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Modified CRM-based Model Reference Adaptive Control with Reduced Peaking Phenomenon

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ABSTRACT

In order to achieve better transient performance of model reference adaptive control (MRAC), a recently proposed idea, named closed-loop reference model (CRM), has been recently proposed. However, there exists a potential peaking phenomenon in the CRM-based MRAC systems due to the introduced feedback gain in the CRM, which may deteriorate the original tracking control response. In this paper, we revisit the CRM-based MRAC design, and provide a new prospective to analyze the bound of peaking value by using L_2 norm and Cauchy-Schwartz inequality. Following the analysis we further provide a potential way to alleviate the peaking phenomenon by improving the parameter estimation error convergence. This has been achieved by introducing a modified adaptive law with a new leakage term, such that exponential convergence of estimation error can be proved via rigorous theoretical analysis. This new framework allows to use large feedback gains in the CRM to improve transient control performance, while the peaking phenomenon can be reduced. A wing rock aircraft model is used as the numerical example to validate the effectiveness of the proposed method and improved performance over the traditional CRM-based MRAC. Simulation results show that the modified CRM- based MRAC system can achieve better control responses.

1. Introduction

In the traditional model reference adaptive control (MRAC), there exists such a conflict that the high adaptive gain in the adaptive laws can help to address the effect of uncertainties in the transient response but may also excite high frequency unmolded dynamics (Gibson, Annaswamy, & Lavretsky, 2013b; Na, Herrmann, & Zhang, 2017; Yucelen & Calise, 2010). Hence, it is well-recognized that high gain learning is not preferable in the control implementation; this could deteriorate the tracking response. In order to solve this problem, several methods have been proposed in the literature (Datta & Ioannou, 2002; Krstić, Kokotović, & Kanellakopoulos, 1993; Zang & Bitmead, 1991). In particular, in reference (Krstić et al., 1993), L_2 norm is used to analyze the tracking error of adaptive systems with disturbances and un-modeled dynamics.

Among different solutions, a new MRAC framework that was subsequently named as the closed-loop reference model (CRM) has recently been reported. In this method, the tracking error is introduced as a feedback into the reference model. Hence, the overall tracking error convergence rate can be improved. CRM-based adaptive control was first proposed in (Lee & Huh, 1997). It is shown that the transient performance in the MRAC system (Narendra & Anuradha, 1989) can be improved by using the modified CRM. In particular, it shows that the CRM-based MRAC system can achieve fast tracking error convergence in the initial period (Lavretsky, 2009; Stepanyan & Krishnakumar, 2011). Due to its outstanding performance in addressing the conflict between the uncertainties of the transient response and the high frequency unmolded dynamics, CRM-based MRAC has recently attracted significant attention, and been used in some practice (Lavretsky, 2006; Stepanyan & Krishnakumar, 2010). However, since the CRM-based adaptive control system introduces a feedback of the tracking error in the reference model, the reference model dynamics are slightly changed compared to the original open-loop reference model (ORM) in (Narendra & Anuradha, 1989). In this case, this modification may cause a potential peaking phenomenon in the CRM-based MRAC designs (Gibson et al., 2013b), which could deteriorate the transient performance of the control system. Consequently, the perfect tracking of the original reference model (this is the control design objective) may be lost.

Although some preliminary solutions have been suggested to address this induced peaking phenomenon (Lavretsky, 2006;

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Stepanyan & Krishnakumar, 2010), the authors assume that the initial state error is zero and the closed-loop system state is independent of the feedback gains in the reference model. Such assumptions may not be true in practice because the initial system state may not be able be measured directly. Hence, it remains an open problem to remedy the peaking phenomenon in the CRM-based MRAC.

In this paper, we first revisit the CRM-based MRAC designs, where the peaking phenomenon is analyzed and the drawbacks of the Lyapunov based analysis is addressed. Then a new analysis of the peaking phenomenon is provided in this paper by using L_2 norm and Cauchy-Schwartz inequality, where the influences of the tracking error and the parameter estimation error on the peaking phenomenon are clarified. It is shown that the peaking value can be reduced if the parameter estimation error can be alleviated. Then inspired by (Na, Herrmann, Ren, & Mahyuddin, 2011), we propose a modified adaptive law where a new leakage term containing the real-time parameter estimation error is incorporated into the traditional adaptive law, such that the estimation error can converge to zero exponentially. Consequently, this modified CRM-based MRAC with new adaptive law can allow using large feedback gain in the CRM to further improve the transient performance while the peaking phenomenon of the control system can be reduced. By using the wing-rock system as a benchmark example, comparative simulation results are provided to show the effectiveness and the improved performance of the proposed modified CRM-based MRAC over the traditional CRM-based MRAC.

The structure of this paper is as follows. Section 2 illustrates the CRM-based MRAC design and the peaking phenomenon. Section 3 contributes to analyze the peaking phenomenon vie a new perspective, and then introduces the modification of the CRM-based MRAC design. Section 4 provides simulation results to show the efficacy of the modified CRM-based adaptive control.

2. Problem preliminaries

2.1 Traditional CRM-based adaptive control

We first recall the recently proposed CRM-based control system. Consider the following uncertain dynamic system.

$$\dot{x} = Ax + B\big(f(x) + u(t)\big) \tag{1}$$

where $x = [x_1, x_2 \cdots x_n]^T \in R^n$ is the system state, $u(t)$ is the control input, $A \in R^{n\times n}$, $B \in R^{n\times m}$ and (A, B) are the system matrices, and the pair (A, B) is controllable, the matrices $(B^T B)$ is invertible. $f(x)$ is unknown system dynamics.

Assumption 1(Na et al., 2011): The unknown dynamics of system (1) can be further rewritten as:

$$f(x) = \theta^T \phi(x) \tag{2}$$

where $\theta \in R^{d\times m}$ is an unknown constant weight matrix. $\phi(x) = [\phi_1(x) \cdots \phi_d(x)]^T \in R^d$ is a known function vector.

