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# Adaptive Parameter Estimation for Multivariable Nonlinear CARMA Systems

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#### ABSTRACT

A novel adaptive parameter identification scheme based on hierarchical identification principle has been presented to identify the parameter information of the multivariable nonlinear controlled autoregressive moving average (CARMA) systems. First, by substituting the nonlinear block into the corresponding linear block, a typical linear regression identification expression is obtained in which the estimated parameters involve parameter vector and bilinear parameter matrix which makes the identification problem difficult. Second, to solve parameter matrix issue, the identification model is changed to two different form estimation models where the estimation models are linear to each parameter vector. In order to interactively identify the parameter vectors, a novel adaptive parameter identification method is proposed to estimate the parameter vector by virtue of hierarchical identification idea. Followed by the parameter convergence is studied by using the stochastic theory. Finally, compared with some publishing identification methods, the developed approach of this paper produce an outstanding identification performance through simulation example.

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# 1. Introduction

System identification is of great importance for the system modeling and control system when the parameters of the practical system are unknown in the control engineering. For example, system identification can provide a model analysis for control system and give a predictive result output (Kim et al., 2015). In the single argument the parameter identifications of the nonlinear systems, parameter identification methods are relatively mature compared to the that of multivariable nonlinear systems (Zhao, 2010; Giri et al, 2009; Pouliquen et al, 2016; Caigny et al, 2009; Martensson et al, 2017; Xiong et al, 2017; El-Koujok et al, 2014). However, the identification algorithms of single argument systems are not suitable for multivariable systems because the latter systems have coupled phenomenon on parameters and possess more complex nonlinear relationships on the parts of systems. These reasons drive us to search some appropriate estimation algorithms. The purpose of system identifications for the multivariable nonlinear systems is to describe the more complex nonlinear systems (Villalva et al, 2007). Therefore, it is interesting to discuss the parameter identification of multivariable systems.

With extensive attention to multivariable systems, a large number of identification approaches for multivariable systems have been published by scholars and engineers (Zhang et al, 2017; Jafari et al, 2014; Kim et al., 2015; Harnischmacher et al, 2007; Han et al, 2010; Ding, 2014; Sato et al, 2017; Wang et al, 2016). By developing the adjustable identification model which is decomposed into two sub-identification models for multi-input single output, two recursive parameter estimation methods are presented to estimate the parameters of the considered systems in (Salhi et al, 2015). In (Zhang et al, 2017), a frequency domain identification method is utilized to identify the multivariable plant model parameters through the usage of consistent estimator and input and output noise covariance matrix in which the estimated parameters need a high computation cost. Furthermore, the frequency identification method is difficult to apply in practical systems because the special input signal conditions are not met in practice. A blind estimation algorithm is studied for multiple channel state space systems which is composed of autoregressive system rather than FIR model, in which each channel model needs to use the cross relation estimation approach through the usage of mutual references among difference channels in (Yu et al, 2016). However, Blind identification model only approximates the whole model or partial model, which is not conducive to the design of the subsequent controller. In (Ding 2014), a hierarchical generalized least-squares scheme is discussed for multivariable systems in present of output error autoregressive noises by using hierarchical identification idea, in which the parameters of concerned systems are separately estimated by using three parameter update law based on interaction estimation theory.

Although, the developed hierarchical generalized method can effectively identify the multivariable systems, the identification accuracy is not so ideal owing to that the measured input and output data are polluted by noise or interference signal. To reduce the effect of noise on the identification data, a novel filter operator is proposed to relieve the noise effect in (Na et al 2014; Na et al 2015). It has been shown that Na' algorithm is effective filter algorithm for a large class of the nonlinear systems. But, we need to modify the corresponding estimation algorithm for the multivariable systems when Na' algorithm is applied to estimate the parameters of multivariable nonlinear systems.

