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Tracking Differentiator Based LOS Rate Estimation for Strapdown Semi-Active Laser Seeker Systems

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ABSTRACT

This paper deals with tracking differentiator (TD) based line-of-sight (LOS) rate estimation for strapdown seeker systems. Strapdown seeker systems need digital calculation method to get the LOS rate in terms of boresight error and attitude. In order to solve noise amplification in boresight error's differential during the calculating process, TD is applied in the LOS rate extraction algorithm. A first-order filter is selected as contrast according to the seeker requirements. The simulation results verify the effectiveness of the estimation algorithm and the results are valuable in noise suppression and engineering application.

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1. Introduction

Strapdown systems are nowadays commonly used in commercial and military applications (aircraft, ships, missiles, etc.). State-of-the-art strapdown systems are based upon Ring Laser Gyroscopes (e.g., Banerjee et al., 2004), Fibre Optic Gyroscopes (e.g., Hakimi et al. 2013) or Hemispherical Resonator Gyroscopes (e.g., Pai et al. 2012). They are using digital electronics and advanced digital filtering techniques.

The Strapdown seeker system eliminates the mechanical complexity of the platform system by having the sensors attached rigidly, or 'strapped down', to the body of the missile. Since line-of-sight (LOS) rate cannot be measured directly, it is usually estimated from some additional measurable quantities based on the dynamic model of LOS rate (e.g., Song et al., 2008; Titterton et al., 2004). The strapdown seeker measures boresight error, which contains LOS and attitude information. Therefore, it is important for the LOS rate computing scheme to dispose seeker's and gyroscope's measurement in the missile.

In recent years, research on sensor signal processing has become a hot topic, and many methods such as intelligent filters (e.g., Jin et al. 2018), multi-sensor fusion (e.g., Zhao et al. 2018), and observer (e.g., Wei et al. 2018) have made progress. Some scholars have studied LOS rate acquisition for strapdown seeker systems. In some research, the extracted LOS signal is differentiated, and filters are

taken to process differential signals. Kalman filtering is widely used in signal processing and has many different types of applications, such as Extended Kalman Filter (EKF) (e.g., Wang et al., 2016; Ananthasayanam et al., 2004), Unscented Kalman Filter (UKF) (e.g., Rui et al., 2014; Dwivedi et al., 2013) and Particle Filter (PF) (e.g., Zhang et al., 2010). In (Sadhu and Ghoshal, 2011), disturbance observer-based technique is applied to design the filter. These methods are difficult to be used in small seeker (e.g. strapdown semi-active laser seeker) due to their large computation. As a nonlinear estimation method, tracking differentiator (TD) can solve the problem of tracking and extracting differential signals from noisy measurement signals (e.g., Jalili and Laxminarayana, 2004; Guo and Zhao, 2011). TD is widely applied engineering signal processing in the field of navigation system (e.g., Tang et al., 2009; Ming et al., 2017), backstepping control (e.g., Wang et al., 2017; Shao and Wang, 2016) and other engineering fields (e.g., Xue et al., 2017).

In this paper, the TD based estimator has been designed to extract LOS rates. Meanwhile available measurements include attitude angles, boresight errors and their angular rates. Boresight error rates will be contaminated by Gaussian noise, if numerical differentiation is applied in the extraction. Also the estimation contains a number of decomposition formulas of derivatives. In this way, TD is a feasible and effective fast path to reach the final scheme. Through the extract scheme, LOS rates have been estimated.

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The paper is organized as follows. At first, we proposed a LOS rate extract scheme for strapdown semi-active laser seeker systems, according to the sensor measurement. Then, TD is applied for extracting boresight error angle rates in the estimator. Next, a simulation case of a TD based LOS rate estimation is described. Some conclusions are given at last.

2. LOS Rate Extract Scheme

2.1 Definition of Coordinate System

Coordinate systems taken to calculate LOS rate are defined as follows.

The *inertial frame* ($O_i-x_iy_iz_i$) has its origin at the center of the Earth and axes which are non-rotating with respect to the fixed stars, defined by the axes O_ix_i , O_iz_i , with O_iz_i coincident with the Earth's polar axis (which is assumed to be invariant in direction).

The *body frame* ($O_b-x_by_bz_b$), depicted in Figure 1, is an orthogonal axis set which is aligned with the roll (γ), pitch (ϑ) and yaw (ψ) axes of the vehicle in which the inertial frame is installed. The origin O_b is the center of the seeker. The axis O_bx_b coincides with the longitudinal axis of the seeker and points toward the forward direction. The axis O_by_b lies in the longitudinal symmetry plane of the seeker, is perpendicular to O_bx_b and points upward. The axis O_bz_b is perpendicular to the $O_bx_by_b$ plane and forms a right-hand coordinate system with O_bx_b and O_by_b .