The original model reference to be tracked is given by:

$$\dot{x}_r(t) = A_r x_r(t) + B_r r \tag{3}$$

where $x_r \in R^n$ is the state of the reference model, $r \in R^m$ is a given bounded control command, $A_r \in R^{n\times n}$ is a Hurwitz system matrix, i.e. all the real parts of its eigenvalues are negative. $B_r \in R^{n\times m}$ is the input matrix. Hence, there exist positive definite matrices $P, Q \in R^{n\times n}$, which make the Lyapunov equation $A_r^T P + P A_r = -Q$ hold.

For CRM-based MRAC system, the tracking error will be added into the reference model. Thus the CRM is as follow:

$$\dot{x}_{cr}(t) = A_r x_{cr}(t) + B_r r + \alpha e_{cr} \tag{4}$$

where $x_{cr} \in R^n$ is the state of the closed-loop reference model, $e_{cr} = x - x_{cr}$ is the tracking error between the closed-loop reference model and the uncertain dynamic system (1). α is a feedback gain selected by the designer.

In the CRM-based MRAC system with the CRM (4), the control input u is designed as:

$$u = K_x x + K_r r + u_a \tag{5}$$

where K_x is feedback gain, K_r is feedforward gain, which fulfill the follow equations:

$$\begin{aligned} A_r &= A + BK_x \\ B_r &= BK_r \end{aligned} \tag{6}$$

u_a is the adaptive feedback given as:

$$u_a = -\hat{\theta}^T \phi(x) \tag{7}$$

where $\hat{\theta}$ is the estimation of the unknown weight matrix θ, which can be obtained through the adaptive law (8)

$$\dot{\hat{\theta}} = \gamma \phi(x) e_{cr}{}^T PB \tag{8}$$

where $\gamma > 0$ is the adaptive gain and $e_{cr} = x - x_{cr}$ is the tracking error between the uncertain dynamic system and the closed-loop reference model given in (4).

Substituting (2), (5)-(8) into system (1), then we can get the closed-loop controlled system as:

$$\dot{x} = A_r x + B_r r + B\tilde{\theta}^T \phi(x) \tag{9}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the estimation error of the unknown weight matrix θ.

The block diagram of the CRM adaptive control architecture is exhibited in Fig. 1.

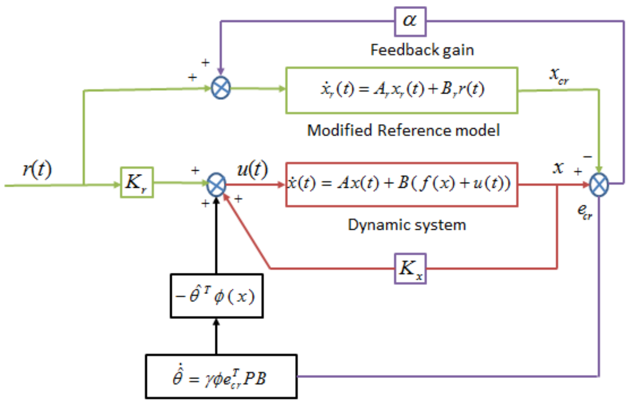


Fig.1. Block diagram of the traditional CRM-based MRAC.

Subtracting (4) from (9), the tracking error of the above CRM-based MRAC system with CRM (4) can be obtained as:

$$\dot{e}_{cr} = (A_r - \alpha I)e_{cr} + B\tilde{\theta}^T \phi(x) \tag{10}$$

where I is an identity matrix. We set $A_r^{'} = A_r - \alpha I$, and then can verify that $A_r^{'}$ is also a Hurwitz system matrix.

For comparison, we also derive from (3) and (9) the tracking error of the original MRAC system with the original reference model (3) as:

$$\dot{e} = A_r e + B\tilde{\theta}^T \phi(x) \tag{11}$$

Comparing (10) with (11), we know the tacking error feedback term αe_{cr} is introduced into the reference model (3), thus the feedback gain α can provide one more parameter to reduce the tracking error without increasing the adaptive gain. Instead, it can shift the eigenvalues of the transform matrix of the closed-loop error equation (10), and thus improve the convergence speed of tracking error. Thus it can avoid the conflict existing in the traditional MRAC system. However, there may exist a potential peaking phenomenon in the CRM-based control system, which has been initially discussed inv(Gibson, Annaswamy, & Lavretsky, 2013a). For clarity, we will discuss this issue in detail in next subsection.

2.2 Peaking phenomenon

Because of the feedback of the tracking error in the CRM (4), the error convergence of CRM-based MRAC in (10) is modified, which, on the other hand, may result in a peaking phenomenon. In this subsection we will analysis the peaking phenomenon and give some potential ways to eliminate the peaking phenomenon.

From equations (3) and (4), we can further obtain their solutions given in (12) and (13)

$$x_r = e^{A_r t} x_r(0) + \int_0^t e^{A_r(t-\tau)} r(\tau) d\tau \tag{12}$$

$$x_{cr} = e^{A_r t} x_{cr}(0) + \int_0^t e^{A_r(t-\tau)} r(\tau) d\tau + \alpha \int_0^t e^{A_r(t-\tau)} e_{cr}(\tau) d\tau \tag{13}$$

In general, the difference between (12) and (13), i.e. $x_{cr} - x_r$, is used to measure the peaking phenomenon. For simplifying the analysis, we assume that $x_r(0)=x_{cr}(0)$. Then, it follows from (12) and (13) that:

$$x_{cr} - x_r = \alpha \int_0^t e^{A_r(t-\tau)} e_{cr}(\tau) d\tau \tag{14}$$

Clearly, this extra error term is due to the difference between the ORM (3) for the standard MRAC and the modified CRM (4) of the CRM-based adaptive control system. Based on (14), it can be verified that the desirable tracking for making the output of CRM x_{cr} the same as the original reference model x_r may not be always achieved.

