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Parametric Lyapunov Equation Approach to Stabilization and Set Invariance Conditions of Linear Systems with Input Saturation

Wenjuan Wu[a,*]

[a] *College of Computer and Information Engineering, Tianjin Normal University, Tianjin 300387, China*

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ABSTRACT

This paper studies the problems of stabilization and sets invariance conditions for linear systems with input saturation nonlinearity. Based on the parametric Lyapunov equation approach, new set invariance conditions can be expressed in terms of linear matrix inequalities. As a result, sufficient conditions are presented for stability analysis. The largest contractively invariant ellipsoids are determined by solving optimization problem with LMI constraints. The simulation results illustrate the effectiveness of the proposed methods.

1. Introduction

The control system with saturated nonlinear constrains is very common in real practical application, as input saturation often leads to degradation of system performance and instability. For this reason, many researchers have studied the topic of input saturation from practical and theoretical aspects (Wang, Ni and Yang 2013; Corradini, Cristofaro and Giannoni 2011; Hu, Xiao, and Friswell 2011) and a lot of methods have been proposed in the literature (Hu, Teel and Zaccarian 2008; De Santis and Isidori, 2001; Hu, Lin and Chen 2002). Low-gain design is important for controller design of systems with input saturation, and low-gain feedback refers to a family of feedback gains that approach zeros as a parameter. The low-gain feedback laws are constructed by eigenstructure assignment in (Lin and Saberi 1994), also a different way of constructing low-gain feedback laws is parameterized Riccati equation (ARE)(Teel 1996), and the value of the low-gain parameter is adjusted on line to achieve global results, instead of semi-global ones. Recently, Zhou proposed a new low-gain design approach in (Zhou, Duan and Lin 2008; Zhou, Duan and Lin 2009), the new approach possesses the advantages of both the eigenstructure assignment approach and the ARE-based approach, and it leads to a feedback gain that is a function of the Lyapunov matrix equation can be easily obtained(Zhou, Duan and Lin 2008).

Parameter uncertainty problem is another common question in real systems, it is well known that parameter uncertainty can affect both the performance and stability of the control systems, and the problem may come from modeling errors, variations in material properties, and changing load environments. So enhancing the robustness against system uncertainties is an important issue.

In literatures, invariant ellipsoids theorem has been widely used in stabilization problem analysis and performance optimization (Wang and Miao 2006; Yang, Sun and Ma 2013; Lu, Lin and Fang 2010). A classical method for establishing set invariance has been by application of the absolute stability analysis tools, such as circle and Popov criteria(Zhang, Yan, Fu and Zhao), where the saturation function is treated as a invariance of level sets of quadratic and Lur'e type Lyapunov functions are established, the method is too conservation. A new sufficient condition for an ellipsoid to be invariant was developed. The condition is less conservative and it can be solved as an optimization problem with LMI constraints (Hu and Lin 2001). However, to the best of our knowledge, the problem of linear systems with input saturation and parameter uncertainty has not been fully investigated and there is still some room to improve.

Motivated by the above observations, in this paper, we study the problem of stability analysis for linear systems under actuator saturation. A set of conditions under which an ellipsoid is contractively invariant with respect to linear system under a saturation low-gain feedback is first established. The determination of the largest contractively invariant ellipsoid that satisfies these conditions is solved as an optimization problem with LMI constraints. With the feedback gain viewed as an additional variable,

* Corresponding author.
E-mail addresses: wuwenjuanzixia@163.com (W. Wu)

this optimization problem can be readily adapted for the design of low-gain feedback.

2. Problem Statement

Consider the following uncertain linear system with input saturation

$$\dot{x} = (A+\Delta A)x + (B+\Delta B)sat(u) \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control input, and sat is the vector valued standard saturation function defined as

$$sat(u) = [\mathrm{sat}(u_1) \quad \mathrm{sat}(u_2) \quad \cdots \quad sat(u_m)]^T,$$

$$sat(u_i) = sign(u_i)\min\{|u_i|, 1\}$$

we have assumed, without loss of generality, unity saturation level. Nonunity saturation levels can be absorbed into the matrices B and u. ΔA and ΔB represent parameter uncertainties, the admissible parameter uncertainties in this paper are assumed to be modeled as

