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Delay-range-dependent Robust H∞ Control Method for a Linear Parameter Varying Uncertain System with Actuator Saturation and State Delay

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ABSTRACT

A robust H_∞ .control for linear parameter varying uncertain system with actuator saturation and state delay is described, in which norm-bonded parameter uncertainties are considered in this paper. An appropriate type Lyapunov functionals is used to investigate the delay-range-dependent stability problem of uncertain actuator saturation system, the conclusions of which is extended to the linear parameter varying (LPV) uncertain systems with actuator saturation. An robust H_∞ .controller can be achieved by solving the corresponding linear matrix inequalities(LMIs), which provides a reference method for control design of time-delay systems with actuator saturation.

1. Introduction

There have been considerable research efforts on robust H_∞ control problem for uncertain parameter varying systems (Moon and Park et al., 2001; Lee and Moon et al., 2004) and time-delay systems (Wu and He et al., 2004; Luo and Ge et al., 2018; Wu, 2018; Meng, 2018). The existing results method for time-delay system deal with stabilization either of delay-independence stabilization or delay-dependence stabilization, which can be can translated into problems of solve LMIs (He and Wu et al., 2004; He and Wang et al., 2007). In recent years, actuator saturation and disturbance problem was analyzed also, simple condition is derived in terms of an auxiliary feedback matrix for determining if given ellipsoid is contractively, and the condition can be expressed as LMIs (e.g., Hu and Lin et al., 2002). Based on Hu's research on saturation and the robust H_∞ control results of time-delay systems and uncertain system, the robust control problem of linear systems with actuator saturation and state delay was studied (Hu and Lin et al., 2002; Cao and Lin et al., 2002) and applied (Zhao and Sun et al., 2009; Fu and Ma et al., 2018).

In this paper, we extend the research results for linear system with actuator saturation state delay of existing literature to the LPV system which considered the uncertainties also. A Lyapunov functional is established and used to investigate the delay-range-dependent stability problem. A robust H_∞ feedback controller can be achieved by solving the corresponding LMIs. This paper is organized as follow. Section 2 is the Problem formulation of uncertain system with actuator saturation and state delay. Section 3 are the proof of robust H_∞ control theory, in which we introduce the robust H_∞ control theory for uncertain linear systems with actuator saturation and state delay firstly and then extend the result to the LPV system. Section 4 gives a concrete example according to the dynamic models of supercavitating vehicles. A brief concluding remark is given in Section 5.

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2. Problem formulation of uncertain system with actuator saturation and state delay

Let us consider the following linear parameter varying uncertain system saturation delayed system (1) as follow:

$$\begin{aligned}\dot{x}(t) &= \left[A(\varphi)+F\Delta(t)E_a\right]x(t)\\ &\quad+\left[A_d(\varphi)+F\Delta(t)E_d\right]x(t-d(t))\\ &\quad+\left[B(\varphi)+F\Delta(t)E_b\right]\sigma(u(t))+B_\omega\omega(t)\end{aligned} \tag{1}$$

where $x(t)=f(t)$, $t\in[-d_2,0]$, and the time-varying continuous function satisfies:

$$d_1 \le d(t) \le d_2 \tag{2}$$

$$\dot{d}(t) \le \mu \tag{3}$$

The uncertain matrices F, E_a, E_b, E_d are constant matrices, $\Delta(t)=diag\{\Delta_1(t),...\Delta_v(v)\}$, with $\Delta_i^T(t)\Delta_i(t) \le I$, $I=1,...v$. And the $A(\varphi)$, $A_d(\varphi)$, $B(\varphi)$ have affine dependence on a time-varying scheduling parameter vector ϕ,which are described by a polytope of vertices u_i, $i=1,2\ ...\ r$ as follow:

$$\begin{bmatrix} A(\varphi) \\ A_d(\varphi) \\ B_\omega(\varphi) \end{bmatrix} = \left\{ \sum_{i=1}^{r} \eta_i \begin{vmatrix} A(u_i) \\ A_d(u_i) \\ B_\omega(u_i) \end{vmatrix} : \eta_i > 0, \sum_{i=1}^{r} \eta_i = 1 \right\} \tag{4}$$

The saturation nonlinearity function is described by (5). where for every $i=1,2\ldots q$,

$$\sigma_i(u(t)) = \begin{cases} u_{imax} & u_i(t) \ge u_{imax} \\ u_i(t) & -u_{imax} \le u_i(t) \ge u_{imax} \\ -u_{imax} & u_i(t) \le u_{imax} \end{cases} \tag{5}$$

Our goal is to design a state-feedback control law as (6), which is asymptotically stable and satisfies H_∞ performance requirements.

$$u(t) = Kx(t) \tag{6}$$

3.Robust H_∞ controller design

3.1 Robust H_∞ design for a linear system with actuator saturation and state delay

In this section, we consider the general saturation system with time-varying delay firstly, which does not contain parameter uncertain. And we will extend the conclusions to the systems with uncertainties in the next section.

Consider system with time-varying delay and actuator saturation as follow

$$\begin{aligned} \dot{x}^T(t) &= A(t)x(t) + A_d(t)x(t-d(t)) \\ \cdots\cdots\cdots &+ B(t)\sigma(u(t)) + B_\omega(t) \end{aligned} \tag{7}$$

where the time-varying continuous function satisfies (2), (3), and the saturation nonlinearity function is described by (5). Lemma 1 utilize the technique of auxiliary feed-back matrices referring to the Hu's idea in [1], which is introduced in this in order to deal with the problem of actuator saturation.