In particular, this difference between the ORM (3) and the CRM (4) will also lead to a potential peaking phenomenon in the CRM-based MRAC system. In fact, based on the Lyapunov stability theorem, the authors of (Gibson et al., 2013b) derived the error convergence bound of (14) as:

$$\|x_{cr} - x_r\| \le \sqrt{\frac{e_{cr}(0)}{2\|A_r\|}} |\alpha|^{\frac{1}{2}} + \sqrt{\frac{\|\tilde{\theta}(0)\|}{2\|A_r\|}} \left(\frac{|\alpha|}{\gamma}\right)^{\frac{1}{2}} \tag{15}$$

Because the closed-loop reference model introduces the tracking error that contains the unknown system dynamics as a feedback, the output of the CRM will also be affected by the unknown system dynamics, which is named peaking phenomenon. Hence, for large gain α in the CRM, a large peaking would appear in $x_{cr} - x_r$ because of the induced exponential term $|\alpha|^{1/2}$. For the detailed analysis on this peaking phenomenon, we refer to (Gibson et al., 2013b).

From (15) we can see that the peaking phenomenon is influenced by the initial tracking error $e_{cr}(0)$, the estimation error of the unknown weight matrix $\tilde{\theta}$, the adaptive gain γ and the feedback gain α in the CRM (4). Hence, we can conclude the following facts:

- Decreasing the initial tracking error $e_{cr}(0)$ can alleviate the peaking phenomenon in the CRM-based control system. However, since in practice it is not always possible to measure the initial system state $x(0)$, we may not be able to set $x_{cr}(0)$ to make $e_{cr}(0)$ zero or small in the control system.
- Decreasing the estimation error $\tilde{\theta}(0)$ is also helpful to eliminate the peaking value. Again, since we cannot know the exact information of the unknown parameter θ, it is not feasible to reduce $\tilde{\theta}(0)$.
- Reducing the feedback gain α can definitely eliminate the peaking phenomenon by reducing the power of the exponential term $|\alpha|^{1/2}$. However, too small α will loose the merits and advantages of CRM over ORM. In fact, the use of α could help to achieve faster convergence of control error e_{cr} and thus improve the transient control response in the MRAC system. Hence, it is preferable to use fairly large feedback gain α.
- For certain α, increasing the adaptive gain γ can reduce the effect of the peaking phenomenon induced by the second term of (15). However, it has been widely recognized that too large gain γ may excite high frequency dynamics and even trigger instability of the adaptive control systems.

From the above observations, we can see that it is generally difficult to find a feasible method to eliminate the peaking phenomenon in the CRM based MRAC, though a practical trade-off strategy has been reported in (Gibson et al., 2013b) by using the Projection algorithm in the adaptive law. That is because the analysis based on Lyapunov theory and the derived error bound in (15) only reflect the influence of the initial tracking error $e_{cr}(0)$ and the initial estimation error $\tilde{\theta}(0)$, while the time-varying and dynamic behavior of $\tilde{\theta}(t)$ and its influence on the system response cannot be addressed. Hence, in the following section, we will try to use Cauchy-Schwartz inequality to revisit the tracking error bound and then provide a new feasible solution to eliminate the peaking phenomenon while keeping the merits of CRM.

3. Modified CRM control system

3.1 Peaking phenomenon analysis from new perspective

In this section, we will provide a new perspective to analyze the peaking phenomenon based on L_2 norm. From this analysis, we can further address the influences of both the initial tracking error $e_{cr}(0)$ and the estimation error $\tilde{\theta}(t)$ on the peaking phenomenon as time increases. Then, we will further propose a modification on the adaptive law to alleviate the peaking phenomenon.

Lemma 1 (Peutemanyz & Aeyelsy, 1988): If A is a Hurwitz matrix, ξ is the maximum real part of the eigenvalues of A, $\xi = \max(real(\lambda(A)))$. Then, we know $\xi < 0$. For any constant $\varepsilon > 0$, and variable $t \ge 0$, we can get:

$$\left\|e^{A(\tau)t}\right\| \le Le^{(\xi+\varepsilon\|A(\tau)\|)t} \tag{16}$$

$$L=\frac{3}{2}(1+\frac{2}{\varepsilon})^{n-1} \tag{17}$$

where $\|\bullet\|$ is the norm of the matrix.

Lemma 2: We can further reformulate Lemma 1 as the following form:

$$\left\|e^{A(\tau)t}\right\| \le Me^{\frac{\xi t}{2}} \tag{18}$$

$$M=\frac{3}{2}(1-4\|A(\tau)\|/\xi)^{n-1} \tag{19}$$

Proof

For simplicity, we can set $\varepsilon=-\xi/(2\|A(\tau)\|$ and substitute it to (16), then it can be verified that (18) with (19) is true.

We first prove the following theorem that states the bound of the tracking error e_{cr}.

Theorem 1: For CRM-based MRAC system (1), (5)-(8) with the CRM (4), the boundary of the tracking error e_{cr} can be obtained as:

$$\|e_{cr}(t)\|_2 \le e_{cr}(0)M'e^{\frac{\xi' t}{2}} + M'\|B\|_2\left\|\tilde{\theta}^T(t)\right\|_2\|\phi(x)\|_2/\sqrt{-\xi'} \tag{20}$$

We use matrix A_r' to represent for $(A_r-\alpha I)$. Thus the variables are defined as $M'=(3/2)(1-4\|A'(\tau)\|/\xi')^{n-1}$ and $\xi'=\max(real(\lambda(A_r')))$.

