2)/2$ area coordinates of n Bernstein basis.

$$B_\lambda^n(t) = B_{\lambda_1,\lambda_2,\lambda_3}^n(\tau_1, \tau_2, \tau_3) = \frac{n!}{\lambda!}\tau^\lambda = \frac{n!}{\lambda_1!\lambda_2!\lambda_3!}\tau_1^{\lambda_1}\tau_2^{\lambda_2}\tau_3^{\lambda_3}$$

$\tau_1, \tau_2, \tau_3 \geq 0$, $|\lambda| = \lambda_1 + \lambda_2 + \lambda_3 = 1$, $|\lambda| = n$。

The following figure shows $B_{0,3,0}^3(\tau), B_{0,2,1}^3(\tau)$ and $B_{1,1,1}^3(\tau)$ images

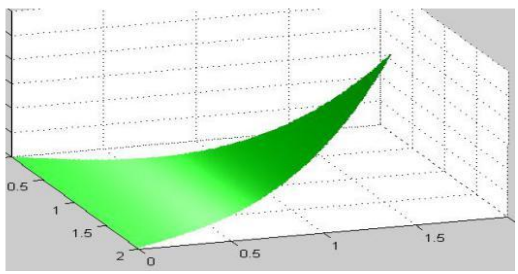

Fig2.4 Bernstein basis of $B_{0,3,0}^3(\tau)$

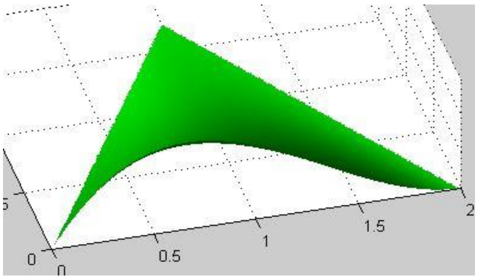

Fig2.5 Bernstein basis of $B^{3}_{0,2,1}(\tau)$

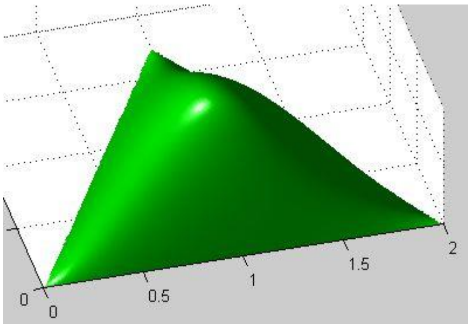

Fig2.6 Bernstein basis of $B^{3}_{1,1,1}(\tau)$

3 The mechanical meaning of one variable Bernstein basis

3.1 Internal force in the length of the surface of Beam

AS shown in Figure 3.1, any length surface is generally believed that there are three components of internal force, the axial force N, shear force Q and bending moment M (Figure 3.3).

The basic method of calculation of the beam cross-section internal forces is a cross-section method, Section method can be drawn to the calculation rule of Section Analysis:

(1) The cross-section beam axial force N is numerically equal to algebraic sum, that all the external forces of cross-section on either side of along the beam axis direction of the tangent to the projection. The axial force is usually tension is positive, the pressure is negative.

(2) The beam of a cross-section of the shear Q is numerically equal to algebraic sum, that all the external forces of cross-section on either side of along the beam axis normal direction to the projection. Shear to the isolation of the cross section make the body clockwise rotation trend is positive, otherwise it is negative.

(3) The cross-section of the beam bending moment M is numerically equal to algebraic sum of torque, that all the external forces during in any of the side of the cross-section centroid. The moment make the down side of the fiber of beam tension is positive, else is negative.