The *quasi-body frame* ($O_b-x_by_bz_b'$) is an orthogonal axis set which is aligned with the pitch (ϑ) and yaw (ψ) axes of the vehicle in which the inertial frame is installed. The definition of $O_b-x_by_bz_b'$ is similar to $O_b-x_by_bz_b$.

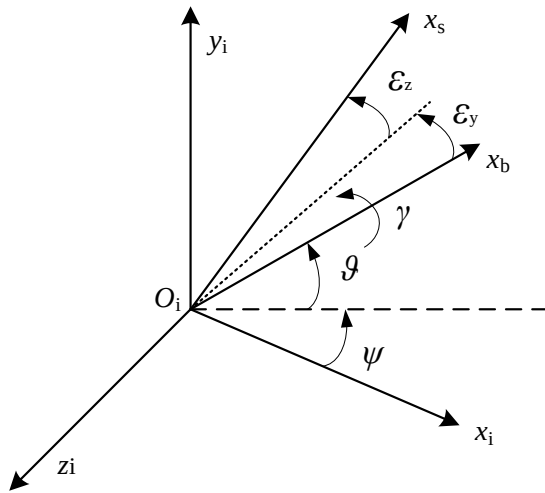


Fig.1. Definitions of the body frame and the line-of-sight frame.

The *line-of-sight frame* ($O_b-x_by_bz_b'$) is an orthogonal axis relative to the quasi-body frame by Euler angles (boresight error) ϵ_y (yaw) and ϵ_z (pitch). O_bx_s points toward the object along the line of sight. The O_by_s axis, which points upward, is on a plane that contains O_bx_s and is perpendicular to O_bx_s and the plane $O_bx_bz_b$ at the same time. O_bz_s is determined by the right-hand rule. In this way, the line-of-sight frame's angular displacements relative to the inertial frame are given by Euler angles q_y (yaw) and q_z (pitch), as indicated in Fig. 2.

2.2 Rotation Matrices

In order to get the LOS rate $\dot{q}_y$, $\dot{q}_z$ in the inertial frame, boresight error and altitude angle are taken into a series of transformations. Here, we define the rotation matrices of the Euler angles as follows:

$$L_x(\cdot) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\cdot) & \sin(\cdot) \\ 0 & -\sin(\cdot) & \cos(\cdot) \end{bmatrix} \quad (1)$$

$$L_y(\cdot) = \begin{bmatrix} \cos(\cdot) & 0 & -\sin(\cdot) \\ 0 & 1 & 0 \\ \sin(\cdot) & 0 & \cos(\cdot) \end{bmatrix} \quad (2)$$

$$L_z(\cdot) = \begin{bmatrix} \cos(\cdot) & \sin(\cdot) & 0 \\ -\sin(\cdot) & \cos(\cdot) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

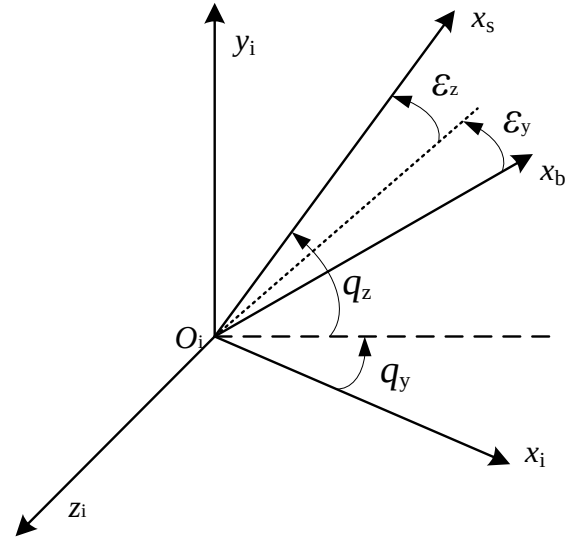


Fig.2. Relations of the line-of-sight frame and the inertial frame.

Thus, the coordinate transformation matrices used in this paper are represented as follows. The rotation matrix from inertial frame to the line-of-sight frame is

$$C_i^s = L_y(q_y)L_z(q_z) \quad (4)$$

The rotation matrix from inertial frame to quasi-body frame is

$$C_i^{b'} = L_y(\psi)L_z(\vartheta) \quad (5)$$

The rotation matrix from quasi-body frame to line-of-sight frame is

$$C_{b'}^s = L_y(\epsilon_y)L_z(\epsilon_z) \quad (6)$$

2.3 Extract Scheme

In order to get the LOS rate $\dot{q}_y$ and $\dot{q}_z$ in the inertial frame, boresight error and altitude angle are taken into a series of transformations.