Proof

The tracking error of CRM based MRAC system is given as in the above (10). Integrating both sides of (10), we can get that:

$$e_{cr}(t)=e^{A_r' t}e_{cr}(0)+\int_0^t e^{A_r'(t-\tau)}B\tilde{\theta}^T(\tau)\phi(x)d\tau \tag{21}$$

According Lemma 2, we can verify that the following inequality holds from (21)

$$\|e_{cr}(t)\|_2 \le M' e^{\frac{\xi' t}{2}} e_{cr}(0) + M'\|B\|_2\left\|\int_0^t e^{\frac{\xi'(t-\tau)}{2}}\tilde{\theta}^T\phi d\tau\right\|_2 \tag{22}$$

By applying the Cauchy-Schwartz inequality, we can further obtain that:

$$\left[\int_0^t e^{\frac{\xi'(t-\tau)}{2}}\tilde{\theta}^T\phi d\tau\right]^2 \le \int_0^t\left[e^{\frac{\xi'(t-\tau)}{2}}\right]^2 d\tau\int_0^t\left[\tilde{\theta}^T\phi\right]^2 d\tau \tag{23}$$

Noticing that the following facts are true

$$0<\int\left[e^{\frac{\xi(t-\tau)}{2}}\right]^2 d\tau \le -\frac{1}{\xi} \tag{24}$$

$$\int_0^t\left[\tilde{\theta}^T\phi\right]^2 d\tau \le \left\|\tilde{\theta}^T\right\|_2^2\|\phi\|_2^2 \tag{25}$$

Then we have:

$$\left\|\int_0^t e^{\frac{\xi(t-\tau)}{2}}\tilde{\theta}^T\phi d\tau\right\|_2 \le \sqrt{-\frac{1}{\xi}\left\|\tilde{\theta}^T\right\|_2^2\|\phi\|_2^2} \tag{26}$$

Substituting (26) into (22), the inequality (20) can be finally validated. This completes the proof. ◇

Then we will use the above bound of e_{cr} to further obtain the boundary of $(x_{cr}-x_r)$ as follows:

Theorem 2: The L2 norm bound of the error $(x_{cr}-x_r)$ between the ORM and CRM is given as:

$$\|x_{cr}-x_r\|_2 \le \frac{M|\alpha|}{\sqrt{-\xi}}e_{cr}(0)M'e^{\frac{\xi' t}{2}}+\frac{MM'|\alpha|}{\sqrt{-\xi}\sqrt{-\xi'}}\|B\|_2\left\|\tilde{\theta}^T(t)\right\|_2\|\phi\|_2 \tag{27}$$

Proof

Applying Lemma 2 on (14), we can obtain that

$$\|x_{cr}-x_r\|_2 \le |\alpha| M\left\|\int_0^t e^{\frac{\xi}{2}(t-\tau)}e_{cr}(\tau)d\tau\right\|_2 \tag{28}$$

According to Cauchy-Schwartz inequality, we can further obtain that:

$$\begin{aligned}\left\|\int_0^t e^{\frac{\xi}{2}(t-\tau)}e_{cr}(\tau)d\tau\right\|_2 &\le \sqrt{\int_0^t e^{\xi(t-\tau)}d\tau\int_0^t e_{cr}{}^2(\tau)d\tau} \\ &=\sqrt{\int_0^t e^{\xi(t-\tau)}d\tau}\,\|e_{cr}(\tau)\|_2\end{aligned} \tag{29}$$

Then by substituting (24) and (25) into (29), we can obtain that:

$$\left\|\int_0^t e^{\frac{\xi}{2}(t-\tau)}e_{cr}(\tau)d\tau\right\|_2 \le \frac{1}{\sqrt{-\xi}}\left[e_{cr}(0)M'e^{\frac{\xi' t}{2}} + M'\|B\|_2\left\|\tilde{\theta}^T(t)\right\|_2\|\phi(x)\|_2/\sqrt{-\xi'}\right] \tag{30}$$

Substituting (30) to (28), the inequality (27) can be validated. This completes the proof. ◇

Remark 1: From (27), we can see that the influence of the initial tracking error $e_{cr}(0)$ will exponentially vanish as time goes to infinity because of the exponential term $e^{\xi' t/2}$, which means the impact of e_{cr} is not the main factor to account for the peaking phenomenon. Instead, the impact of $\tilde{\theta}$ is immune along the increase of time; that is the unknown system dynamics of $\tilde{\theta}$ play a more important role in creating the peaking phenomenon. Thus, it is essential for reducing the peaking phenomenon in the CRM-based MRAC system. This new perspective provides a potential way to eliminate the peaking phenomenon by decreasing the bound of $\tilde{\theta}$.

3.2 Modified adaptive law

From the alternative analysis of peaking phenomenon, we know that one potential way to alleviate the peaking phenomenon is to reduce the estimation error $\tilde{\theta}$. In this section, inspired by previous result reported in (Na et al., 2011; Na et al., 2017), we will provide a new adaptive law, which can effectively make the estimated parameter $\hat{\theta}$ converges to its real value θ exponentially, which can allow the estimation error $\tilde{\theta}$ converging to zero fast.

The modified control structure is given in the following Fig.2.

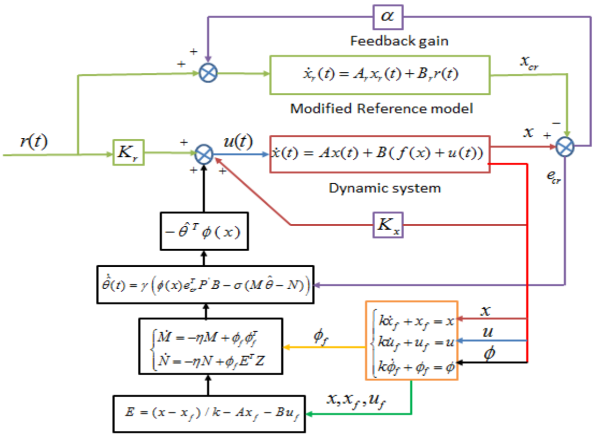


Fig.2. Block diagram of the modified CRM-based MRAC.