Lemma 1. Consider system (7) with time-varying delay and actuator saturation, existing a auxiliary matrix $H \in R^{m\times n}$, and feedback control K in (6), for every (8) and (9), so that the following equations (10) and (11) hold.

$$x \in \xi(P,1) \in \psi(H) = \left\{ x \in R^n : p^T x^p \le 1 \right\} \tag{8}$$

$$\xi(P,1) \in \psi(H) = \left\{ x \in R^n : |h_i x| \le 1, i \in [1,m] \right\} \tag{9}$$

$$2x^T(t)TB\sigma(Kx(t)) \le 2\dot{x}^T(t)BW(v,K,H)x(t) \tag{10}$$

$$2\dot{x}^T(t)TB\sigma(Kx(t)) \le 2x^T(t)BW(v,K,H)x(t) \tag{11}$$

where matrix set as follow:

$$BW(v,K,H) = \{W \in R^{q\times n} : W = [v_i k_i + (1-v_i)h_i\}$$

$$v \in \psi(v), \psi(v) = \left\{ v \in R^q : v_i = 0 \ \ or \ \ v_i = 1 \right\}$$

$$\xi(P,1) = \left\{ X \in R^n : x^T P x \le 1 \right\}$$

and k_i, h_i is the ith row of K, H.

Proof. Lemma 1 follow Hu's idea, and the proof similar with Hu's theorem 1 in literature [1].

Theorem 1. Consider system with time-varying delay and actuator saturation in (7), where the time-varying continuous function satisfies (2), (3), and the saturation nonlinearity function is described by (5). Given scalars $\gamma > 0$, $\rho > 0$, $0 \le d_1 \le d_2$, $\mu > 0$ and let $d_{12} = d_2 - d_1$. Then the closed-loop control in (6) is asymptotically stable and satisfies $\|Z_\omega \infty\| < \gamma$ for all nonzero $\omega \in L_2[0,\infty)$ under zero initial condition if there exist matrices $P > 0$, $T > 0$ $Q_i > 0$, $i=1,2,3$, N_i, S_i, M_i, $i=1,2,3,4,5$, $W(v,K,H)$, satisfying (12).

$$\Pi_H = \begin{bmatrix} \hat{\Pi} & d_2 N & d_{12} S & d_{12} N \\ * & -d_2 N & 0 & 0 \\ * & * & -d_1 d_2 (Z_1 + Z_2) & 0 \\ * & * & * & -d_{12} Z_2 \end{bmatrix} \tag{12}$$

where,

$$\hat{\Pi} = \begin{bmatrix} \Pi_{11} + C_1^T C_1 & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & \Pi_{16} \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} & 0 \\ * & * & \Pi_{33} & \Pi_{34} & \Pi_{35} & 0 \\ * & * & * & \Pi_{44} & \Pi_{45} & 0 \\ * & * & * & * & \Pi_{55} & \Pi_{56} \\ * & * & * & * & * & -\gamma^2 \end{bmatrix}$$

$$N^T = \left[N_1^T, N_2^T, N_3^T, N_4^T, N_5^T, 0 \right]$$

$$M^T = \left[M_1^T, M_2^T, M_3^T, M_4^T, M_5^T, 0 \right]$$

$$S^T = \left[S_1^T, S_2^T, S_3^T, S_4^T, S_5^T, 0 \right]$$

$$\Pi_{11}=\sum_{i=1}^{3}Q_i+N_1+N_1^T+AT+TBW_v+\left(AT+TBW_v\right)^T$$
$$\Pi_{12}=N_2^T-N_1+S_1-M_1+A_dT$$
$$\Pi_{13}=N_3^T+M_1$$
$$\Pi_{14}=N_4^T-S_1$$
$$\Pi_{15}=P-T+N_5^T+\left(AT+TBW_v\right)^T$$
$$\Pi_{16}=TB_\omega$$
$$\Pi_{22}=(\mu-1)Q_3-2N_2-2N_2^T-M_2-M_2^T+S_2+S_2^T$$
$$\Pi_{23}=-N_3^T+S_3^T+M_2-M_2^T$$
$$\Pi_{24}=-N_4^T+S_4^T+M_2-M_4^T$$
$$\Pi_{25}=TA_d-N_5^T+S_5^T-S_2-M_5^T$$
$$\Pi_{33}=-Q_1+M_3+M_3^T$$
$$\Pi_{34}=-S_3+M_4^T$$
$$\Pi_{35}=M_5^T$$
$$\Pi_{44}=-Q_2-S_4-S_4^T$$
$$\Pi_{45}=-S_5^T$$
$$\Pi_{55}=d_2Z_1+d_{12}Z_2-2T$$
$$\Pi_{56}=TB_\omega$$

Proof. Construct a Lyapunov functional for the system as (13)

$$\begin{aligned}V(t)&=x^T(t)Px(t)+\int_{t-d_1}^{t}x^T(s)Q_1x(s)ds\\&+\int_{t-d_2}^{t}x^T(s)Q_2x(s)ds\\&+\int_{t-d_3}^{t}x^T(s)Q_3x(s)ds\\&+\int_{-d_2}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z_1\dot{x}(s)dsd\theta\\&+\int_{-d_2}^{-d_1}\int_{t+\theta}^{t}\dot{x}^T(s)Z_1\dot{x}(s)dsd\theta\end{aligned}\tag{13}$$

where $P>0$, $Q_i>0$, $i=1,2,3$, $Z_i>0$, $i=1,2$ are matrices to be determined. Then we have

$$\begin{aligned}\dot{V}(t)&=\dot{x}^T(t)Px(t)+x^T(t)P\dot{x}(t)+x^T(t)Q_1x(t)\\&-x^T(t-d_1)Q_1x(t-d_1)+x^T(t)Q_2x(t)\\&-x^T(t-d_2)Q_2x(t-d_2)+x^T(t)Q_3x(t)\\&-\left(1-\dot{d}(t)\right)x^T(t-d(t))Q_3x(t-d(t))\\&+d_2\dot{x}^T(t)Z_1\dot{x}(t)-\int_{t-d_2}^{t}\dot{x}^T(s)Z_1\dot{x}(s)ds\\&+d_{12}\dot{x}^T(t)Z_2\dot{x}(t)-\int_{t-d_2}^{t-d_1}\dot{x}^T(s)Z_2\dot{x}(s)ds\end{aligned}$$

