Contents lists available at YXpublications

International Journal of Applied Mathematics in Control Engineering

Journal homepage: http://www.ijamce.com

A New Belief Interval-Valued Soft Set Theoretic Approach to Decision Making Problems

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ARTICLE INFO

Article history:
Received 22 February 2020
Accepted 25 April 2020
Available online 29 April 2020

Keywords:
Belief interval-valued soft set
Soft belief value
Soft belief degree
Parameter reduction
Group decision-making

ABSTRACT

In this paper, new symmetrical notions of soft belief value and soft belief degree are proposed, which are on the basis of belief interval-valued soft set as an improved approach to make decisions. Comparing with previous approaches, the improved approach is easier to calculate and understand when solving same decision problems and obtaining the same correct results. Another advantage is that can compare horizontally and vertically among different parameters and different objects. Furthermore, the paper proposes a rule of parameter reduction developed in accordance with the new concepts and numerical examples employed as evidence of the reduction. Finally, it puts forward a decision method for group decision-making according to soft belief value and soft belief degree and an example to it.

1. Introduction

Molodstov (Molodstov, 1999) introduced soft set as an instrument to solve uncertainty problems in 1999. Then, Maji et al. (Maji, 2009; Maji et al., 2002) put forward diverse operations on it and studied decision theory. They also proposed the fuzzy soft set such that they used a unique number to indicate the subordinate degree of the object to each parameter. Furthermore, Maji (Maji, 2009) and Yang et al. (Yang et al., 2009) considered using interval to present the subordinate degree of the object to each parameter, and they separately proposed interval-valued fuzzy soft sets and intuitionistic fuzzy soft sets. In the following years, Dey et al. (Dey et al., 2015) and Feng et al. (Feng, F., 2011; Feng, F. et al., 2010; Feng, F. et al., 2011; Feng, Q. and Zhou, 2014) studied soft sets in various directions. Recently, many related concepts of soft set and decision making (e.g., Nguyen et al., 2014; Jiang et al., 2011; Zhang et al., 2012; Liu and Yang, 2013; Xue et al., 2017; Khan et al., 2019; Muhammad et al., 2019; Yang and Yao, 2019; Li and Chen, 2019; Zheng et al., 2019) have been considered to solve different kinds of uncertainty problems. As the result of the development of soft set theory, Vijayabalajin et al. (Vijayabalajin and Ramesh, 2019) proposed belief interval-valued soft set(BIVSS) and introduced soft belief power and soft recommend value to make multi-attribute decisions. Moreover, group decision making was combined with some concepts like interval-value and fuzzy via Xu et al. (Xu and Chen, 2007) and Zhou et al. (Zhou et al., 2018). Akram et al. (Akram et al., 2019) put forward some group decision making algorithms on the basis of hesitant N-soft sets. In the context, we will propose another new method to make decisions on BIVSS and an algorithm according to new concepts to make group decisions on the basis of two or more BIVSSs.

Parameter reduction has been discussed in previous studies (e.g., Sani et al., 2018; Zhan and Alcantud, 2019; Rong et al., 2019; Zhao, et al., 2019). Maji et al. (Maji et al., 2002) took into account the foremost level reduction soft set with the aid of rough set method. Yet, Chen et al. (Chen et al., 2005) reminded about the mistakes of the previous reduction and proposed another concept of parameter reduction, which was similar to the one for the rough set. Kong et al. (Kong et al., 2008; Kong et al., 2015; Kong et al., 2011) proposed normal parameter reduction of the soft set and corresponding algorithm. They also studied the normal parameter reduction soft set in different contexts, which is a great contribution of soft set reduction. Afterwards, Ma et al. (Ma et al., 2014) considered four reduction algorithms about interval-valued fuzzy soft sets. They analyzed and compared the four algorithms of parameter reduction.

Dempster (Dempster, 1967) and Shafer (Shafer, 1976) introduced Dempster-Shafer theory (DST) as a new effective tool to study uncertainty problems. Hence, Sambuc et al. (Sambuc et al., 1975) put forward φ-flow function to express DST, and Zadeh (Zadeh, 1967) developed an equivalent notion named interval valued fuzzy sets. Intuitionistic fuzzy sets are relevant to other undetermined models, which were studied by Deschrijver et al. (Deschrijver et al., 2007). Xiao (Xiao, 2018) generalized DST to fuzzy soft set and

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made significant contribution to medical diagnosis problem.