$$[\Delta A \quad \Delta B] = DF[E_1 \quad E_2] \tag{2}$$

where D, E_1 and E_2 are known real constant matrices. F is an unknown matrix satisfying

$$F^T F \le I \tag{3}$$

Parametric Lyapunov equation based low-gain design relies on the solution to the following parametric ARE:

$$A^T P + PA - PBB^T P = -\gamma P$$

where A and B are two given constant matrices and γ is a scalar. With the solution $P(\gamma)$, a state feedback law can be constructed as follows:

$$u = -B^T P(\gamma)x \tag{4}$$

Based on the parametric Lyapunov equation method, some properties will be given in Lemma 1.

Lemma 1(Zhou and Lin 2009) Assume that (A, B) is controllable. Let

$$\gamma > -2\min\{\mathrm{Re}(\lambda(A))\}$$

where $\mathrm{Re}(\lambda(A))$ denotes the set of the real parts of the eigenvalues of A. Then the ARE has a unique positive definite solution $P(\gamma) = W^{-1}(\gamma)$, where $W(\gamma)$ is the unique positive solution to the following Lyapunov equation:

$$W\left(A + \frac{\gamma}{2}I_n\right)^T + \left(A + \frac{\gamma}{2}I_n\right)W = BB^T$$

moreover, $\frac{d}{d\gamma}P(\gamma) > 0, \forall \gamma > 0$ and

$$tr(B^T P(\gamma)B) = 2tr(A) + n\gamma$$

For matrix $H \in R^{m\times n}$, let

$$L(H) = \{x \in R^n : |h_i x| \le 1, i \in [1, m]\} \tag{5}$$

where h_i represents the i-th row of matrix H. $L(H)$ represents the region in R^n where Hx does not saturate.

Let D be the $m \times n$ diagonal matrices whose diagonal elements are either 1 or 0. There are 2^m elements in D. Suppose that these elements of D are labeled as $D_i, i \in [1, 2^m]$. Denote $D_i^- = I - D_i$. We can see, $D_i^- \in D$ if $D_i \in D$.

Lemma 2(Hu and Lin 2001)

Let $F, H \in R^{m\times n}$. Then, for any $x \in L(H)$, it holds that

$$sat(Fx) \in co\{D_s Fx + D_s^- Hx, s \in [1, 2]\} \tag{6}$$

where co stands for the convex hull.

By Lemma 2, we can see that control input can get into the nonlinear area. The saturated control can be treated as a series of linear convex hull form in this method.

To present our main results, we need the following Lemmas.

Lemma 3(Boyd, Ghaoui and Feron, 1994) (Schur Complement Lemma) Let the partitioned matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

be symmetric. Then $S < 0$ if and only if

$$S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$$

or

$$S_{22} < 0, S_{11} - S_{12} S_{11}^{-1} S_{12}^T < 0$$

Lemma 4(Boyd, Ghaoui and Feron, 1994) Let D, E and Y be real matrices with appropriate dimension, with Y satisfying $Y = Y^T$,then

$$Y + DFE + E^T F^T D^T < 0$$

for all $F^T F \le I$, if and only if there exists a scalar such that

$$Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0$$

3 Main results

3.1. Stability analysis

In this paper, we will solve the problem of the system with actuator saturation by using the auxiliary matrix method. Next we will consider the low-gain feedback stabilization, and two theorems will be given.

Theorem 1 Assume (A,B) is controllable, the ARE has a unique positive definite solution $P(\gamma)$, for a given $P_\gamma \in R^{n\times n}$ and $H \in R^{m\times n}$, F satisfying (3) and $x_0 \in \varepsilon(P_\gamma,1)$, such that

$$\Xi = \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ * & -\varepsilon I \end{bmatrix} < 0 \tag{7}$$

where

$$P_\gamma^{-1} = Q_\gamma$$

$$\begin{aligned}\Gamma_1 &= Q_\gamma^T A^T + Q_\gamma^T (D_s(-B^T P) + D_s^- H)^T B^T + AQ_\gamma \\ &\quad + B(D_s(-B^T P)Q_\gamma + D_s^- HQ_\gamma) + \varepsilon DD^T\end{aligned}$$

$$\Gamma_2 = \left(E_1 Q_\gamma + E_2\left((D_s(-B^T P(\gamma)) + D_s^- H)\right)Q_\gamma\right)^T$$

and $\varepsilon(P_\gamma,1) \subset L(H)$, then the close-loop system is asymptotically stable. Moreover, $\varepsilon(P_\gamma,1)$ is a contractively invariant set.