Let

$$\zeta(t)=\begin{bmatrix}x(t)\\x(t-d(t))\\x(t-d(1))\\x(t-d(2))\\\dot{x}(t)\\\omega(t)\end{bmatrix}$$

Note that the following equations are true for any appropriately dimensioned matrices $T>0$ and N, S, M, $i=1,\dots,5$

$$2\zeta^T(t)\left[x(t)-x(t-d(t))-\int_{t-d}^{t}\dot{x}(s)ds\right]=0$$
$$2\zeta^T(t)S\left[x(t-d(t))-x(t-d_2)-\int_{t-d_2}^{t-d(t)}\dot{x}(s)ds\right]=0$$
$$2\zeta^T(t)M\left[x(t-d_1)-x(t-d(t))-\int_{t-d(t)}^{t-d_1}\dot{x}(s)ds\right]=0$$
$$2\left[x^T(t)T+\dot{x}^T(t)T\right]\begin{bmatrix}-\dot{x}(t)+Ax(t)+A_dx(t-d(t))\\+B\sigma(Kx(t))+B_\omega w(t)\end{bmatrix}=0$$

Thanks to (10) and (11) in lemma 1, for every $x\in\xi(P,1)$ (10) holds,

$$\begin{aligned}&2[x^T(t)T+\dot{x}(t)T]\begin{bmatrix}-\dot{x}(t)T+Ax(t)+A_dx(t-d(t))\\+B\sigma(Kx(t))+B_\omega w(t)\end{bmatrix}\\&\le x^T(t)T\begin{bmatrix}-\dot{x}(t)+Ax(t)+A_dx(t-d(t))+\\A_dx(t-d(t))+\mathrm{BW}(v,\mathrm{K},\mathrm{H})+B_\omega w(t)\end{bmatrix}\\&+2\dot{x}^T(t)T\begin{bmatrix}-\dot{x}(t)+Ax(t)+A_dx(t-d(t))+\\A_dx(t-d(t))+\mathrm{BW}(v,\mathrm{K},\mathrm{H})+B_\omega w(t)\end{bmatrix}\end{aligned}\tag{14}$$

Then using these relations (14) and some algebraic manipulations, we can obtain the following inequality.

$$\begin{aligned}\dot{V}(t)&\le\zeta^T(t)\left(\Pi+d_2NZ_1^{-1}N^T+d_{12}SZ_3^{-1}S^T+d_{12}MZ_2^{-1}M^T\right)\zeta(t)\\&-\int_{t-d(t)}^{t}\left[\zeta^T(t)N+\dot{x}(s)Z_1\right]Z_1^{-1}[\zeta^T(t)N+\dot{x}(s)Z_1]^Tds\\&-\int_{t-d(T)}^{t-d_1}\left[\zeta^T(t)M+\dot{x}^T(s)Z_2\right]Z_2^{-1}[\zeta^T(t)M+\dot{x}^T(s)Z_2]^Tds\\&-\int_{t-d_2}^{t-d(t)}\left[\zeta^T(t)S+\dot{x}^T(s)Z_3\right]Z_3^{-1}[\zeta^T(t)S+\dot{x}^T(s)Z_3]^Tds\end{aligned}$$

where, $Z_3=Z_1+Z_2$.

Let,

$$\tilde{\Pi}=\Pi+d_2NZ_1^{-1}N^T+d_{12}SZ_3^{-1}S^T+d_{12}MZ_2^{-1}M^T\tag{15}$$

where,

$$\Pi=\begin{bmatrix}\Pi_{11}&\Pi_{12}&\Pi_{13}&\Pi_{14}&\Pi_{15}&\Pi_{16}\\ *&\Pi_{22}&\Pi_{23}&\Pi_{24}&\Pi_{25}&0\\ *&*&\Pi_{33}&\Pi_{34}&\Pi_{35}&0\\ *&*&*&\Pi_{44}&\Pi_{45}&0\\ *&*&*&*&\Pi_{55}&\Pi_{56}\\ *&*&*&*&*&0\end{bmatrix}$$

Because the last three integrals in (15) is all less than 0, we obtain (16)

$$\dot{V}(t) \le \zeta^T(t)\tilde{\Pi}\zeta^T(t) \tag{16}$$

By the Schur complement, (16) is equivalent to (17)

$$\begin{bmatrix} \Pi & d_2N & d_{12}S & d_{12}M \\ * & -d_2Z_1 & 0 & 0 \\ * & * & -d_{12}(Z_1+Z_2) & 0 \\ * & * & * & -d_{12}Z_2 \end{bmatrix} \le 0 \tag{17}$$

Next, we shall establish the H∞ performance of system under zero initial condition. Consider the following index:

$$J = \int_0^\infty \left[Z^T(t)Z(t) - \omega^T(t)\omega(t) \right] \mathrm{d}t$$

Then, we have (14), for any nonzero $\omega \in L_2[0,\infty)$.

$$J \le \int_0^\infty \left[Z^T(t)Z(t) - \omega^T(t)\omega(t) + \dot{V}(t) \right] \mathrm{d}t \tag{18}$$

and

$$Z^T(t)Z(t) - \omega^T(t)\omega(t) + \dot{V}(t) \le \zeta^T(t)\Pi_H\zeta(t) \tag{19}$$

If $\Pi_H < 0$ we have $J < 0$, i.e. $Z_\omega\infty < \gamma$. Hence H_∞ is guaranteed for all nonzero $\omega \in L_2[0,\infty)$. Under zero initial condition, and the performance of system is established.