For this purpose, we define the filtered variables as:

$$\begin{cases} k\dot{x}_f + x_f = x \\ k\dot{u}_f + u_f = u \\ k\dot{\phi}_f + \phi_f = \phi \end{cases} \tag{31}$$

where $k > 0$ is a constant filter parameter. If k is small enough, we can get that $x_f \to x,\ u_f \to u,\ \phi_f \to \phi$. It can also be seen from (31) that x_f, u_f, ϕ_f can be easily implemented by a low-pass filter operation $1/(ks+1)$ on the variables x, u, ϕ.

We further define an auxiliary term $E \in R^n$ as:

$$E = (x - x_f)/k - Ax_f - Bu_f \tag{32}$$

which can be online calculated via the variables given in (31).

Next we define auxiliary matrix $M \in R^{d\times d}$ and vector $N \in R^{d\times m}$ as:

$$\begin{cases} \dot{M} = -\eta M + \phi_f \phi_f^T, & M(0) = 0 \\ \dot{N} = -\eta N + \phi_f E^T Z, & N(0) = 0 \end{cases} \tag{33}$$

where $\eta > 0$ is a design parameter, and $Z = B(B^T B)^{-1} \in R^{n\times m}$ exists for any $\det(B^T B) \neq 0$.

Now, we can verify the following fact:

Lemma 3: For the variables defined in (33) with variables given in (31), we have $N = M\theta$.

Proof

We apply the filter operation $1/(ks+1)$ on both sides of system (1), then we can obtain that:

$$\frac{s}{ks+1}[x] = \frac{1}{ks+1}\left([Ax] + [Bu] + [B\theta^T\phi]\right) \tag{34}$$

where $[\bullet]$ denotes the variable after Laplace transform. According to (31), we can get the following equalities through Laplace transform.

$$\begin{cases} [x_f] = \dfrac{1}{ks+1}[x] \\ [u_f] = \dfrac{1}{ks+1}[x] \\ [\phi_f] = \dfrac{1}{ks+1}[\phi] \end{cases} \tag{35}$$

Substituting (35) to (34) with (31), we can further obtain that:

$$B\theta^T[\phi_f] = \frac{1}{k}([x] - [x_f]) - A[x_f] - B[u_f] \tag{36}$$

Applying the inverse Laplace Transform on (36) we can obtain that:

$$B\theta^T\phi_f = \frac{1}{k}(x - x_f) - Ax_f - Bu_f \tag{37}$$

Considering (37) with (33), we can get that:

$$\dot{N} = -\eta N + \phi_f \phi_f^T \theta B^T Z_2 \tag{38}$$

Then the integration of $\dot{M}$ and $\dot{N}$ can be calculated along (33) and (38) as:

$$\begin{cases} M = \int_0^t e^{-\eta(t-\tau)} \phi_f(\tau)\phi_f^T(\tau)d\tau \\ N = \int_0^t e^{-\eta(t-\tau)} \phi_f(\tau)\phi_f^T(\tau)d\tau\theta \end{cases} \tag{39}$$

From (39), we can see that the following relationship between M and N is true:

$$N = M\theta \tag{40}$$

This completes the proof. ◇

It is shown in Lemma 3 and (40) that auxiliary matrix M and vector N contain the information of the unknown parameter θ. Then by using auxiliary matrix M and vector N, a new adaptive law will be designed and applied to the CRM-based MRAC system to alleviate the peaking phenomenon. The new adaptive law is designed as:

$$\dot{\hat{\theta}}(t) = \gamma\left(\phi(x)e_{cr}^T P' B - \sigma(M\hat{\theta} - N)\right) \tag{41}$$

where P' Q are positive definite matrices that make the Lyapunov equation $(A_r')^T P' + P' A_r' = -Q$ hold with A_r' being a Hurwitz system matrix shown in (10).

We now validate the convergence of the CRM-based adaptive control system with the adaptive control law (41).

Theorem 3: Consider the controlled system (1) with the modified reference model (4), the adaptive law is given by (41). If the regressor vector $\phi_f(x)$ is persistent excited (PE), then all signals in the closed-loop system are bounded, and the tracking error $e_{cr}(t)$ and the parameter estimation error $\tilde{\theta}(t)$ converge to zero *exponentially*.

Proof

It can be verified that the tracking error of CRM-based MRAC with (4), (5) and the adaptive law (41) are the same as that given in (10). Hence, we choose the Lyapunov function as $V = e_{cr}{}^T P' e_{cr} + tr(\tilde{\theta}^T\gamma^{-1}\tilde{\theta})$. Moreover, we know $\tilde{\theta} = \theta - \hat{\theta}$, thus the follow equality can be validated

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}}(t) = -\gamma\left(\phi(x)e^T P' B - \sigma(M\hat{\theta} - N)\right) \tag{42}$$

According to the fact that $\phi_f(\tau)\phi_f^T(\tau)$ is nonnegative, thus we can deduce that $M \geq 0$ as strictly proved in (Na et al., 2011; Na et al., 2017). Moreover, from (40), we can obtain that:

$$M\hat{\theta} - N = -M\tilde{\theta} \tag{43}$$

Calculating the derivative of V along (10), (43) and (42), we can obtain that:

$$\begin{aligned} \dot{V}(e_{cr}, \tilde{\theta}) &= -e_{cr}^T((A_r')^T P' + P' A_r')e_{cr} + 2e_{cr}^T P' B\tilde{\theta}^T\phi(x) \\ &\quad - 2tr(\tilde{\theta}^T\phi e_{cr}^T P' B) - 2\sigma tr(\tilde{\theta}^T M\tilde{\theta}) \\ &= -e_{cr}^T Q e_{cr} - 2\sigma tr(\tilde{\theta}^T M\tilde{\theta}) \\ &\leq -\lambda V \end{aligned} \tag{44}$$

where $\lambda = \min\{\lambda_{\min}(Q)/\lambda_{\max}(P'),\ 2\sigma\xi'/\lambda_{\max}(\Gamma^{-1})\}$ determines the convergence speed.