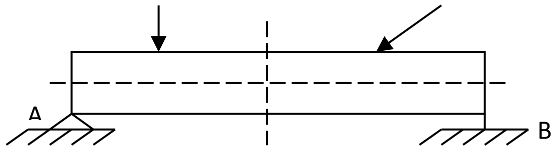

Fig3.1 Sign-span statically indeterminate bean

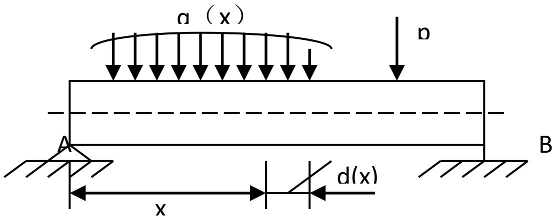

Fig3.2 Loads of simple support bean

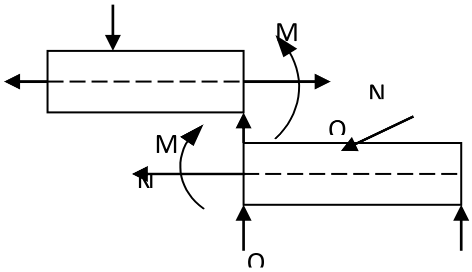

Fig3.3 Internal force component of the beam

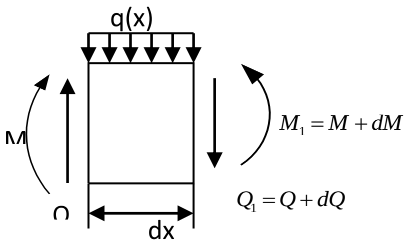

Fig3.4 Differential sectional diagram

3.2 Differential relation between the load and internal force

Shown in Figure 3.1 is simply supported beam, take the x-axis coincident with the beam axis and define the right is positive. Take the load perpendicular to the rod axis and define the down is positive. Remove isolated body from the beam for segmentation dx, Internal forces and load set on the differential as Figure 3.4 shown. Part of the load q (x) (load set degree in dx is can be constant), and set to go around some of its force Q, Q_1 and torque M, M_1, under a state of equilibrium. So we can see

$$Q_1 = Q + dQ \text{ and } M_1 = M + dM$$

As shown in Figure 3.4 of the segment, we can come from balance equation $\sum y = 0$ out

$$Q - (Q + dQ) - q(x)dx = 0$$

Then

$$\frac{dQ}{dx} = -q(x) \qquad (3\text{-}1)$$

The right side of the micro-segment as the cross-section centroid moment center, the torque balance equation is

$$\sum M = 0, \qquad M - (M + dM) + Qdx - q(x)dx\frac{dx}{2} = 0$$

Then spent high order differential

$$\frac{dM}{dx} = Q \qquad (3\text{-}2)$$

Combine conditions（3-1）and（3-2）

$$\frac{d^2M}{dx^2} = -q(x) \qquad (3\text{-}3)$$

Satisfy conditions（3-1）and（3-2）is differential relations of M, Q, q (x). The geometric meaning of these formulas are: in a point the shear tangent equal to the degree of load at the point, but the sign is opposite; the tangent of the bending moment diagram at a point equal to the point of the shear, the bending moment diagram of a point on the second derivative equal to the degrees of load at the point, but the sign is opposite.

3.3. The relationship between moment and the Bernstein basis

(1) The beam section of the load q (x) = 0, Q image is a horizontal straight line, as Bernstein basis B_0^0, the moment image shows is an oblique straight line as the Bernstein basis of B_0^1 or B_1^1.

Figure 3.5-a shown a period of no-load beam AB. For solving differential equations of the shear equation (3-1) we can get $Q(x) = C$ (C is a real number). AS the positive and negative in mechanics represent the direction. The size of the shear is exactly shown of the Bernstein basis B_0^0, as shown in Figure 3.5-b, when $C = \pm 1$.

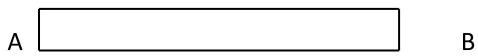

Fig3.5-a No-load beam

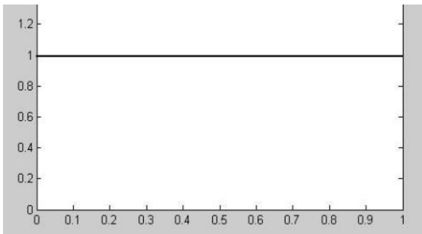

Fig3.5-b The Bernstein basis of B_0^0

If we integrate shear through condition (3-2), we can get the symbol Moment of expression $M(x) = \pm x + b$ (b is a real number). When is positive and b = 0 the Bernstein basis is B_1^1, when the sign of x is negative and b = 1the Bernstein basis is B_0^1, as shown in Figure 3.5-c

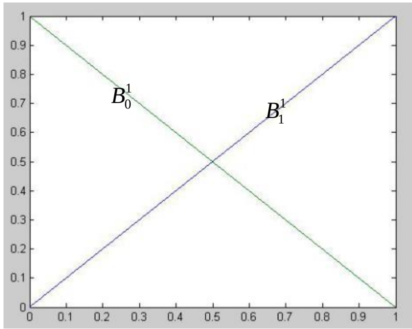

Fig3.5-c Bernstein basis when n=1

(2)The section of the beam with a uniformly distributed load (q (x) is constant), Q-images is an oblique straight line, and bending moment image is a parabola, as the Bernstein basis of B_0^2, B_1^2 or B_2^2.