Proof: At first, (A,B) is controllable. According to Lemma 1, the ARE has a unique positive definite solution $P(\gamma)$. By lemma 2, for every $x \in \varepsilon(P_\gamma,1)$,

$$\begin{aligned}&sat\left((-B^T P(\gamma))x\right) \in \\ &co\left\{D_s(-B^T P(\gamma))x + D_s^- Hx, s \in [1,2]\right\}\end{aligned}$$

It follows that

$$\begin{aligned}&(A+\Delta A)x + (B+\Delta B)sat\left((-B^T P(\gamma))x\right) \in \\ &co\left\{(A+\Delta A)x + (B+\Delta B)(D_s(-B^T P(\gamma)) \right. \\ &\left. + D_s^- H)x + x, s \in [1,2]\right\}\end{aligned}$$

Define a Lyapunov function

$$V(x) = x^T P_\gamma x$$

The derivative of this Lyapunov function along the trajectory of the closed-loop system is given by

$$\begin{aligned}\dot{V} &= 2x^T P_\gamma \dot{x} \\ &= 2x^T P_\gamma ((A+\Delta A)x + (B+\Delta B)sat(-B^T P(\gamma)x)) \\ &= 2x^T P_\gamma \sum_{i=1}^{2^m} \eta_i \left((A+\Delta A)x + (B+\Delta B)(D_i(-B^T P(\gamma)) + D_i^- H)x\right) \\ &\le 2x^T P_\gamma \sum_{i=1}^{2^m} \eta_i \left((A+\Delta A) + (B+\Delta B)(D_i(-B^T P(\gamma)) + D_i^- H)\right)x \\ &\le \sum_{i=1}^{2^m} \eta_i x^T \Xi x\end{aligned}$$

By using (2), we have

$$\begin{aligned}\Xi &= \left(A + B(D_s(-B^T P(\gamma)) + D_s^- H)\right)^T P_\gamma \\ &\quad + P_\gamma\left(A + B(D_s(-B^T P(\gamma)) + D_s^- H)\right) \\ &\quad + \left(\Delta A + \Delta B(D_s(-B^T P(\gamma)) + D_s^- H)\right)^T P_\gamma \\ &\quad + P_\gamma\left(\Delta A + \Delta B(D_s(-B^T P(\gamma)) + D_s^- H)\right) \\ &= \left(A + B(D_s(-B^T P(\gamma)) + D_s^- H)\right)^T P_\gamma \\ &\quad + P_\gamma\left(A + B(D_s(-B^T P(\gamma)) + D_s^- H)\right) \\ &\quad + \left(DFE_1 + DFE_2(D_s(-B^T P(\gamma)) + D_s^- H)\right)^T P_\gamma \\ &\quad + P_\gamma\left(DFE_1 + DFE_2(D_s(-B^T P(\gamma)) + D_s^- H)\right)\end{aligned}$$

By using Lemma 4, we obtain

$$\begin{aligned}&\left(DFE_1 + DFE_2(D_s(-B^T P(\gamma)) + D_s^- H)\right)^T P_\gamma \\ &\quad + P_\gamma\left(DFE_1 + DFE_2(D_s(-B^T P(\gamma)) + D_s^- H)\right) \\ &= P_\gamma DF\left(E_1 + E_2\left((D_s(-B^T P(\gamma)) + D_s^- H)\right)\right) \\ &\quad + \left(E_1 + E_2\left((D_s(-B^T P(\gamma)) + D_s^- H)\right)\right)^T F^T D^T P_\gamma \qquad (8) \\ &\le \varepsilon P_\gamma DD^T P_\gamma^T \\ &\quad + \frac{1}{\varepsilon}\left(E_1 + E_2\left((D_s(-B^T P(\gamma)) + D_s^- H)\right)\right)^T \\ &\quad \left(E_1 + E_2\left((D_s(-B^T P(\gamma)) + D_s^- H)\right)\right)\end{aligned}$$