By using Lyapunov stability theorem, we can conclude that the closed-loop control system is stable and converge exponentially. We have both the tracking error and estimation error are bounded, i.e., $e_{cr}, \tilde{\theta} \in L_\infty$. Moreover, we can verify from that $\tilde{\theta} \in L_2$, $e_{cr} \in L_2$ from (44). On the other hand, from (10) and (42) that $\dot{e}_{cr}, \dot{\theta} \in L_\infty$ are true. Hence, based on Barbalat's lemma, we can obtain $e_{cr} \to 0$ and $\tilde{\theta} \to 0$, as $t \to \infty$. The proof is completed. ◇

From (43), we can see that through introducing the new leakage term $\sigma(M\hat{\theta} - N)$, the modified adaptive law (41) will contain the real-time parameter estimation error $\tilde{\theta}$, which can guarantee the convergence of $\hat{\theta}$ to its real value $\hat{\theta}$ exponentially. This new adaptive law can not only guarantee the convergence of the estimation error $\tilde{\theta}$ apart from the control system, but also can alleviate the peaking phenomenon by achieving faster convergence speed of the tracking error e_{cr} and eliminating the influence of estimation error $\tilde{\theta}$. We will validate this claim later in the simulations.

Remark 2: The traditional adaptive law (8) can only guarantee the convergence of the tracking error, i.e. $e_{cr} \to 0$, as $t \to \infty$. It can not eliminate the impact of the unknown estimation error $\tilde{\theta}$ because only the boundedness of $\tilde{\theta}$ can be proved by using this adaptive law. However, it is verified and claimed in Theorem 3 that both $e_{cr} \to 0$ and $\tilde{\theta} \to 0$ for $t \to \infty$ can be guaranteed by applying the new adaptive law (41). From the analysis given in subsection 3.1, we know that the estimation error $\tilde{\theta}$ plays a dominant role in creating the peaking phenomenon. Hence, by using this new adaptive law, we can reduce the bound of $\tilde{\theta}$ very fast, thus the peaking phenomenon due to the use of feedback gain α in the CRM can be alleviated effectively.

4. Simulation results

By using the wing rock aircraft model (Singh, Wells, & Yirn, 1995), extensive simulations are provided in this section to validate the effectiveness of the modified scheme. The wing-rock dynamics are given by:

$$\begin{bmatrix} \dot{\delta} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u + f(x)] \tag{45}$$

where δ represents for the roll angle of the wing-rock dynamics, p stands for the roll rate of wing-rock dynamics and $f(x)$ denotes the unknown system dynamics which are given by:

$$f(x) = \theta_1 x_1 + \theta_2 x_2 + \theta_3 |x_1| x_2 + \theta_4 |x_2| x_2 + \theta_5 x_1^3 \tag{46}$$

where the parameters θ_i (i=0, …5) are: $\theta_1 = 0.0748, \theta_2 = 0.263$, $\theta_3 = -0.0749, \theta_4 = 0.0011, \theta_5 = 0.0026$.

To achieve satisfactory tracking response, the reference model matrices to be tracked are designed as:

$$A_r = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \quad B_r = \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} \tag{47}$$

where $\omega_n = 1\,rad/s$ is the natural frequency of the system, $\zeta = 0.707$ is the damping ratio. Considering this reference model we can calculate the MRAC gains as $K_x = [-1 \quad -1.414]$ $K_r = 1$. The filter parameter used in the modified adaptive law (41) is set as k=0.001, the learning rate is $\gamma = diag([100,100,100,100,100])$. The initial conditions of simulations are: $\theta(0) = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$, and $x(0) = [0.5 \quad 0]^T$. The external command $r(t)$ used and the input of the reference model is a square-wave with the period 30 seconds and its amplitude is 30 rad.

For comparison, the traditional CRM based MRAC shown in (Lavretsky, 2006; Stepanyan & Krishnakumar, 2010) is also simulated, where the adaptive law (8) is used instead of (41). The other simulation parameters (e.g. initial conditions, the learning gain and the feedback gains in the CRM) are the same as modified control scheme.

For showing the effect of different feedback gains α of CRM on the control response, different cases are simulated. Hence, the following four cases are considered:

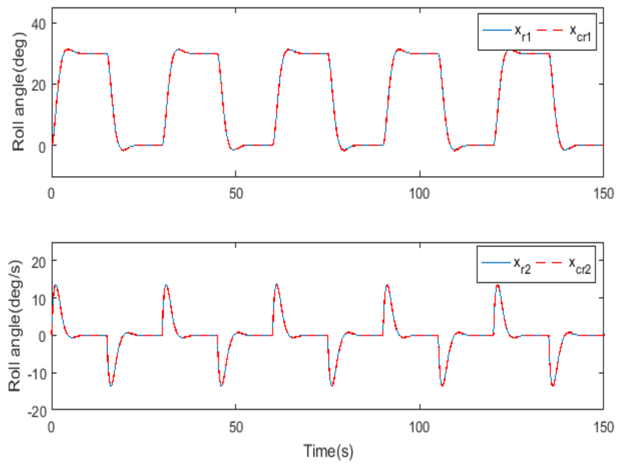
Case 1): Traditional CRM control system with adaptive law (8) and gain $\alpha = diag([100,100])$.