3.2 Robust H_∞ design for parameter uncertainties systems with actuator saturation and state delay

In this section, we will extend the conclusions of the previous section to the parameter uncertainty system.

consider parameter uncertainties system with saturation delayed as (20),

$$\begin{aligned} \dot{x}(t) &= \left[A(t) + F\Delta(t)E_a \right] x(t) \\ &\quad + [A_d(t) + F\Delta(t)E_d] x(t - d(t)) \\ &\quad + \left[B(t) + F\Delta(t)E_b \right] \sigma_1(u(t)) + B_\omega(t) \end{aligned} \tag{20}$$

where the time-varying continuous function satisfies (2), (3), and the saturation nonlinearity function is described by (5). $x(t) = \phi(t)$, $t \in [-d_2, 0]$ and F, E_a, E_b, E_d are constant matrices, and $\Delta(t) = \mathrm{diag}\{\Delta_1(t),...\Delta_1(v)\}$, with $\Delta_i^T(t)\Delta_i(t) \le I$, $I = 1,...v$ denotes time-varying. Before proceeding further, we introduce lemma 2 in this in order to deal with the parameter uncertainties, which will be used in proof of Theorem 3.

lemma 2. Let F, E_a, E_b, E_d, be real matrices of appropriate dimensions, then for any real matrix $\Lambda = diag\{\lambda_1 I, ..., \lambda_r I\}$, inequality (21) holds.

$$F\Delta E + E^T\Delta^T F^T + F\Lambda F^T + E^T\Lambda^{-1}E \tag{21}$$

Lemma2 holds because of the fact:

$$\left(\Lambda^{\frac{1}{2}}F^T - \Lambda^{\frac{1}{2}}\Delta E \right)^T \left(\Lambda^{\frac{1}{2}}F^T - \Lambda^{\frac{1}{2}}\Delta E \right) \ge 0 \tag{22}$$

Theorem 2. Consider parameter uncertainties system with time-varying delay and actuator saturation as (20), Given scalars $\gamma > 0$, $\rho > 0$ $0 \le d_1 \le d_2$, $\mu > 0$, and let $d_{12} = d_2 - d_1$, the time-varying continuous function satisfies (2) and (3), Then the closed-loop system (6) is asymptotically stable and satisfies $\|Z_\omega\infty\| < \gamma$ for all nonzero $\omega \in L_2[0,\infty)$ under zero initial condition if there exist matrices $P > 0$, $T > 0$, $Q_i > 0$, $i = 1,2,3$ N_i, S_i, M_i, $i = 1,2,3,4,5$, $W(v,K,H)$, $W(v,K,H)$ and Diagonal matrix $\Lambda > 0$ satisfying (23).

$$\begin{bmatrix} f_{11} & \bar{E} \\ * & -\Lambda \end{bmatrix} < 0 \tag{23}$$

where

$$\begin{aligned} f_{11} &= \Pi_H + \bar{F}\Lambda\bar{F}^T \\ \bar{E} &= E_1^T + E_1^T \\ \bar{F} &= F_1 + F_2 \\ E_1 &= \left[E_aT + TE_bW_v \;\; E_dT \;\; 0\;0\;0\;0 \right] \\ E_2 &= \left[E_aT + TE_bW_s \;\; E_dT \;\; 0\;0\;0\;0 \right] \\ F_1 &= \left[F^T \;\; 0\;0\;0\;0\;0 \right] \\ F_2 &= \left[0\;0\;0\;0\; F^T\; 0 \right] \end{aligned}$$

Proof. Replaced the parameter of Theorem1as follow:

$$\begin{aligned} A_d &= A_d(t) + F\Delta(t)E_d \\ B &= B(t) + F\Delta(t)E_b \end{aligned}$$

Then the system (20) is asymptotically stable, only if,

$$\Pi_H + \overline{F_1}\Delta\overline{E_1} + \bar{E}_1^T + \Delta\bar{F}_2^T + \overline{F_2}\Delta\overline{E_2} + \bar{E}_1^T\Delta\bar{F}_2^T \tag{24}$$

$$\Pi_H + \bar{F}\Lambda\bar{F}^T + \bar{E}_1^T + \Delta\bar{F}_2^T + \overline{F_2}\Delta\overline{E_2} + \bar{E}_1^T\Delta\bar{F}_2^T \tag{25}$$

By the Schur complement, (24) and (25) is equivalent to (23), this is the complete proof.

3.3 Robust H_∞ control of linear parameter varying uncertain system with actuator saturation and state delay

In this section, we extend the result of theorem 1 and 2 to the systems linear parameter varying uncertain system with actuator saturation and state delay. We consider the general linear parameter varying saturation system with time-varying delay which does not contain parameter uncertain in theorem 3 firstly, and we will extend the conclusions to the systems with uncertainties in the theorem 4.