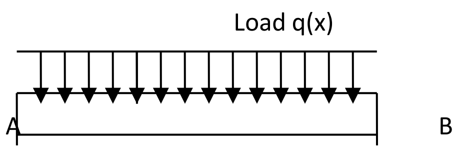

Fig3.6-a Uniform load beam

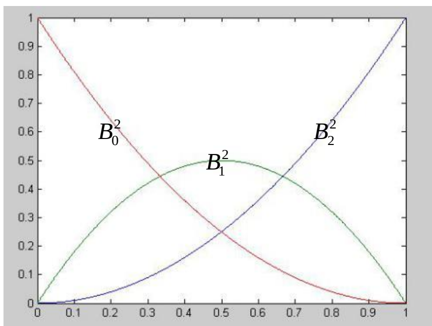

Fig3.6-b Bernstein basis when n=2

When $q(x)=-2$, through the condition (3-1) we can integrate $Q(x)=2x+C$ (C is a real number). When C = 0, again integrate $Q(x)$ and available $M(x)=x^2+D$ (D is a real number), when D = 0, we can get $M(x)=x^2$ as the Bernstein basis of B_2^2. Similarly, when $q(x)=-2$ and to control the values of C and D, we can get the Bernstein basis of B_0^2; when $q(x)=4$ and to control the values of C and D, we can get the Bernstein basis of B_1^2.

(3) If the beam section have linear distribution of the load q(x)=kx+b, the Q-image is a parabola, and bending moment image is a cubic parabola, as the Bernstein basis of B_0^3, B_1^3, B_2^3 or B_3^3 (see [16]).

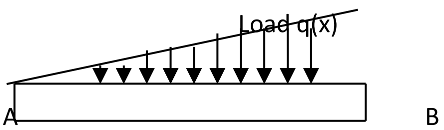

Fig3.7-a Linear load beam

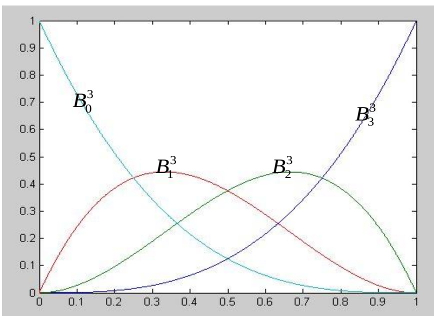

Fig3.7-b Bernstein basis when n=3

When $q(x)=-6x$, through the condition (3-1) we can integrate $Q(x)=3x^2+C$ (C is a real number). When C = 0, again integrate $Q(x)$ and available $M(x)=x^3+D$ (D is a real number), when D = 0, we can get $M(x)=x^3$ as the Bernstein basis of B_3^3. Similarly, when $q(x)=6-18x$ and to control the values of C and D, we can get the Bernstein basis of B_2^3; when $q(x)=18x-12$ and to control the values of C and D, we can get the Bernstein basis of B_1^3; when $q(x)=6-6x$ and to control the values of C and D ,we can get the Bernstein basis of B_0^3.

4 The mechanical meaning of dimensional Bernstein basis

We use the area coordinates $u,v,w(u+v+w=1)$ study dimensional Bernstein basis.

$$
\begin{array}{ccccccc}
 & & & u^3 & & & \\
 & & 3u^2v & & 3u^2w & & \\
 & 3uv^2 & & 6uvw & & 3uw^2 & \\
v^3 & & 3v^2w & & 3vw^2 & & w^3
\end{array}
$$

Label

$$
\begin{array}{ccccccc}
 & & & 300 & & & \\
 & & 210 & & 201 & & \\
 & 120 & & 111 & & 102 & \\
030 & & 021 & & 012 & & 003
\end{array}
$$