Inspired by (Na et al 2014, 2015) and (Ding 2014), the focus of this paper is to develop an adaptive estimation approach identifying the parameters of the multivariable nonlinear CARMA systems, whose structure is show in Fig.1, by using filter operator and hierarchical identification principle. One of the contributions of this paper is to apply the filter operator filtering the noise effect on the identification data based on two constructed identification models, in which the estimation output expressions are linear to the corresponding parameter vectors; The other one is to develop an identification schemes interactive estimating the parameter vectors by using hierarchical identification idea, in which three adaptive parameter update laws are presented to obtain the estimated parameters through the usage of the corresponding consistent estimators. Moreover, the convergence of identification algorithm is discussed by using the stochastic theory. The developed algorithm can effectively estimate the unique parameter values rather than the coupled parameter values. Comparative examples are offered to test the usefulness of the proposed algorithm.



Fig.1. Structure of multivariable CARMA systems

The content of this paper is listed as follows. Problem formulation for multivariable CARMA systems is offered in Section2. The constructed adaptive identification scheme is derived in Section 3. In Section 4, the convergence of the presented scheme is discussed. In Section 5, simulation examples are given and followed by Section 6 offers some interesting conclusions.

#### 2. Problem statement

In this section, the estimation model is provided through the usage of the interior relation between the nonlinear element and linear subsystem element. The identification scheme for the estimation model will be given in the next section by using the filter operator and hierarchical identification idea.

According to Fig.1, the multivariable CARMA systems can be

described by the following mathematical expression

$$\alpha(q)y(t) = \boldsymbol{\beta}(q)\boldsymbol{x}(t) + \lambda(q)v(t)$$
(1)

where y(t) is measurable system scalar output signal;

v(t) represents scalar noise term with zero mean.

$$x_i(t) = f_i[u_i(t)], i = 1, \dots, m$$
; the inner vector  $\mathbf{x}(t)$ 

and input signal  $\boldsymbol{u}(t)$  can be defined by

$$\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_m(t) \end{bmatrix}, \quad \boldsymbol{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}$$

The expression  $\alpha(q)$ ,  $\beta(q)$ ,  $\lambda(q)$  are polynomials with backward shift operator  $q^{-1}u(t) = u(t-1)$  and described

by

$$\alpha(q) = 1 + \alpha_1 q^{-1} + \alpha_2 q^{-2} + \dots + \alpha_n q^{-n},$$
  

$$\beta(q) = [\beta_1, \beta_2, \dots, \beta_m],$$
  

$$\beta_i(q) = \beta_{i1} q^{-1} + \dots + \beta_{in} q^{-n}, i = 1, \dots, m,$$
  

$$\lambda(q) = 1 + \lambda_1 q^{-1} + \lambda_2 q^{-2} + \dots + \lambda_n q^{-n}.$$

The nonlinear element  $f_i$  can be approximated by some basis functions

$$x_{i}(t) = f_{i}[u_{i}(t)] = \sum_{j=1}^{p} \gamma_{ij} f_{ij}[u_{i}(t)]$$
(2)

where  $\gamma_{ij}$  denotes the coefficients of basis functions; p is number of the functions. If the p is sufficiently large, the nonlinear element can be effectively approximated.

Assumption: The orders m, n are known and the initial state of systems are zero, i.e., y(t) = 0, u(t) = 0 and v(t) = 0when  $t \le 0$ ; The linear elements are stable, minimum phase. Substituting (2) into (1), we have

$$y(t) = \sum_{i=1}^{m} \beta_i(q) \sum_{j=1}^{p} \gamma_{ij} f_{ij}[u_i(t)] + \sum_{k=1}^{n} (-\alpha_k y(t-k) + \lambda_k v(t-k)) + v(t)$$
(3)

This paper aims to develop an adaptive identification scheme for the considered identification model (3) based on the filtering technique and hierarchical identification idea, to identify the parameters of (3), i.e.,  $\beta_{ik}$ ,  $\gamma_{ij}$ ,  $\alpha_k$  and  $\lambda_k$  from measured input and output data and to test the estimation performance through the usage of the illustrated example.

### 3. Adaptive estimation algorithm

In this section, an adaptive identification approach is presented to estimate the parameters of (3).