In the computing scheme, the LOS rate in the line-of-sight frame is transformed to quasi-body frame and then transformed to inertial frame by the rotation matrix $(C_i^{b'}C_{b'}^s)^{-1}$.

The unit vector of LOS in the line-of-sight frame is $\vec{r}_s$, i.e., $\vec{r}_s = (1, 0, 0)^T$. The projection under the inertial frame is expressed as

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = (C_I^S)^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}. \quad (7)$$

From (4) and (7), we can express LOS angles in the inertial frame q_y and q_z in terms of LOS projection as follows:

$$q_y = \tan^{-1}\left(-\frac{z_i}{x_i}\right), \quad (8)$$

$$q_z = \sin^{-1}(y_i). \quad (9)$$

In terms of (8) and (9), LOS rates in the inertial frame are deduced as

$$\dot{q}_y = \frac{z_i \dot{x}_i - x_i \dot{z}_i}{x_i^2 + z_i^2}, \quad (10)$$

$$\dot{q}_z = \frac{\dot{y}_i}{\sqrt{1 - y_i^2}}. \quad (11)$$

In order to get values of $\dot{q}_y$ and $\dot{q}_z$, the projection (x_i, y_i, z_i) needs to be computed by boresight error and missile altitude according to sensors' output, the process is as follows:

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} A_4 E_4 - A_2 \sin \varepsilon_y - \sin \psi E_3 \\ \sin \vartheta E_4 + \cos \vartheta \sin \varepsilon_y \\ -A_3 E_4 + A_1 \sin \varepsilon_y - \cos \psi E_3 \end{bmatrix} \quad (12)$$

$$\begin{aligned} \dot{r}_{ix} = & \frac{d\vartheta}{dt} (-A_2 E_4 - A_4 \sin \varepsilon_y) \\ & + \frac{d\psi}{dt} (-A_3 E_4 + A_1 \sin \varepsilon_y - \cos \psi E_3) \\ & + \frac{d\varepsilon_y}{dt} (-A_4 E_2 - A_2 \cos \varepsilon_y + \sin \psi E_1) \\ & + \frac{d\varepsilon_z}{dt} (-A_3 E_3 - \sin \psi E_4) \end{aligned}, \quad (13)$$

$$\begin{aligned} \dot{r}_{iy} = & \frac{d\vartheta}{dt} (\cos \vartheta E_4 - \sin \vartheta \sin \varepsilon_y) \\ & + \frac{d\varepsilon_y}{dt} (-\sin \vartheta E_2 + \cos \vartheta \cos \varepsilon_y), \\ & - \frac{d\varepsilon_z}{dt} \sin \vartheta E_3 \end{aligned}, \quad (14)$$

$$\begin{aligned} \dot{r}_{iz} = & \frac{d\vartheta}{dt} (A_1 E_4 + A_3 \sin \varepsilon_y) \\ & + \frac{d\psi}{dt} (-A_4 E_4 + A_2 \sin \varepsilon_y + \sin \psi E_3) \\ & + \frac{d\varepsilon_y}{dt} (A_2 E_2 + A_1 \cos \varepsilon_y + \cos \psi E_1) \\ & + \frac{d\varepsilon_z}{dt} (A_3 E_3 - \cos \psi E_4) \end{aligned}, \quad (15)$$

$$\text{where } \begin{cases} A_1 = \sin \vartheta \sin \psi \\ A_2 = \sin \vartheta \cos \psi \\ A_3 = \cos \vartheta \sin \psi \\ A_4 = \cos \vartheta \cos \psi \end{cases}, \quad \begin{cases} E_1 = \sin \varepsilon_y \sin \varepsilon_z \\ E_2 = \sin \varepsilon_y \cos \varepsilon_z \\ E_3 = \cos \varepsilon_y \sin \varepsilon_z \\ E_4 = \cos \varepsilon_y \cos \varepsilon_z \end{cases}, \quad \frac{d\vartheta}{dt} = \omega_z,$$