Consider the time-delay LPV system (26) with affine dependence on scheduling variables.

$$\dot{x}(t) = A(\varphi) + A_d x(t - d(t)) + B\sigma(u(t)) + B_\omega(\varphi)\omega(t) \quad (26)$$

where the time-varying continuous function satisfies (2), (3), and the saturation nonlinearity function is described by (5). $x(t) = f(t), t \in [-d_2, 0]$, And the $A(\varphi)$, $A_d(\varphi)$, $B(\varphi)$, have affine dependence on a time-varying scheduling parameter vector ϕ ,which are described by a polytope of vertices u_i , $i = 1,2 \ldots r$ as (4).

Theorem 3. Consider system with time-varying delay and actuator saturation in (26), Given scalars $\gamma > 0$, $\rho > 0$, $0 \le d_1 \le d_2$, $\mu > 0$, and let $d_{12} = d_2 - d_1$, the time-varying continuous function satisfies (2) and (3), Then the closed-loop control (5) is asymptotically stable and satisfies $Z_\omega \infty < \gamma$ for all nonzero $\omega \in L_2[0, \infty)$ under zero initial condition if there exist matrices $P > 0$, $T > 0$, $Q_i > 0$, $i = 1,2,3$. N_i , S_i , M_i , $i = 1,2,3,4,5$, $W(v, K^i, H)$, $W(v, K^i, H)$. For every μ_i satisfying (12) with $A \to A(\mu_i)$, $A_d \to A_d(\mu_i)$, $B_\omega \to B_\omega(\mu_i)$ and $K = \sum_{i=1}^{r} \eta_i K^i$, where η_i were defined in equation (4).

Proof. Note that the system (26) have affine dependence on scheduling parameters $A(\varphi)$, $A_d(\varphi)$, $B_\omega(\varphi)$, the matrix inequalities in (12) have affine dependence on A , A_d , B_ω , and the Variable relations satisfying (5). Then, replace with $A \to A(\varphi), A_d(\varphi) \to A_d(\varphi)$, and $B_\omega \to B_\omega(\mu_i)$, equations (31) are convex combinations of the vertex inequalities, which are defined vertex values ($A(\mu_i), A_d(\mu_i), B_\omega(\mu_i)$) of the parameter vector ($A(\varphi)$, $A_d(\varphi)$, $B_\omega(\psi)$). so if (8) hold for every ($A(\mu_i), A_d(\mu_i), B_\omega(\mu_i)$) $i = 1,2,\ldots,r$,it will be hold for ($A(\mu_i), A_d(\mu_i), B_\omega(\mu_i)$), and then the system (26) is asymptotically stable and satisfies $Z_\omega \infty < \gamma$ for all nonzero $\omega \in L_2[0, \infty)$ under zero initial condition.

In next theorem 4, we'll extend the result of theorem 3 to the parameter uncertain LPV system with actuator saturation and state delay as (1)

Theorem 4. Consider system with time-varying delay and actuator saturation in (1), Given scalars $\gamma > 0$, $\rho > 0$, $0 \le d_1 \le d_2$, $\mu > 0$, and let $d_{12} = d_2 - d_1$. Then the closed-loop control (6) is asymptotically stable and satisfies $Z_\omega \infty < \gamma$ for all nonzero $\omega \in L_2[0, \infty)$ under zero initial condition if there exist matrices $P > 0$, $T > 0$, $Q_i > 0$, $i = 1,2,3$ N_i , S_i , M_i , $i = 1,2,3,4,5$, $W(v, K^i, H)$, $W(v, K^i, H)$, $i = 1,2,\ldots,r$, for every μ_i satisfying (23) with $A \to A(\mu_i)$, $A_d \to A_d(\mu_i)$, and $B_\omega \to B_\omega(\mu_i)$. and $K = \sum_{i=1}^{r} \eta_i K^i$, where η_i were defined in equation (4).

The proof is straightforward by applying theorem 2 and theorem 3.

4. Example

This section uses the supercavitation vehicle as an example to verify theorem 4.

4.1. The dynamic model of supercavitation vehicle

Supercavitation is a higher stage of cavitation, and it is a hydrodynamic process by which an underwater body is almost entirely enveloped in a layer of gas. Figure 1 shows a supercavitating vehicle traveling underwater. It can be seen that the whole of the vehicle is surrounded by a cavity. Only small regions of the vehicle are in contact with water. The condition is similar to traveling in air. Because the density and viscosity of the gas is dramatically lower than that of water, a supercavitating vehicle with proper design is able to achieve a tremendous reduction in skin friction drag and exhibit very high speed under water.

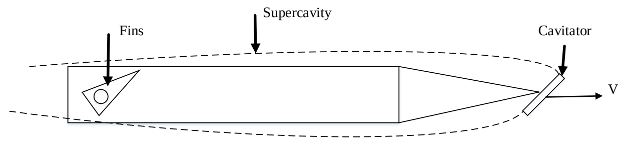

Fig. 1. Supercavitating Vehicle

Different from fully-wetted vehicles, a supercavitating vehicle is enveloped by gas surface and absence of the buoyant. The pressure center of the vehicle is typically located well forward with respect to the center of gravity. This requires a particular method to effecting hydrodynamic control. Control of the supercavitating vehicle presents a number of special challenges.

In the literature the supercavitating vehicle dynamic models were developed in the constant velocity condition based on dynamics of supercavity and hydrodynamic force of vehicle (Shao and Mesbahi et al., 2003; Vanek and Bokor et al., 2007; Vanek and Bokor et al., 2006). Despite these success, no model of Delay-range-dependent robust H_∞ control method for a linear parameter varying uncertain system with actuator saturation and state delay problem for supercavitating vehicles is available in the open literature.