Based on the define some variables, (3) can be written as follows

$$y(t) = \sum_{i=1}^{m} \boldsymbol{\beta}_{i}^{T} \boldsymbol{F}_{i}(t) \boldsymbol{\gamma}_{i} + \boldsymbol{\psi}^{T}(t) \boldsymbol{\zeta} + v(t)$$
(4)

where the variables are summarized as follows:

$$\boldsymbol{\beta}_{i} = \begin{bmatrix} \boldsymbol{\beta}_{i1} \\ \boldsymbol{\beta}_{i2} \\ \vdots \\ \boldsymbol{\beta}_{in} \end{bmatrix}, \quad \boldsymbol{\gamma}_{i} = \begin{bmatrix} \boldsymbol{\gamma}_{i1} \\ \boldsymbol{\gamma}_{i2} \\ \vdots \\ \boldsymbol{\gamma}_{ip} \end{bmatrix},$$
$$\boldsymbol{\alpha} = \begin{bmatrix} \boldsymbol{\alpha}_{1} \\ \boldsymbol{\alpha}_{2} \\ \vdots \\ \boldsymbol{\alpha}_{n} \end{bmatrix}, \quad \boldsymbol{\lambda} = \begin{bmatrix} \boldsymbol{\lambda}_{1} \\ \boldsymbol{\lambda}_{2} \\ \vdots \\ \boldsymbol{\lambda}_{n} \end{bmatrix}, \quad \boldsymbol{\zeta} = \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\lambda} \end{bmatrix},$$
$$\boldsymbol{F}_{i}(t) = \begin{bmatrix} f_{i1}[u_{i}(t-1)] & \cdots & f_{ip}[u_{i}(t-1)] \\ f_{i1}[u_{i}(t-2)] & \cdots & f_{ip}[u_{i}(t-2)] \\ \vdots & \vdots \\ f_{i1}[u_{i}(t-n)] & \cdots & f_{ip}[u_{i}(t-n)] \end{bmatrix},$$
$$\boldsymbol{\psi}(t) = [-y(t-1), \cdots, -y(t-n), v(t-1), \cdots, v(t-n)]^{T}.$$

Note that the first term on the right side of (4) includes the bilinear parameter on the product of  $\beta$  and  $\gamma$ . If we directly develop an estimation algorithm to estimate the bilinear parameter, it will obtain some coupled parameters and may be a heavy computational cost for identification algorithm. One of the effective solutions is hierarchical identification idea to handle the above difficult (Ding et al, 2005). Then, two constructed identification models are defined by the following expression, respectively.

$$\mathbf{y}(t) = \boldsymbol{\beta}^{T} \boldsymbol{F}_{\gamma}(t) + \boldsymbol{\psi}^{T}(t)\boldsymbol{\zeta} + \mathbf{v}(t)$$
(5)

where the variables are defined by

$$\boldsymbol{F}_{\gamma}(t) = \begin{bmatrix} \boldsymbol{F}_{1}(t)\boldsymbol{\gamma}_{1} \\ \boldsymbol{F}_{2}(t)\boldsymbol{\gamma}_{2} \\ \vdots \\ \boldsymbol{F}_{m}(t)\boldsymbol{\gamma}_{m} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}, \boldsymbol{\beta}_{m} \end{bmatrix}^{T},$$

and the other one model is defined by

$$\mathbf{y}(t) = \mathbf{F}_{\beta}(t)\mathbf{\gamma} + \mathbf{\psi}^{T}(t)\boldsymbol{\zeta} + \mathbf{v}(t)$$
(6)

where the variables are listed as follows

$$\boldsymbol{\gamma} = \begin{bmatrix} \boldsymbol{\gamma}_1 \\ \boldsymbol{\gamma}_2 \\ \vdots \\ \boldsymbol{\gamma}_m \end{bmatrix}, \quad \boldsymbol{F}_{\beta}(t) = \begin{bmatrix} \boldsymbol{\beta}_1^T \boldsymbol{F}_1(t), \cdots, \boldsymbol{\beta}_m^T \boldsymbol{F}_m(t) \end{bmatrix}^T$$

It is note that when the defined variables  $F_{\gamma}(t)$  and  $F_{\beta}(t)$ 

are seen as the two information vectors, then (5) and (6) are linear to the parameter vectors  $\boldsymbol{\beta}$  and  $\boldsymbol{\gamma}$ , respectively. The parameter vector  $\boldsymbol{\zeta}$  can be estimated by using (5) or (6). To obtain unique identification model,  $\|\boldsymbol{\beta}\|$  is set as a constant (e.g.,  $\|\boldsymbol{\beta}\|=1$ ) (Wang et al, 2015).