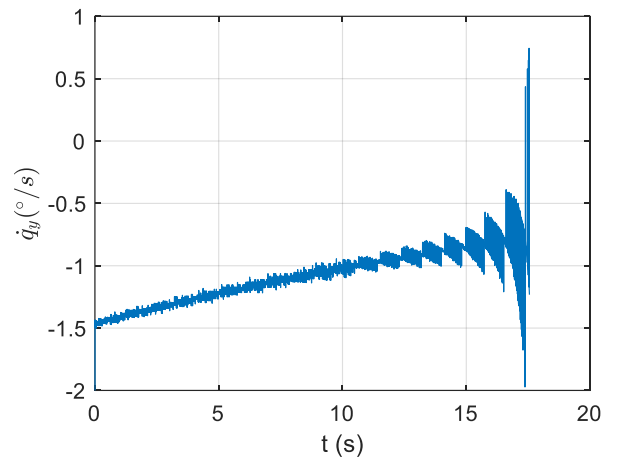
$$\frac{d\psi}{dt} = \frac{\omega_y}{\cos \vartheta}.$$

In (13)-(15), $\frac{d\varepsilon_y}{dt}$ and $\frac{d\varepsilon_z}{dt}$ can't be directly measured, but can be processed in terms of ε_y and ε_z .

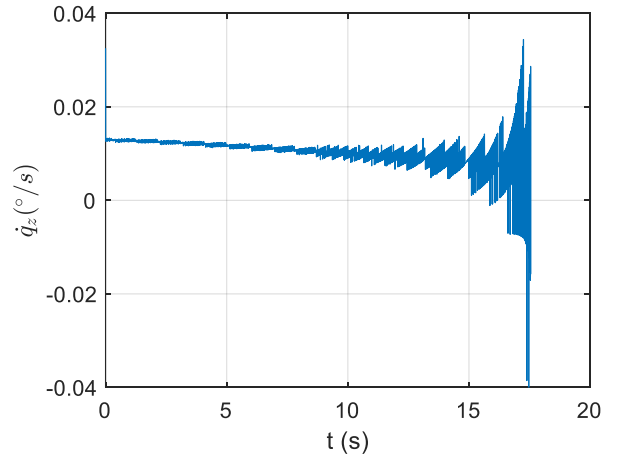
3. Problem description

Boresight errors ε_y and ε_z are the outputs of strapdown seeker system, according to which $\frac{d\varepsilon_y}{dt}$ and $\frac{d\varepsilon_z}{dt}$ can be calculated in the LOS rate extract scheme. The numerical differential method of the boresight error brings white noise and exhibits a significant gradient change, and the result cannot be used, as shown in Fig. 3.

Under this circumstance, a suitable filter with rapid response should be designed for the signal of boresight error.



(a) Numerical differentiation result of $\dot{q}_y$



(b) Numerical differentiation result of $\dot{q}_z$

Fig.3. Numerical differentiation result of boresight errors.

4. Tracking differentiator

TD is a feasible engineering method getting tracking signal and differential signal based on original signal.

The expression of discrete TD is:

$$\begin{cases} x_1(k+1) = x_1(k) + h x_2(k) \\ x_2(k+1) = x_2(k) + h f_{st}(x_1(k) - u(t), x_2(k), r, h_0) \end{cases} \quad (16)$$

where

$$f_{st}(x_1 - u, x_2, r, h_0) = \begin{cases} -ra & |a| \leq d \\ -r \cdot \text{sign}(a) & |a| > d \end{cases}, \quad (17)$$

$$a = \begin{cases} x_2 + \frac{c}{h_0} & |c| < d_0 \\ x_2 + \frac{\text{sign}(c)(a_0 - d)}{2} & |c| > d_0 \end{cases}, \quad (18)$$

$$\begin{cases} d = rh_0 \\ d_0 = dh_0 \\ c = x_1 - u(t) + h_0 x_2 \\ a = \sqrt{d^2 + 8r|c|} \end{cases} \quad (19)$$

Here u means the input, x_1 is the tracking signal of u , x_2 is the differential signal of u , h is the time interval of discrete signal u , h_0 and r are the parameters of the filter. r represents the tracking speed of the input signal. The larger r is, the faster x_1 tracks the input signal u , but when u is contaminated by noise, the signal x_1 is contaminated by more noise. In order to filter out the noise contained in x_1 , the appropriate h_1 should be selected to obtain a good filtering effect. However, the larger h_1 gains, the greater the phase loss of the x_1 tracking signal u .

In the swapdown semi-active laser seeker, the output of the seeker ($\varepsilon_y, \varepsilon_z$) is filtered by differential tracker to require LOS rate (q_y, q_z), shown in Fig.4.

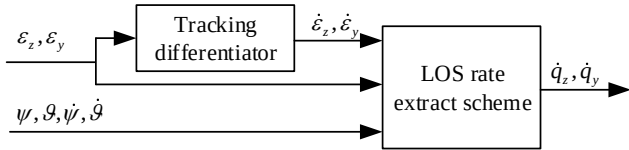


Fig.4. Process of LOS rate acquisition.