Case 2): Modified CRM control system with adaptive law (41) and gain $\alpha = diag([100,100])$.

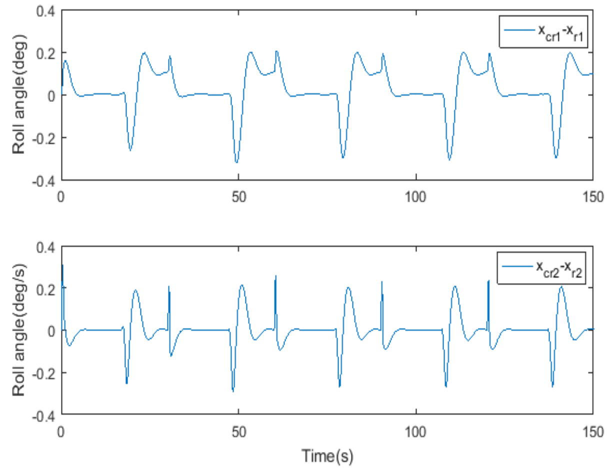
Case 3): Traditional CRM control system with adaptive law (8) and gain $\alpha = diag([1000,1000])$.

Case 4): Modified CRM control system with adaptive law (41) and gain $\alpha = diag([1000,1000])$.

Simulation results of the above cases are provided in Fig.3-Fig.6. respectively. It is shown in Fig.3 and Fig.4 that satisfactory steady-state control responses can be achieved by using both the traditional CRM-based MRAC and the modified CRM-MRAC with improved adaptive law. However, comparing Fig. 3 with Fig.4, we can see that it is quite obvious that the peaking phenomenon (the error $(x_{cr} - x_r)$ between the ORM and CRM) in Fig. 3 is worse than that in Fig.4. Hence, the transient control response of the proposed modified CRM-based MRAC can be improved (i.e. the tracking error $(x - x_r)$ between the system output and the original reference model can vanish very fast, leading to better transient control response). This is because the estimation error $\tilde{\theta}$ can converges to zero in an exponential manner, and thus help to reduce the bound of $(x_{cr} - x_r)$ as claimed in Theorem 2.