To enhance the estimation accuracy, the system data is filtered by using filter operator. Then, the filtered signals  $F_{\gamma f}(t)$ ,  $F_{\beta f}(t)$ ,

 $\boldsymbol{\psi}_{f}(t)$  and  $y_{f}(t)$  can be expressed by

$$y_{f}(t) = \frac{k}{k+h} y_{f}(t-1) + \frac{h}{k+h} y(t)$$
(7)

$$\boldsymbol{F}_{\gamma f}(t) = \frac{k}{k+h} \boldsymbol{F}_{\gamma f}(t-1) + \frac{h}{k+h} \hat{\boldsymbol{F}}_{\gamma}(t) \qquad (8)$$

$$\boldsymbol{F}_{\beta f}(t) = \frac{k}{k+h} \boldsymbol{F}_{\beta f}(t-1) + \frac{h}{k+h} \hat{\boldsymbol{F}}_{\beta}(t) \qquad (9)$$

$$\boldsymbol{\psi}_{f}(t) = \frac{k}{k+h} \boldsymbol{\psi}_{f}(t-1) + \frac{h}{k+h} \hat{\boldsymbol{\psi}}(t)$$
(10)

where the constants k, h > 0.  $\hat{F}_{y}(t)$  denotes the estimate of  $F_{y}(t)$ .

 $\hat{F}_{\beta f}(t)$  and  $\hat{\psi}_{f}(t)$  have similar representations.

Then the following regressive matrices M1(t), M2(t), M3(t) and variables N1(t), N2(t), N3(t) are defined by

$$M1(t) = \frac{1}{1+Lh}M1(t-1) + \frac{h}{1+Lh}F_{\gamma f}(t)F_{\gamma f}^{T}(t) \quad (11)$$

$$N1(t) = \frac{1}{1 + Lh} y_f(t) / F_{\gamma f}(t) M 1(t-1) + \frac{h y_f(t) F_{\gamma f}^{T}(t)}{1 + Lh}$$
(12)

$$M2(t) = \frac{1}{1+Lh}M2(t-1) + \frac{n}{1+Lh}F_{\beta f}(t)F_{\beta f}^{T}(t)$$
(13)

$$N2(t) = \frac{1}{1+Lh} y_f(t) / F_{\beta f}(t) M 2(t-1) + \frac{h y_f(t) F_{\beta f}(t)}{1+Lh}$$
(14)

$$M3(t) = \frac{1}{1+Lh}M3(t-1) + \frac{h}{1+Lh}\psi_{f}(t)\psi_{f}^{T}(t) \quad (15)$$

$$N3(t) = \frac{1}{1+Lh} y_f(t) / \psi_f(t) M 3(t-1) + \frac{h y_f(t) \psi_f^{-1}(t)}{1+Lh}$$
(16)

The corresponding adaptive laws are described by

$$\boldsymbol{\gamma}(t) = \boldsymbol{\gamma}(t-1) - \Gamma W_1(t) \tag{17}$$

$$\boldsymbol{\beta}(t) = \boldsymbol{\beta}(t-1) - \Gamma W_2(t) \tag{18}$$

$$\boldsymbol{\zeta}(t) = \boldsymbol{\zeta}(t-1) - \Gamma W_3(t) \tag{19}$$

where the variable can be defined by

$$W_{1}(t) = M1(t)\gamma(t-1) - N1(t) + v(t)$$
 (20)

$$W_2(t) = M2(t)\beta(t-1) - N2(t) + v(t) \quad (21)$$

$$W_3(t) = M3(t)\zeta(t-1) - N3(t) + v(t)$$
 (22)

where  $\boldsymbol{\beta}(t)$  is the estimation of  $\boldsymbol{\beta}(t)$ ,  $\boldsymbol{\gamma}(t)$  is the estimation  $\boldsymbol{\gamma}(t)$ ,

 $\boldsymbol{\zeta}(t)$  is the estimation of  $\boldsymbol{\zeta}(t)$  .