The equation of state is $x = [z, \omega, \theta, q]$ and the equations of motion for the pitch-plane dynamics of the supercavitating vehicle from Dzielski and Kurdila (2003) are given as follows:

$$\dot{Z} = \omega - V\theta \quad (27)$$

$$\dot{\theta} = q \quad (28)$$

$$M_I = A_I \begin{bmatrix} \dot{\omega} \\ \dot{q} \end{bmatrix} + B_I \begin{bmatrix} \delta_f \\ \delta_c \end{bmatrix} + F_{grav}^{\wedge} + F_{plane}^{\wedge} \begin{bmatrix} 1 \\ L \end{bmatrix} \quad (29)$$

where

$$A_I = CV \begin{bmatrix} \dfrac{-n}{mL} & \dfrac{-n}{m} + \dfrac{7}{9C} \\ \dfrac{-n}{m} & \dfrac{-nL}{n} + \dfrac{17}{36C} L \end{bmatrix}$$

$$B_I = CV^2 \begin{bmatrix} \frac{-n}{mL} & \frac{1}{mL} \\ \frac{-n}{m} & 0 \end{bmatrix} \tag{30}$$

$$M_I = \begin{bmatrix} \frac{7}{9} & \frac{17}{36}L \\ \frac{17}{36}L & \frac{11}{60}R^2 + \frac{133}{405}L^2 \end{bmatrix}$$

$$F_{grav}^{\wedge} = \begin{bmatrix} \frac{7}{9} \\ \frac{17}{36}L \end{bmatrix} \tag{31}$$

and $F_{plane}^{\wedge}$ is the normalized value of the planing force F_{plane} defined as follows:

$$\begin{aligned} F_{plane}^{\wedge} &= \frac{F_{plane}}{\pi\rho m R^2 L} \\ &= -\frac{V^2}{mL}\left(\frac{1+h'}{1+2h'}\right)\left[1-\left(\frac{R'}{h'+R'}\right)^2\right]\alpha_{plane} \end{aligned} \tag{32}$$

(z - depth; v - velocity level; ω - vertical velocity; θ - angle of pitch; q - rate of pitch; δ_c - angle of cavitator; δ_f - angle of fins.)

where

$$\begin{aligned} C &= \frac{1}{2}C_x\left(\frac{R_n}{R}\right)^2 \\ &= \frac{1}{2}C_{x0}\left(1+\sigma\right)\left(\frac{R_n}{R}\right)^2 \end{aligned} \tag{33}$$

$$R' = \frac{R_c - R}{R}$$

$$R_c = R_n\left[0.82\frac{(1+\sigma)}{\sigma}\right]^{\frac{1}{2}} K_2 \tag{34}$$

$$\begin{aligned} K_1 &= \frac{L}{R_n}\left(\frac{1.92}{\sigma}-3\right)^{-1} - 1 \\ K_2 &= \left[1-\left(1-\frac{1.5\sigma}{1+\sigma}\right)K_1^{\frac{40}{17}}\right]^{\frac{1}{2}} \end{aligned} \tag{35}$$

Considering the memory effect of cavity–vehicle interaction, the immersion depth h' and planing angle α_{plane} are functions of both instant- and delayed-state variables, and they are modeled in Vanek et al. (2007) as follows:

$$h' = \begin{cases} \frac{1}{R}\left[z(t)+\theta(t)L - z(t-\tau) + R - R_c\right] & \text{bottom contant} \\ 0 & \text{inside cavity} \\ \frac{1}{R}\left[R - R_c - z(t) - \theta(t)L + z(t-\tau)\right] & \text{top contact} \end{cases} \tag{36}$$

$$\alpha_{plane} = \begin{cases} \theta(t) - \theta(t-\tau) + \frac{\omega(t-\tau) - \dot{R}_c}{V} & \text{bottom contant} \\ 0 & \text{inside cavity} \\ \theta(t) - \theta(t-\tau) + \frac{\omega(t-\tau) - \dot{R}_c}{V} & \text{top contact} \end{cases} \tag{37}$$

with the following planing conditions:

$$\begin{cases} \text{bottom contant} & if\ R_c - R < z(t) + \theta(t)L - z(t-\pi) \\ \text{inside cavity} & \text{oterwise} \\ \text{top contact} & if\ R_c - R > z(t) + \theta(t)L - z(t-\pi) \end{cases} \tag{38}$$

where $\tau = \frac{L}{V}$.

Order $\lambda_1(h') = \left(\frac{1+h'}{1+2h'}\right)\left[1-\left(\frac{R'}{h'+R'}\right)^2\right]$

We further define

$$\lambda_2 = \begin{cases} -\frac{\dot{R}_c}{V} & \text{bottom contact} \\ 0 & \text{inside cavity} \\ \frac{\dot{R}_c}{V} & \text{top contact} \end{cases} \tag{39}$$

$$\lambda_3 = z(t) + \theta(t)L - z(t-\tau) \tag{40}$$

and define variables

$$\pi_1 = \frac{V^2}{mL}\lambda_1 \text{ and } \pi_2 = \frac{\lambda_2}{\lambda_3} \tag{41}$$

If planing occurs, i.e., there is bottom contact or top contact, by the definitions of λ_1, λ_2, λ_3, π_1 and π_2, we have

$$\begin{aligned} F_{plane}^{\wedge} &= -\frac{V^2}{mL}\left(\frac{1+h'}{1+2h'}\right)\left[1-\left(\frac{R'}{h'+R'}\right)^2\right]\alpha_{plane} \\ &= -\pi_1\alpha_{plane} \\ &= -\pi_1\left[\theta(t) - \theta(t-\tau) + \frac{\omega(t-\tau)}{V} + \lambda_2\right] \\ &= -\pi_1\left\{\theta(t) - \theta(t-\tau) + \frac{\omega(t-\tau)}{V} + \right. \\ &\qquad \left. \pi_2\left[z(t) - \theta(t)L - z(t-\tau)\right]\right\} \end{aligned} \tag{42}$$

$$F_{plane}^{\wedge} = \begin{bmatrix} -\pi_3 \\ -\pi_1 - \pi_3 L \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} z(t) \\ \theta(t) \\ \omega(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} \pi_3 \\ \pi_1 \\ -\frac{\pi_1}{V} \\ 0 \end{bmatrix}\begin{bmatrix} z(t-\tau) \\ \theta(t-\tau) \\ \omega(t-\tau) \\ q(t-\tau) \end{bmatrix} \tag{43}$$