As triangulation of the triangle ABC in Figure 4.1 shown, we set up the point O during triangle, the area enclosed by the AB edge and O is u, the area enclosed by the BC side and O is v and the edge of AC and point O surrounded the area is w. Figure 4.2, set the area of the triangle ABC is S, so the coordinates

$$A=(x_1,y_1), B=(x_2,y_2), C=(x_3,y_3), O=(x,y)$$

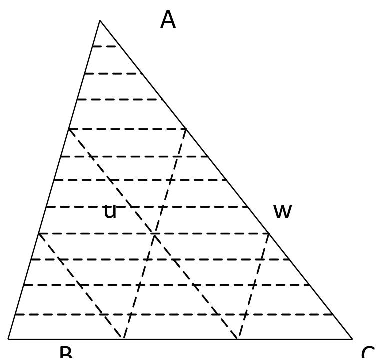

Fig4.1 Triangulation diagram

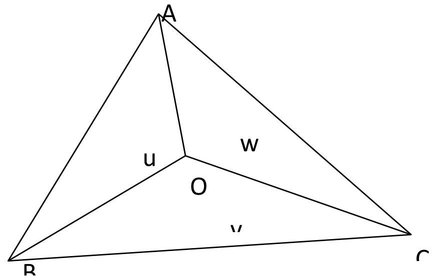

Fig4.2 Area distribution diagram

(1) $B^3_{0,3,0}(\tau)$, $B^3_{3,0,0}(\tau)$ and $B^3_{0,0,3}(\tau)$ are a class of functions, we take $B^3_{0,3,0}(\tau)$ as example and trilateral triangle T for n times. When u take the timing as take points in the AB side of the parallel lines, the basis function degradation of t^3 (t is the relative area of the triangle OBC), by (3-3), we can get $q(t)=-6t$ by $M(t)=t^3$ second guide. For w taken from time to time, the basis function is still degradation of $M(t)=t^3$ and we can get $q(t)=-6t$ by $M(t)=t^3$ second guide. For the v take time, Bernstein basis degenerates to $M(t)=v^3$ (v constant) and get $q(t)=0$ for a given derivation. Integrated above trilateral combination (2-1) shows the load distribution on the triangle is $q(t)=-12t=-12v$, by (2-2) we can push

$$q(x,y)=-\frac{6}{S}((x_3y_1-x_1y_3)+(y_3-y_1)x+(x_1-x_3)y)$$

That is, when we applied $q(x,y)$ in the Triangle area the size of the load, the bending moment picture shows $B^3_{0,3,0}(\tau)$ in Fig2.4.

(2) $B^3_{2,1,0}(\tau)$, $B^3_{2,0,1}(\tau)$, $B^3_{1,2,0}(\tau)$, $B^3_{1,0,2}(\tau)$, $B^3_{0,2,1}(\tau)$ and $B^3_{0,1,2}(\tau)$ are classes of functions, we take $B^3_{0,2,1}(\tau)$ as example and trilateral triangle T for n times deciles. When u takes given constant, the Bernstein basis degradation of $M(t)=3t^2(1-u-t)$ (set of triangles OBC relative area t), and its second guide was $q(t)=18t+6u-6$. When v take timed, the Bernstein basis degenerates to $M(t)=3tv^2$ (v is a constant, t is the relative area of the triangle OBC) and its second derivative are $q(t)=0$. For the w take timed, the Bernstein basis degradation to $M(t)=3t^2w$ and its second derivative are $q(t)=-6w$.(see [17-20]) Therefore

$$q(x,y)=18v+6u-6w-6$$

By (2-2) we can get $q(x,y)=18\tau_1+6\tau_2-6\tau_3-6$, When to impose a load of this size, the bending moment of picture shown as $B^3_{0,2,1}(\tau)$ in Fig 2.5.

(3) For $B^3_{1,1,1}(\tau)$ we trilateral triangle T as equal portions of n times. When u take the time Bernstein basis degradation as $M(t)=6ut(1-u-t)$ (u is a fixed value, t is the relative area of the triangle OBC), its second derivative was $q(t)=12u$. Because of rotation symmetry of $B^3_{1,1,1}(\tau)$, if v take timed we get $q(t)=12v$, and w take timed we get $q(t)=12w$. As condition $u+v+w=1$ When the triangle area imposed by the load of 12 units, the bending moment picture shows $B^3_{1,1,1}(\tau)$ in Fig 2.6.

6 Conclusions

In this paper we study the Bernstein basis, and combined knowledge of a statically indeterminate beam. Ultimately we are able to figure out the Bernstein basis of physical background. We can recognize the Bernstein basis functions from the mechanical level. In the triangular domain we use the triangulation method simplified dimension. By some mechanical knowledge we get bending moment diagram as exactly Bernstein basis in triangular domain, when we take specific load. Lay to that knowledge, we can take the foundation for geometric modeling.

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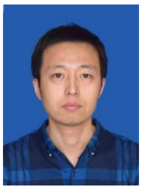

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