Note that the  $\psi(t)$  can be replaced by using the estimation value  $\psi(t)$ , which is written as follows:

$$\boldsymbol{\psi}(t) = [-y(t-1), \cdots, -y(t-n), \hat{v}(t-1), \cdots, \hat{v}(t-n)]^T$$
$$\hat{v}(t) = y(t) - \boldsymbol{\beta}^T(t) \boldsymbol{F}_{\gamma f}(t) + \boldsymbol{\psi}_f^T(t) \boldsymbol{\zeta}(t-1)$$

Then, (7), (8), (11), (12) and (17) construct an adaptive estimation scheme to estimate the parameter  $\gamma$ ; (7), (9), (13), (14) and (18) constitute an adaptive estimation approach to estimate the parameter  $\boldsymbol{\beta}$ ; (7), (10), (15), (16) and (19) construct an adaptive estimation method to estimate the parameter  $\boldsymbol{\zeta}$ .

## 4. Convergence

In this section, the convergence of the developed scheme is briefly introduced. Then, the following assumptions and theorem are offered.

Assume that  $\{v(t), \mathcal{F}_t\}$  is a bounded martingale, in which the  $\sigma$  algebra sequence  $\{\mathcal{F}_t\}$  is constituted by the noise  $\{v(s) \mid s \leq t\}$ , and the noise  $\{v(t)\}$  satisfies the following conditions (Goodwin et al, 1984): (A1)  $E[v(t) \mid \mathcal{F}_{t-1}] = 0, a.s.$ , (A2)  $E[\parallel v(t) \parallel^2 \mid \mathcal{F}_{t-1}] = \sigma_v^2(t) \leq \sigma_v^2 < \infty, a.s.$ 

(A3) 
$$\limsup_{t\to\infty}\frac{1}{t}\sum_{i=1}^t ||v(i)||^2 \leq \sigma_v^2 < \infty, a.s.$$

**Theorem 1**: For the adaptive algorithm in (7)-(22), if (A1)-(A3) hold and  $F_{\gamma f}(t)$ ,  $F_{\beta f}(t)$  and  $\psi_f(t)$  are also persistently exciting.

Then, the persistently exciting conditions are written as

$$\frac{1}{N} \sum_{j=1}^{N-1} \boldsymbol{F}_{\gamma f}(t) \boldsymbol{F}_{\gamma f}^{T}(t) \geq C_{1} \boldsymbol{I}$$
$$\frac{1}{N} \sum_{j=1}^{N-1} \boldsymbol{F}_{\beta f}(t) \boldsymbol{F}_{\beta f}^{T}(t) \geq C_{2} \boldsymbol{I}$$
$$\frac{1}{N} \sum_{i=1}^{N-1} \boldsymbol{\psi}_{f}(t) \boldsymbol{\psi}_{f}^{T}(t) \geq C_{3} \boldsymbol{I}$$

where  $C_i$ , i = 1, 2, 3 are positive constants. I denotes unit matrix. *Proof:* The similar proof of Theorem 1 can be found in (Na et al, 2014).

#### 5. Examples

In this section, the proposed scheme and several published estimation algorithms are applied to identify the parameters of the considered system.

The persistent excitation input signal is random signal u(t) with the zero mean and unit variance, while the white noise with zero mean is chosen as noise term v(t) which is irrelevant with input signal. The length of sample is set to N=1000. The initial values of the filtered variable and auxiliary models are set to 0.001. Moreover, to validate the usefulness of the proposed scheme, the

different signal-noise ratio (SNR) values are considered.

To show the advantage of the presented scheme, two popular the identification methods are chosen as comparison algorithms.

- (1) H-ESG: Extended stochastic gradient algorithm based on hierarchical identification idea in (Wang et al, 2016). The parameter estimation for multivariable CARMA systems are implemented based on the following initial parameter values. The initial parameter of γ is γ(0) = [0.1,0,0,0.48]<sup>T</sup>, β(0) = [0.299,0.664,0.00001,0.49]<sup>T</sup>, the parameter ζ(0) = 10[0,0.077,0.141,0.111]<sup>T</sup>.
- (2) Proposed algorithm in Section 3: The initial parameter can be chose as follows: k = 20, L = 80, h = 0.001,  $\Gamma = 100 diag([5.63, 0.64, 3.12, 0.5])$ , the initial parameter of  $\beta$  is  $\beta(0) = [4, 1, 2.2, 0.61]^T$ , the initial parameter of  $\gamma$  is  $\gamma(0) = [7.99, 5.71, 6.74, 7.85]^T$ , the initial parameter of  $\zeta$  can be chose as follows  $\zeta(0) = 10[0.134, 0.084, 0.121, 0.11]^T$ .
- (3) D-LS: Decomposition based recursive least squares in (Liu et al, 2014). The parameter  $\beta(0) = [0.01, 0.08, 0.02, 0.1]^T$ ,  $\gamma(0) = [0.9, 0.85, 0.9, 0.5]^T$ , the initial parameter value  $\zeta(0) = [0.000001, 0.179, 0.00001, 0.01]^T$ .  $P = 10^6 I$ .