The LPV model with time-delay is obtained. The $\sigma(c)$ and $\sigma(f)$ should not be too large, let's say 25°. And the supercavitating vehicle model can be expressed as

$$\dot{x}(t) = Ax(t) + A_d x(t-\tau) + B\sigma(u) \tag{44}$$

with $x=\begin{bmatrix} z & \theta & \omega & q\end{bmatrix}^T$ and $u=\begin{bmatrix}\sigma_c & \sigma_f\end{bmatrix}^T$

$$A=\begin{bmatrix} \begin{matrix}0 & 0\\ 0 & 0\end{matrix} & \begin{matrix}0 & 0\\ 0 & 0\end{matrix} \\ M_I^{-1}\begin{bmatrix}\pi_3 & \pi_1 & \dfrac{-\pi_1}{V}\\ -\pi_3 L & \pi_1 L & \dfrac{-\pi_1 L}{V}\end{bmatrix} & \begin{matrix}0 & 0\\ 0 & 0\end{matrix}\end{bmatrix} + \begin{bmatrix}0 & -V & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & & \\ 0 & 0 & \multicolumn{2}{c}{M_I^{-1}A_I}\end{bmatrix} \tag{45}$$

$$A_d=\begin{bmatrix} & 0 & 0 & 0 & 0\\ & 0 & 0 & 0 & 0\\ M_I^{-1}\begin{bmatrix}\pi_3 & \pi_1 & \dfrac{-\pi_1}{V}\\ -\pi_3 L & \pi_1 L & \dfrac{-\pi_1 L}{V}\end{bmatrix} & & & & \begin{matrix}0\\0\end{matrix}\end{bmatrix} \tag{46}$$

$$B=\begin{bmatrix}0 & 0\\ 0 & 0\\ M_I^{-1} & B_I\end{bmatrix} \tag{47}$$

The parameters are shown in table 1

Tab 1. System parameters for simulations

Parameter	Value and units
g	9.81 m s^{-2}
m	2
n	0.5
R_n	0.0191 m
R	0.0508 m
R_c	0.0902 m
$\dot{R}_C$	-3.2965
L	1.8 m
V	75 m s^{-1}
σ	0.03
C_{xO}	0.82

4.2. Control design

According to section 4.1, a time-delay saturated system is obtained. Among them, the supercavitation vehicle is disturbed by uncertain factors such as ocean current, such as formula (23) in theorem 2.

According to theorem 4, using LMI toolbox in matlab to calculate the k value.

$$\begin{aligned} K^1&=\begin{bmatrix}-0.0197 & -0.0292 & 0.8953 & 0.9856\\ -0.0064 & -0.0075 & 0.2899 & 0.2153\end{bmatrix}\\ K^2&=\begin{bmatrix}18.8030 & 5.2158 & 9.7020 & 5.6036\\ 3.8567 & 1.3030 & 14.4852 & 1.4916\end{bmatrix}\\ K^3&=\begin{bmatrix}-0.4280 & -3.6069 & 308.7058 & 5.6036\\ -0.1451 & -1.2668 & 104.7102 & 0.4667\end{bmatrix}\\ K^4&=\begin{bmatrix}-34.2192 & 1.4684 & 651.5831 & 6.7270\\ -15.5092 & -0.2262 & 226.5051 & 1.7827\end{bmatrix}\end{aligned} \tag{48}$$

and the parameters η_i can be calculated as

$$\begin{aligned}\eta_1&=(1-\beta_1)(1-\beta_2)\\ \eta_2&=(1-\beta_1)\beta_2\end{aligned} \tag{49}$$

$$\begin{aligned}\eta_3&=\beta_1(1-\beta_2)\\ \eta_4&=\beta_1\beta_2\end{aligned} \tag{50}$$