5. Simulation

We demonstrate the good performance of the proposed LOS rate estimator compared with the conventional estimator through a trajectory simulation under a representative terminal-guide mortar projectile.

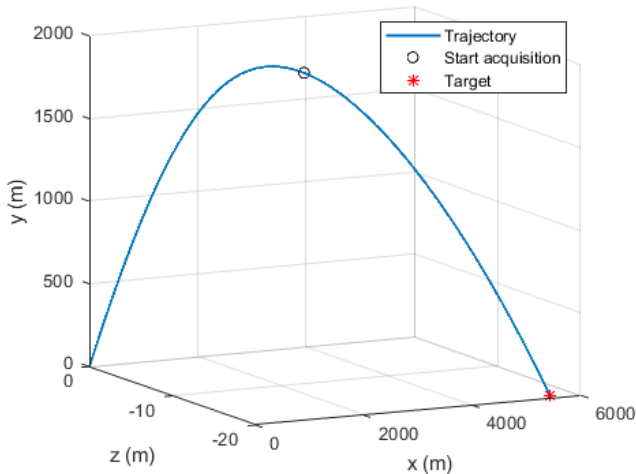


Fig.5. A high-angle fire trajectory.

5.1 High-Angle Fire

A high-angle fire trajectory is taken into simulation, shown in Fig.5. The terminal-guide mortar projectile starts guiding at the start acquisition point, and the strapdown seeker begins to acquire the boresight errors of the target.

First, we compare the two estimators filtering the signal of boresight error. The conventional estimator consisted of a differentiator and a RC filter, shown in Fig.6.

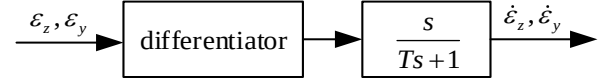
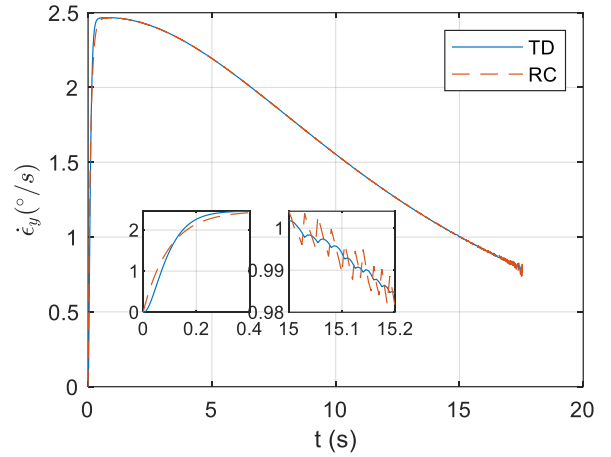
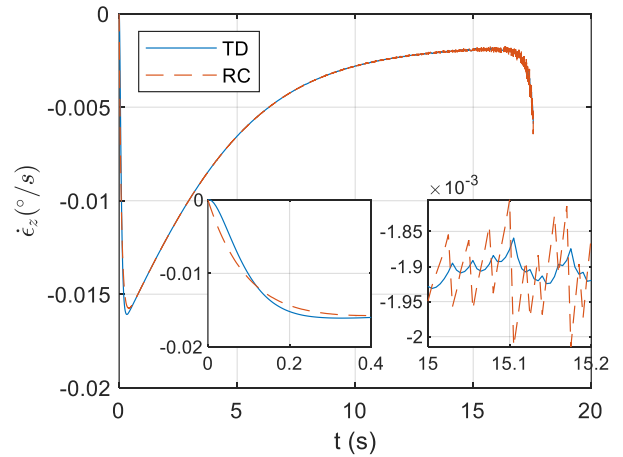


Fig.6. Framework of conventional estimator.



(a) $\dot{\varepsilon}_y$ after estimation.



(b) $\dot{\varepsilon}_z$ after estimation.

Fig.7. Boresight error's rate after estimation.