Comparative estimation curves by the considered estimation algorithms with SNR= 2.7088 (the variance of noise is 0.01) are compared in Figs.2-4. The estimated parameters can reach their true values after 500 samples. Specifically speaking, the estimated result by proposed algorithm and H-ESG algorithm can convergence to their expected values while the D-LS method produces some serious vibrations and gives close to their true values. Compared estimation results also show that the proposed algorithm has an outstanding estimation result



**Fig.2.** Comparative estimation histories for  $\gamma$  with SNR= 2.7088



**Fig.3.** Comparative estimation histories for  $\beta$  with SNR= 2.7088



**Fig.4.** Comparative estimation histories for  $\zeta$  with SNR= 2.7088 To display the accuracy of the parameter estimation errors, the processes of the estimation errors  $\delta$  by identification algorithms

$$(\delta = \sqrt{\frac{\|\boldsymbol{\beta}(t) - \boldsymbol{\beta}\|^2 + \|\boldsymbol{\gamma}(t) - \boldsymbol{\gamma}\|^2 + \|\boldsymbol{\zeta}(t) - \boldsymbol{\zeta}\|^2}{\|\boldsymbol{\beta}\|^2 + \|\boldsymbol{\gamma}\|^2 + \|\boldsymbol{\zeta}\|^2}}) \text{ are displayed in}$$

Fig.5. It can be clearly seen that the estimation errors by identification algorithms produce the satisfactory results. Compared with the H-ESG and D-LS algorithms, the presented method produces faster convergence speed and higher estimation accuracy.



Fig.5. Comparative estimation errors with SNR= 2.7088

In order to illustrate the identification performance of three estimation approaches, the model verifications by identification algorithms are implemented based on the estimation results. Due to the limitation of the space, the model validation by presented approach is only offered. The model validation curve is shown in Fig.6. From Fig.6, we know that the predictive output can track the actual output with small model error, which shows that the proposed method produces the better model validation result and has an excellent identification performance than the H-ESG and D-LS algorithms under the other test condition constant.



#### Fig.6. model validation with SNR= 2.7088

To further demonstrate the effectiveness of the presented approach, the SNR is reduced to SNR=1.1276 (the variance of noise is 0.25), which means the considered system is added to strong noise. Parameter estimations by three identification methods are plotted in Figs.7-9. From Figs.7-9, we observe that the proposed method produces some closer to the real values while the H-ESG algorithm give close to the their real values, but D-LS algorithm has

strong oscillation phenomenon which is far away from their real values. Compared with the shown estimation results by Figs.2-4, although the estimation performance is deteriorating with the increase of noise, the parameter estimation curves by H-ESG and presented algorithm tend to near real values.



**Fig.7.** Comparative estimation for  $\gamma$  with SNR= 1.1276



**Fig.8.** Comparative estimation for  $\beta$  with SNR= 1.1276



**Fig.9.** Comparative estimation for  $\zeta$  with SNR= 1.1276

Estimation errors by the considered algorithms are depicted in Fig.10 and model validation by proposed approach is shown in Fig.11. According to Fig.10, One can find that the proposed approach has better estimation accuracy and convergence rate comparing to the H-ESG and D-LS algorithm. From Fig.11, we see that the output of proposed method can capture the actual output, which demonstrate the effectiveness of the presented algorithm.



Fig.10. Comparative estimation errors for the algorithm with SNR = 1.1276



Fig.11. Model validation for the algorithm with SNR= 1.1276

From the above comparison results, it can be seen that the proposed algorithm can achieve better results than the H-ESG and D-LS algorithms under the different SNR values.

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