$$\begin{aligned}\Xi &= \left(A + B(D_s(-B^T P(\gamma)) + D_s^- H)\right)^T P_\gamma \\ &\quad + P_\gamma\left(A + B(D_s(-B^T P(\gamma)) + D_s^- H)\right) \\ &\quad + \varepsilon P_\gamma DD^T P_\gamma^T \\ &\quad + \frac{1}{\varepsilon}\left(E_1 + E_2\left((D_s(-B^T P(\gamma)) + D_s^- H)\right)\right)^T \\ &\quad \left(E_1 + E_2\left((D_s(-B^T P(\gamma)) + D_s^- H)\right)\right)\end{aligned}$$

By the Schur complement Lemma, we get

$$\Xi = \begin{bmatrix} \Gamma & \left(E_1 + E_2\left((D_s(-B^T P(\gamma)) + D_s^- H)\right)\right)^T \\ \left(E_1 + E_2\left((D_s(-B^T P(\gamma)) + D_s^- H)\right)\right) & -\varepsilon I \end{bmatrix} < 0 \tag{9}$$

where

$$\Gamma = \left(A + B(D_s(-B^T P(\gamma)) + D_s^- H)\right)^T P_\gamma$$
$$+ P_\gamma \left(A + B(D_s(-B^T P(\gamma)) + D_s^- H)\right) + \varepsilon P_\gamma DD^T P_\gamma^T$$

Pre-multiplying and post-multiplying (9) by

$diag\left\{P_\gamma^{-1}, I\right\} = diag\left\{Q_\gamma, I\right\}$, we obtain

$$\Gamma = Q_\gamma^T A^T + Q_\gamma^T (D_s(-B^T P) + D_s^- H)^T B^T + AQ_\gamma$$
$$+ B(D_s(-B^T P)Q_\gamma + D_s^- HQ_\gamma) + \varepsilon DD^T$$

Then the (9) can be written as

$$\Xi = \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ * & -\varepsilon I \end{bmatrix} < 0 \qquad (10)$$

where

$$\Gamma_1 = Q_\gamma^T A^T + Q_\gamma^T (D_s(-B^T P) + D_s^- H)^T B^T + AQ_\gamma$$
$$+ B(D_s(-B^T P)Q_\gamma + D_s^- HQ_\gamma) + \varepsilon DD^T < 0$$

$$\Gamma_2 = \left(E_1 Q_\gamma + E_2 \left((D_s(-B^T P(\gamma)) + D_s^- H)\right) Q_\gamma\right)^T$$

In view (10), we have

$$\dot{V} < 0, \forall x \in \varepsilon\left(P_\gamma, 1\right) \setminus \{0\}$$

which indicates that $\varepsilon\left(P_\gamma, 1\right)$ is contractively invariant, and the closed-loop is asymptotically stable.

If $\Delta A = \Delta B = 0$, in the Case, the system reduces to the general system.

3.2. Estimation of the domain of attraction

The estimation of the domain of attraction boils down to the determination of the largest invariant ellipsoid $\varepsilon\left(P_\gamma, 1\right)$. We measure the $\varepsilon\left(P_\gamma, 1\right)$ with respect to a shape reference set X_R by the largest α, and we have $\alpha X_R \subset \varepsilon\left(P_\gamma, 1\right)$. Thus the determination of the largest $\varepsilon\left(P_\gamma, 1\right)$ can be formulated into an optimization problem.

Here are two optimization problems of the Case:

Case: $\Delta A \neq 0, \Delta B \neq 0$

$$\sup_{P(\gamma)>0, P_\gamma, H} \alpha$$
$$\begin{aligned} s.t \quad & (a)\, \alpha X_R \subset \varepsilon\left(P_\gamma, 1\right) \\ & (b) \quad (7) \\ & (c)\, \varepsilon\left(P_\gamma, 1\right) \subset L(H) \end{aligned}$$

If X_R is an ellipsoid $X_R = \left\{x \in R^m : x^T R x \leq 1\right\}$, then constraint (a) is equivalent to $\alpha^2 P_\gamma \leq R$.