The expression of discrete RC filter is:

$$\frac{dy(k+1)}{dt} = [y(k+1) - y(k)] / h_0, \quad (20)$$

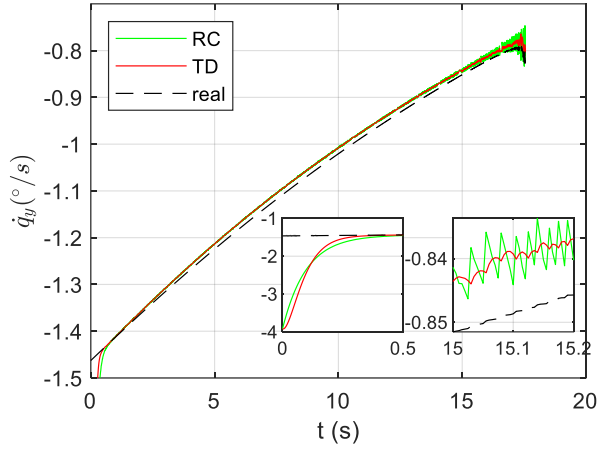
$$x'_2(k+1) = \alpha \frac{dy(k+1)}{dt} + (1 - \alpha)x'_2(k), \quad (21)$$

where y is the input signal and α is the filter parameter. In this simulation, the parameters of the filter are selected as follows: $h_0=0.05, r=1000, \alpha=0.05$.

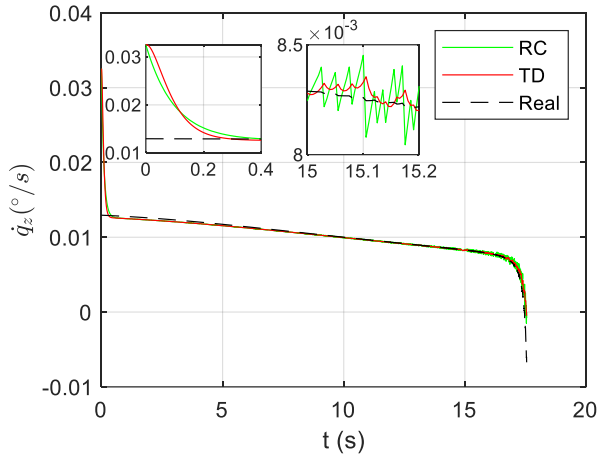
The simulation result ($\dot{\varepsilon}_y, \dot{\varepsilon}_z$) processed by the two estimators are shown in Fig.7.

As shown in Fig.7, the filtering results are basically same at the numerical level. On the one hand, the output of TD has faster tracking performance. On the other hand, TD has stronger immunity in the later stage of increasing noise.

Next we apply the two estimators in the proposed LOS rate extract scheme. For fair comparison, the parameter of filters set to make sure time delay phenomenon at the same level. The simulation results of LOS rate ($\dot{q}_y, \dot{q}_z$) are shown in Fig.8 and Fig.9.



(a) $\dot{q}_y$ through extract scheme.



(b) $\dot{q}_z$ through extract scheme.

Fig.8. LOS rates through extract scheme.

From the simulation results in Fig.8, the curve of LOS rate is similar to the boresight errors' filtering result. TD has a shorter tracking time and a smaller shock range than RC filter. Errors after TD and RC are basically the same level, because two filters have same time delay. The error between the TD results and real values of the LOS rate is less than 0.05 degree/s in $O_{x|y}$ and 5×10^{-4} degree/s in $O_{x|z}$, which is acceptable in engineering application.

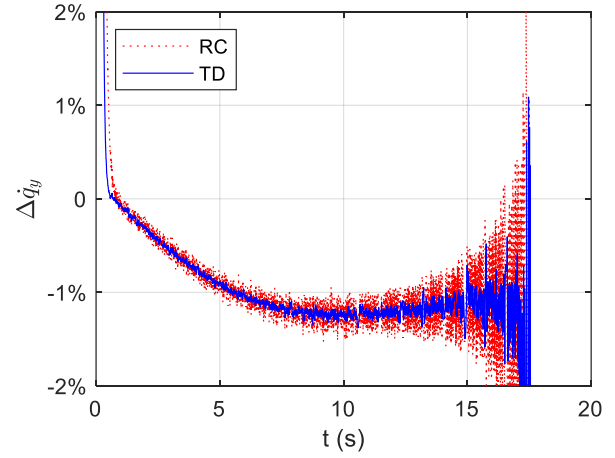
Fig.9 shows the percentage errors of LOS rate using TD and RC filters. The yaw and pitch's errors are both lower than 3%. TD based estimation has less fluctuation than RC filter, in another way TD can suppress the noise better.