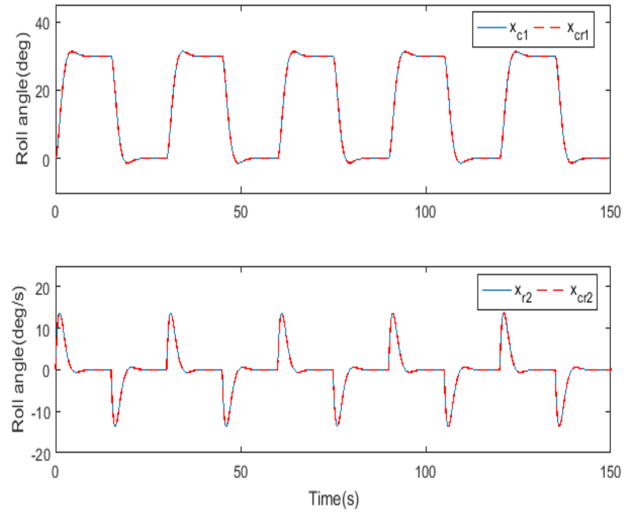


(a) Tracking profile of state variables

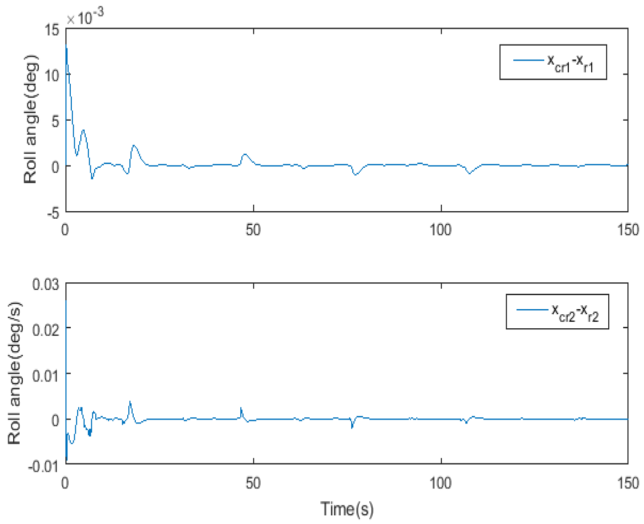


(b) The peaking value

Fig.3. Traditional CRM-based MRAC with feedback gain $\alpha = diag([100,100])$



(a) Tracking profile of state variables

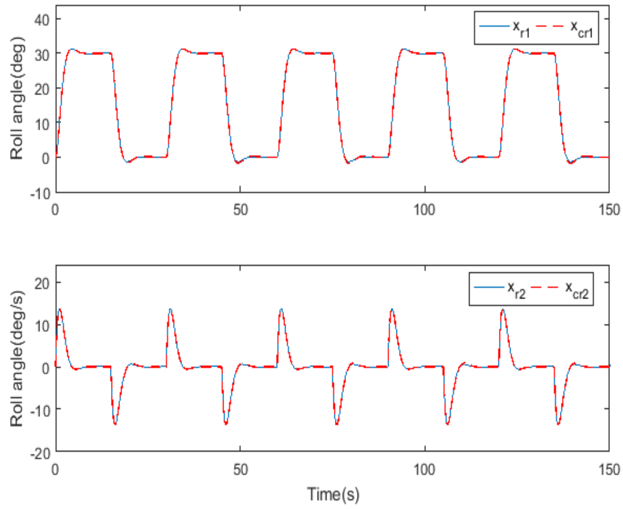


(b) The peaking value

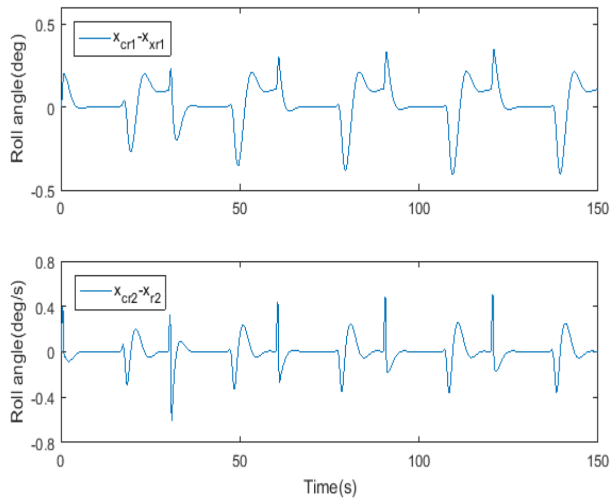
Fig.4. Modified CRM-based MRAC with feedback gain $\alpha = diag([100,100])$

The above observation is more obvious when we further increase the feedback gain in the CRM to $\alpha = diag([100,100])$. As analyzed in (15), the bound of $(x_{cr} - x_r)$ can be enlarged for large feedback gain due to the induced term $|\alpha|^{1/2}$, which may aggravate the peaking phenomenon. In fact, the peaking value of $(x_{cr} - x_r)$ shown in Fig. 5 is almost double of that shown in Fig.3. However, from Fig.6, it can be seen that the modified CRM-based MRAC with new adaptive law (41) can retain almost the same peaking response as that shown in Fig.4, which means that the convergence of $\tilde{\theta}$ can effectively alleviate the peaking phenomenon of the CRM control system, which is claimed in Theorem 2. Hence, the modified CRM scheme can allow us to use a large feedback gain α in the CRM, which will help to improve the transient convergence of the tracking error $e_{cr}(t)$ as pointed out in (Lavretsky, 2006; Stepanyan & Krishnakumar, 2010). Nevertheless, it is noted in Fig. 4 and Fig.6 that in the initial period (for first 5 sec) there exist smaller peaks in $(x_{cr} - x_r)$ because of the unavoidable initial tracking error $e_{cr}(0)$. However, the peaking values in Fig. 4 and Fig.6 are smaller than that shown in Fig.3 and Fig.5.

The simulation results shown in these figures all verify the effectiveness of the proposed modified adaptive law, when it is incorporated into the CRM-based MRAC control.

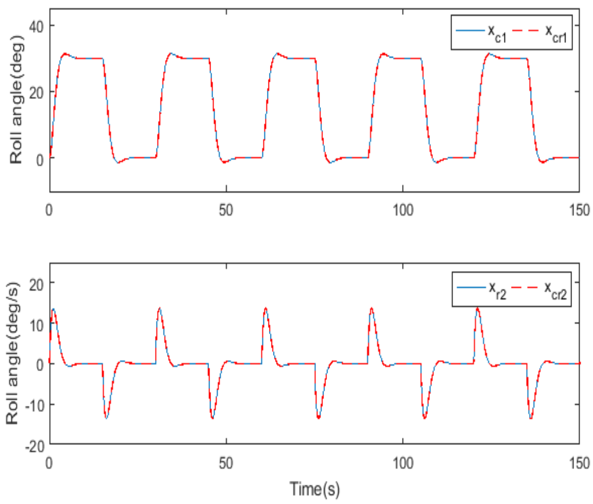


(a) Tracking profile of state variables

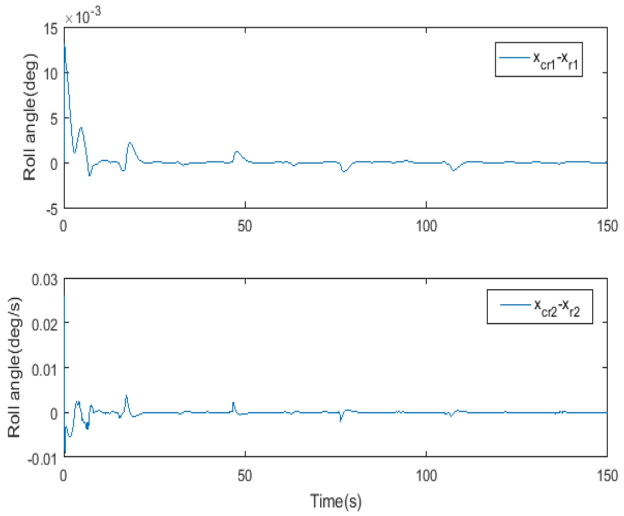


(b) The peaking value

Fig.5. Traditional CRM-based MRAC with feedback gain $\alpha = diag([1000,1000])$



(a) Tracking profile of state variables



(b) The peaking value

Fig.6. Modified CRM-based MRAC with feedback gain $\alpha = diag([1000,1000])$

5. Summary

Peaking phenomenon has been recognized as a critical problem for the recently proposed CRM-based MRAC system. The aim and contribution of this paper is to provide a new perspective for analyzing the peaking phenomenon by using the L_2 norm and Cauchy-Schwartz inequality. Based on the provided analysis, this paper further provides a modified adaptive law with exponential convergence to alleviate the bound of peaking value in the CRM-based MRAC system. In this new framework, sufficiently large feedback gain in the CRM is allowed to achieve improved transient response. Comparative simulation results are provided to verify the theoretical claims. Both theoretical analysis and simulation results validate the effectiveness and the improved response of the modified CRM-based MRAC system over traditional CRM-based MRAC.

Acknowledgements

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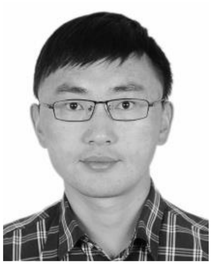

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