By Schur complement Lemma, the constraint (a) is further equivalent to

$$\begin{bmatrix} \frac{1}{\alpha^2} R & I \\ I & P_\gamma^{-1} \end{bmatrix} \leq 0$$

The constraint (c) is equivalent to

$$h_i P_\gamma^{-1} h_i^T \leq 1 \Leftrightarrow \begin{bmatrix} 1 & h_i P_\gamma^{-1} \\ P_\gamma^{-1} h_i^T & P_\gamma^{-1} \end{bmatrix} \geq 0 .$$

Let $Q_\gamma = P_\gamma^{-1}$ and $\frac{1}{\alpha^2} = k$, then the two optimization problems can be written as the following LMI problems:

Case : $\Delta A \neq 0, \Delta B \neq 0$

$$\inf_{Q_\gamma, H} k$$
$$\begin{aligned} s.t \quad & (a) \begin{bmatrix} kR & I \\ I & Q_\gamma \end{bmatrix} \leq 0 \\ & (b) \quad (7) \\ & (c) \begin{bmatrix} 1 & h_i Q_\gamma \\ Q_\gamma h_i^T & Q_\gamma \end{bmatrix} \geq 0 \end{aligned}$$

4 Numerical simulation examples

In this section, we will give two numerical examples to demonstrate the effectiveness of the present results

Considering the mathematical model of pitch/yaw channel control system of missile is as follows:

$$\dot{x} = A(t)x + B(t)sat(u)$$

$$A(t) = \begin{bmatrix} a_1 & a_2 & \frac{J_x - J_y}{J_z}\omega_x & a_3\omega_x \\ 1 & a_4 & 0 & -\frac{\omega_x}{57.3} \\ \frac{J_z - J_x}{J_y}\omega_x & a_5\omega_x & a_6 & a_7 \\ 0 & \frac{\omega_x}{57.3} & 1 & a_8 \end{bmatrix},$$

$$B(t)=\begin{bmatrix} b_1 & 0 \\ b_2 & 0 \\ 0 & b_3 \\ 0 & b_4 \end{bmatrix}$$

where $x=\left[\omega_z\ \alpha\ \omega_y\ \beta\right]^T$, $u=\left[\delta_z\ \delta_y\right]^T$, and

$$A=\begin{bmatrix} -1.8780 & -2.1298 & -5.2356 & 1.9895 \\ 1.0000 & -1.5060 & 0 & -6.9808 \\ 5.2356 & -2.0593 & -1.9500 & -3.7606 \\ 0 & 6.9808 & 1.0000 & -0.7710 \end{bmatrix},$$

$$B=\begin{bmatrix} -1.5787 & 0 \\ -0.2430 & 0 \\ 0 & -1.4948 \\ 0 & -0.1910 \end{bmatrix},$$

The initial condition is

$$x(0)=\left[0.9566 \quad 0 \quad 0.56063 \quad 0\right]^T.$$

Case : $\Delta A\neq 0, \Delta B\neq 0$:

$$\left(\left[\Delta A \quad \Delta B\right]=DF\left[E_1 \quad E_2\right]\right)$$

$$E1=\begin{bmatrix} -1.1302 & 33.6195 & 53.4600 & 3.7794 \\ 0.0018 & -0.7010 & 0.0250 & -0.0144 \\ -0.3248 & 3.5318 & 3.7399 & 2.4304 \\ -0.0010 & -0.3385 & 0.1790 & -0.0027 \end{bmatrix},$$

$$E2=\begin{bmatrix} -1.9425 & 0.2172 \\ 0.0153 & 0.0072 \\ -0.4989 & -0.0044 \\ -0.0386 & 0.0041 \end{bmatrix}$$

$$D=\begin{bmatrix} -0.10 & -0.1298 & -0.2356 & 0.9895 \\ 0.01 & -0.5060 & 0 & -0.9808 \\ 0.2356 & -0.593 & -0.9500 & -0.07606 \\ 0 & 0.9808 & 0.01 & -0.7710 \end{bmatrix}$$