5.2 Low-Angle Fire

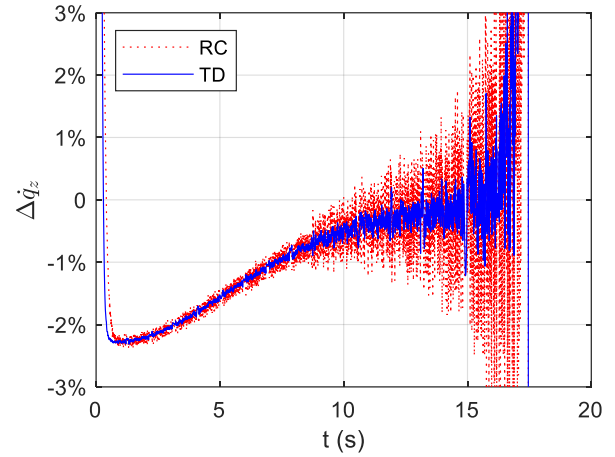
A low-angle fire trajectory is taken into simulation in this part,

shown in Fig.10.

The simulation result ($\dot{\varepsilon}_y, \dot{\varepsilon}_z$) processed by the two estimators are shown in Fig.11.



(a) Percentage error of $\dot{q}_y$.



(b) Percentage error of $\dot{q}_z$.

Fig.9. error of LOS rate through extract scheme.

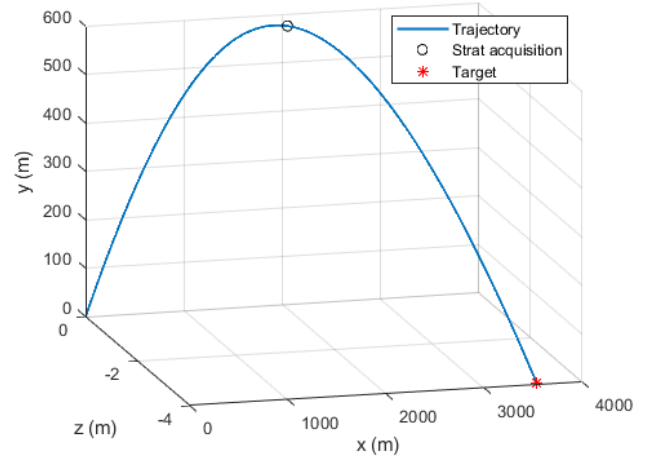


Fig.10. A low-angle fire trajectory.

As shown in Fig.11, the filtering results in the low-angle fire trajectory show the same effects as the high-angle fire. The simulation results of LOS rate ($\dot{q}_y, \dot{q}_z$) are shown in Fig.12 and Fig.13.

Fig.12 and Fig.13 show TD based estimator performs well in LOS rate acquisition. The error between the TD results and real values of the LOS rate is less than 0.02 degree/s in $O_i x_i y_i$ and 5×10^{-5} degree/s in $O_i x_i z_i$. TD based estimation suppress the noise in the terminal-guide mortar projectile's LOS rate.

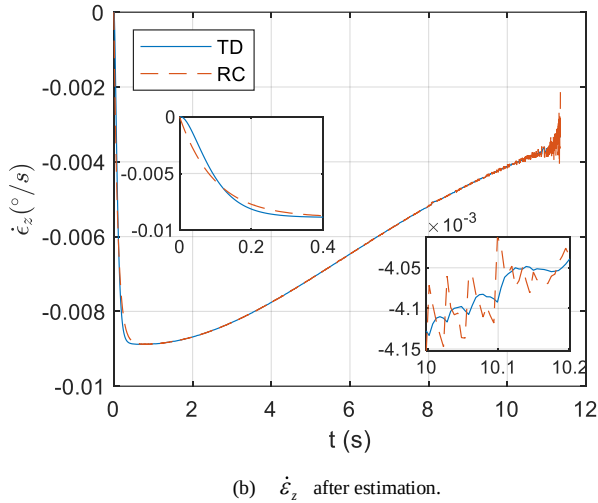
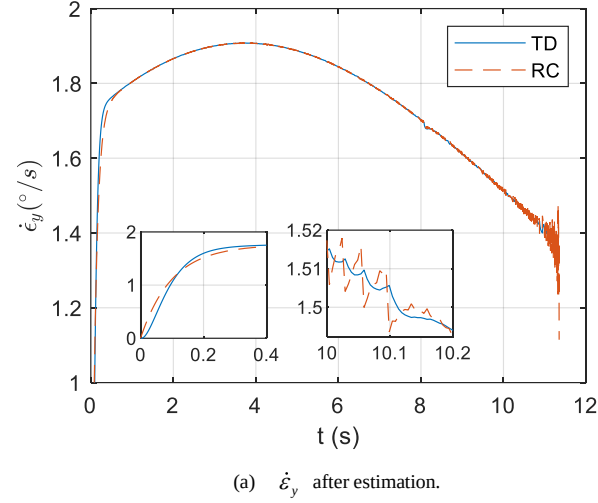


Fig.11. Boresight error's rate after estimation.