with β_1 and β_2 defined as

$$\beta_1=\frac{\pi_1}{865.625}\ ,\ \beta_2=\frac{\pi_3}{966.73} \tag{51}$$

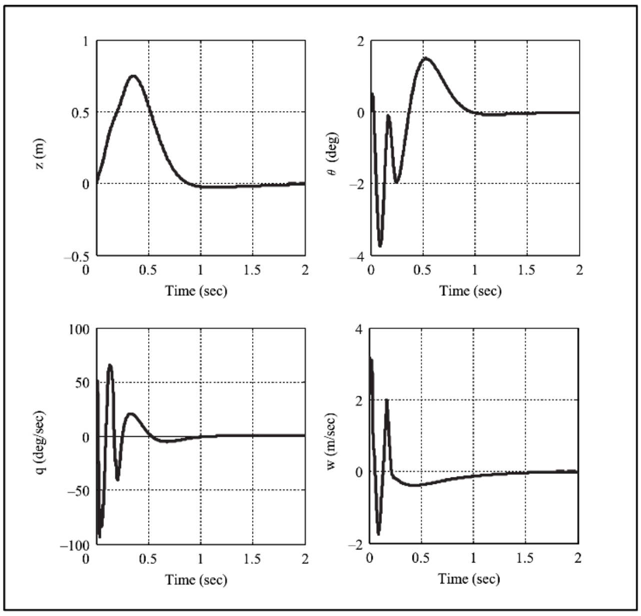

Fig. 2. System control input curve

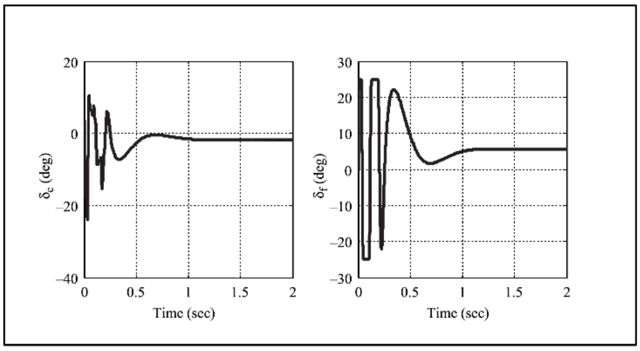

Fig. 3. System state response curve

The simulink simulation is shown below under the control rate of (48). In the initial value state, the control input signals are shown in Figure 2. And the input signals approach stability within 1 second above. The system response curve is shown in figure 3. We draw a conclusion from it that the system can operate stably at 25° saturation.

5.Summary

The control synthesis problem for a linear parameter varying uncertain system with actuator saturation and state delay is investigated in this paper. A delay-range-dependent Lyapunov function and auxiliary feedback have be considered to guaranteed a closed-loop performance, and the H_∞ performance has been established via a Lyapunov approach. The robust H_∞ controller can be achieved by solving the corresponding LMIs, which provides a reference method for control design of time-delay systems with actuator saturation.

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References

Moon,Y.,S., Park,P., Kwon,W.,H., 2001. Delay-dependent robust control stabilization of uncertain state-delay syatems. J.INT,Control. 14,1447-1455.

Lee,Y.,S.,Moon,Y.,S.,Kwon,W.,H., 2004. Delay-dependent robust H-inf control for uncertain systems with a state-delay. J.Automatica. 40,65-72.

Wu, M., He, Y., She, J. H., Liu, G. P. 2004. Technical communique: delay-dependent criteria for robust stability of time-varying delay systems.Automatica.40(8), 1435-1439.

Luo, Ge, Wang, Guan. 2018. Time Delay Estimation-based Adaptive Sliding-Mode Control for Nonholonomic Mobile Robots. J.International Journal of Applied Mathematics in Control Engineering, 1-8.

Wu. 2018. Parametric Lyapunov Equation Approach to Stabilization and Set Invariance Conditions of Linear Systems with Input Saturation. J.International Journal of Applied Mathematics in Control Engineering, 79-84.

Meng. 2018. Adaptive Parameter Estimation for Multivariable Nonlinear CARMA Systems. J.International Journal of Applied Mathematics in Control Engineering, 96-102.

He,Y.,Wang,Q.,G.,Lin,C.,2007. Delay-dependent stability for systms with time-varying delay. J.Automatica. 43,371-376.

Hu, T.S., Lin, Z.L.,Chen,B.M.,2002. An analysis and design method for linear systems subject to actuator saturation and disturbance. J.Automatica. 38,351-359.

Hu, T., Lin, Z., Chen, B. M. 2002. Brief An analysis and design method for linear systems subject to actuator saturation and disturbance. Pergamon Press, Inc.

Cao, Y. Y., Lin, Z., Hu, T. 2002. Stability analysis of linear time-delay systems subject to input saturation.J. IEEE Transactions on Circuits & Systems I Fundamental Theory & Applications.9(2), 233-240.

Zhao,Y.,B.,Sun,.W.,C.,Gao,H.,J.,2009. Robust control synthesis for seat suspension systems with actuator saturation and time-varying input delay.J.Journal of Sound & Vibration.329(21), 4335-4353.

Fu, L., Ma, Y. 2018. H-inf memory feedback control for uncertain singular markov jump systems with time-varying delay and input saturation.J.Computational & Applied Mathematics. 3, 1-24.

J. Dzielski, A. Kurdila. A Benchmark Control Problem for Supercavitating Vehicles and an Initial Investigation of Solutions. Journal of Vibration and Control, 2003, 12: 791 804

Shao, Y., Mesbahi, M., and Balas, G., 2003. 'Planing, switching, and supercavitating flight control, in Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit, AIAA 2003-5724, Austin, TX, USA.

Vanek, B., Bokor, J., Balas, G., and Arndt, R., 2007. Longitudinal motion control of a high-speed supercavitation vehicle, Journal of Vibration and Control 13(2), 159–184.

Vanek, B., Bokor, J., and Balas, G., 2006. Theoretical aspects of high-speed supercavitation vehicle control, in Proceedings of the American Control Conference, Minneapolis, MN, USA, pp. 5263–5268.

Vanek, B., Bokor, J., and Balas, G., 2006. High-Speed Supercavitation Vehicle Control. AIAA Guidance, Navigation, and Control Conference and Exhibit. Keystone, Cotorado, 6446

Dzielski, J. and Kurdila, A., 2003, A benchmark control problem for supercavitating vehicles and an initial investigation of solutions, Journal of Vibration and Control 9(7), 791–804.

Vanek, B., Bokor, J., Balas, G., and Arndt, R., 2007, Longitudinal motion control of a high-speed supercavitation vehicle, Journal of Vibration and Control. 13(2), 159–184.

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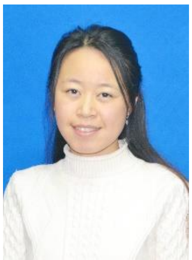

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