If $\gamma =$ 3.9994 ,

$$P(\gamma)=\begin{bmatrix} 0.8034 & -0.0856 & -0.0119 & 0.2188 \\ -0.0856 & 0.1554 & -0.1901 & -0.0078 \\ -0.0119 & -0.1901 & 0.7392 & -0.0779 \\ 0.2188 & -0.0078 & -0.0779 & 0.1591 \end{bmatrix},$$ let the

shape reference set be given by $R=I$. The solutions of the optimization problem can be obtained by LMI:

$$P=\begin{bmatrix} 3.3932 & 1.5794 & 0.1220 & -0.5368 \\ 1.5794 & 2.6609 & 0.3634 & -0.0091 \\ 0.1220 & 0.3634 & 3.4211 & 1.7600 \\ -0.5368 & -0.0091 & 1.7600 & 2.8663 \end{bmatrix}.$$

$$H=\begin{bmatrix} 1.1389 & -0.1952 & -0.0842 & 0.3430 \\ 0.0397 & -0.2769 & 0.9722 & -0.1933 \end{bmatrix}.$$

$$k=0.0330, \quad \alpha=0.1818.$$

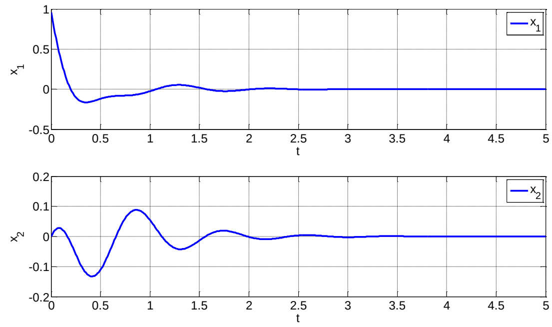

Fig. 1. The states x_1 and x_2 for Case

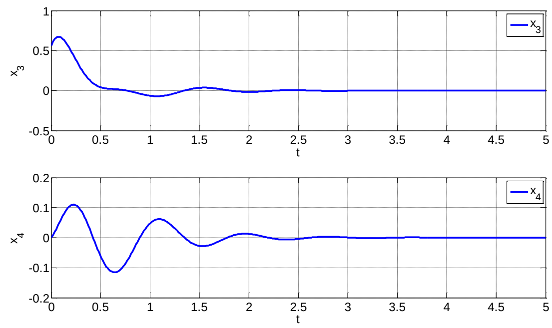

Fig. 2. The states x_3 and x_4 for Case

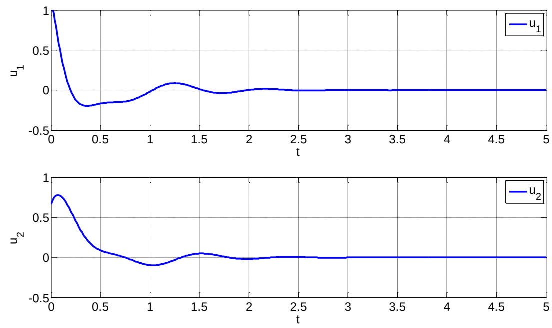

Fig. 3. Control inputs u_1 and u_2 for Case

Figs. 1, 2, 5 and 6 show that the proposed controller guarantees that the systems are asymptotically stable, The control inputs u_1 and u_2 for two Cases are shown in Figs. 3. From these figures, we can observe that control input u_1 is saturate at the beginning of the simulation time.

5 Conclusions

In this paper, some new results of the parametric Lyapunov equation based low-gain design feedback laws were established. By constructing a Lyapunov function, two inequalities for a class of linear systems with input saturation have been designed such that the closed-loop systems are asymptotically stable. Also the estimate domain of attraction problem can be converted into optimization problem with LMI constraints. Finally, the simulation results are given to demonstrate the effectiveness of the results.

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Wenjuan Wu received the B.S. degree from Yuncheng University in 2009, received M.S. degree from Yanshan University in 2012, and received Ph.D. degree in Center for Control Systems and Guidance Technology, Harbin Institute of Technology. From 2016, she became a Lecturer at Tianjin Normal University, her research interests include stability analysis of stochastic dynamical system, fuzzy neural networks, delayed differential systems, nonlinear control, adaptive control and fault tolerant control. She has published several papers in national journals in China.