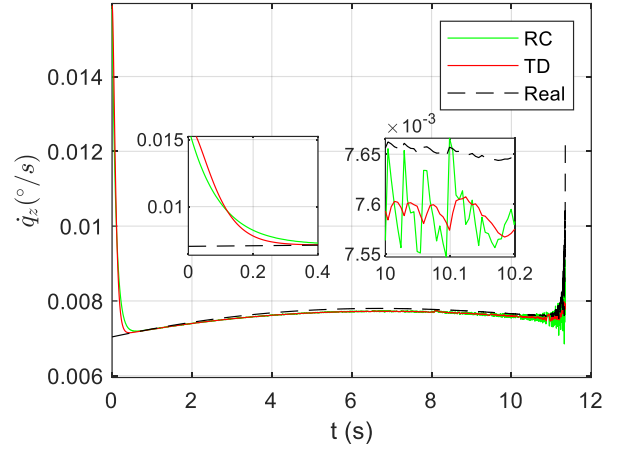
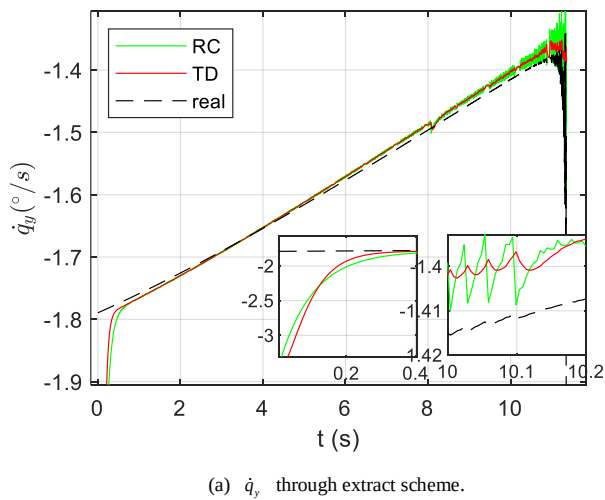


Fig.12. LOS rates through extract scheme.

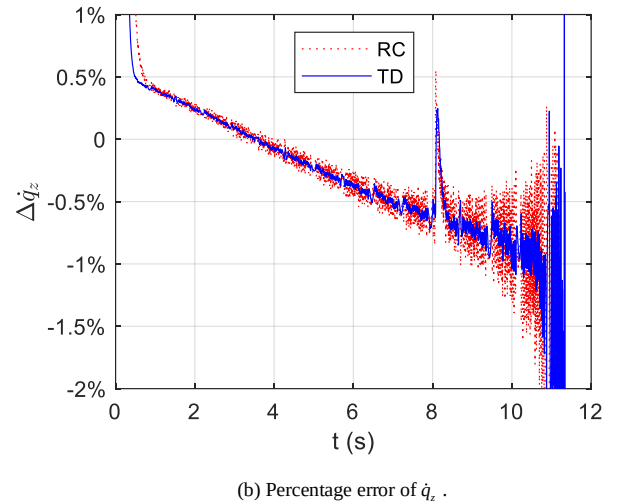
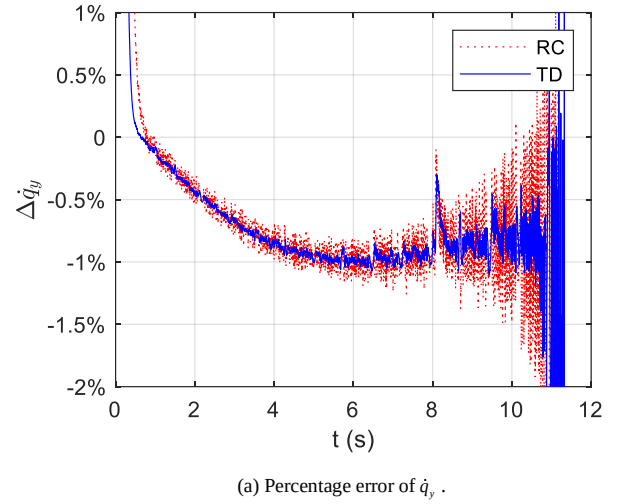


Fig.13. error of LOS rate through extract scheme.

6. Summary

We have proposed a LOS rate extract scheme using TD for strapdown semi-active laser seeker systems. In this scheme, LOS rate is calculated by attitude from sensors in the missile body and boresight error from the strapdown seeker. We take TD to solve the noise in differential of boresight error signal. A RC filter is used in

the estimator to be TD's comparison. The simulation results show that TD can effectively suppress noise. It has engineering application value to improve guidance accuracy.

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