For the intuitionistic fuzzy set, Hong et al. (Hong et al., 2000) considered a modified interval representation

$$\left[\mu(x_i),1-\upsilon(x_i)\right]$$

substituting

$$\langle \mu(x_i),\upsilon(x_i)\rangle$$

According to (Sani et al., 2017), an approach of solving DST with intuitionistic fuzzy was put forward by Dymova et al. (Dymova and Sevastjanov , 2010), then some operations were defined by them. Dymova et al. (Dymova and Sevastjanov , 2012) introduced the relationship between Atanassov Intuitionistic Fuzzy set(A-IFS) (e.g., Atanassov,, 1986; Dimitris et al., 2008; Wu et al., 2011; Song et al., 2018) and DST. They defined the basic assignment function. Vijayabalaji and Ramesh (Vijayabalajin and Ramesh, 2019) generalized the idea of the belief interval-value set developed from DST and soft set to present the belief interval valued soft set(BIVSS) and related operations and then proposed the soft belief power and soft recommend value on BIVSS as approaches to deal with multi-attribute decision making problems.

In the above literature, the approaches to address the belief interval-valued decision making problems are complicated and difficult. Furthermore, the parameter reduction of BIVSS has not been introduced. To solve these problems, we performed the following research. In Section 2, the fundamental notions with regard to the intuitionistic fuzzy set, intuitionistic fuzzy soft set, and belief interval-valued soft set are discussed. Section 3 puts forward new concepts named the soft belief value and soft belief degree based on BIVSS and the corresponding algorithm to solve decision making problems, which refers to the problems of investment and complementary and alternative medicine (CAM) mentioned in the paper. Then, we compare our approach with that of Vijayabalaji et al. (Vijayabalajin and Ramesh, 2019) to solve multi-attribute decision making problems, which indicate that our approach is easier to calculate and understand. Different objects and different parameters also can be compared horizontally and vertically by our approach. In Section 4, we present an algorithm for parameter reduction of BIVSS according to new concepts such as the soft belief value and soft belief degree. Some examples are also provided to illustrate the algorithm. In Section 5, a decision method for group decision making such as medical diagnosis by utilizing the soft belief value and soft belief degree and an example for explaining them are put forward. Finally, the concluding section discusses the results and states further research directions and limitations.

2. Preliminaries

In this section, Suppose $U=\{l_1,l_2,\dots l_n\}$ is a finite universe set, $E=\{s_1,s_2,\dots s_m\}$ is the set of parameters and $S\subseteq E$.

2.1 Intuitionistic fuzzy sets

Definition 1. (Atanassov et al., 1986). An intuitionistic fuzzy set A can be represented as.

$$A=\left\{\langle l_i,\mu_A(l_i),\upsilon_A(l_i)\rangle | l_i\in U\right\}$$

subject to $0\le \mu_A(l_i)+\upsilon_A(l_i)\le 1$ for every $l_i\in U$.

$$\pi_A(l_i)=1-\left(\mu_A(l_i)+\upsilon_A(l_i)\right)$$

is called the hesitation degree of $l_i\in U$. Hong et al. (Hong et al., 2000). Introduced

$$\left[\mu_A(l_i),1-\upsilon_A(l_i)\right]$$

representing the intuitionistic fuzzy set A to substitute the previous representation. There is an advantage that the representation

$$\left[\mu_A(l_i),1-\upsilon_A(l_i)\right]$$

express a normal interval because the right boundary is bigger than the left boundary. The basic concepts of the intuitionistic fuzzy set in accordance with DST can be redefined. Dymova and Sevastjanov (2010) (Vijayabalajin and Ramesh, 2019) proposed the triplet

$$\pi_A(l_i)=1-\left(\mu_A(l_i)+\upsilon_A(l_i)\right)$$

to represent the basic assignment function. That is

$$Bel_A(l_i)=\mu_A(l_i)$$

and

$$Pl_A(l_i)=\mu_A(l_i)+\pi_A(l_i)=1-\upsilon_A(l_i).$$

Definition 2. (Dymova and Sevastjanov, 2010). An intuitionistic fuzzy set A can be represented as.

$$A=\left\{\langle l_i,Bl_A(l_i)\rangle | l_i\in U\right\}$$

where $Bl_A(l_i)=\left[Bel_A(l_i),Pl_A(l_i)\right]$ represents the belief interval and $Bel_A(l_i)=\mu_A(l_i)$ and $Pl_A(l_i)=1-\upsilon_A(l_i)$ represent the degrees of belief and plausibility.

2.2 Intuitionistic fuzzy soft sets

Definition 3. (Molodstov, 1999). A mapping $F:S\to P(U)$, indicated by (F,S), is a soft set on U, and $P(U)$ represents the power set of U.

Definition 4. (Roy and Maji, 2007). A mapping $G:S\to \mathscr{P}(U)$, indicated by (G,S) is a fuzzy soft set on U, and $\mathscr{P}(U)$ represents the set of all fuzzy sets over U.

Definition 5. (Maji, 2009). A mapping $I:S\to \mathcal{P}(U)$, indicated by (I,S) is a intuitionistic fuzzy soft set on U, and $\mathcal{P}(U)$ represents the set of all intuitionistic fuzzy sets over U.

2.3. Belief interval-valued soft set

Definition 6. (Vijayabalajin and Ramesh, 2019). $\mathbb{P}(U)$ is a set of all belief interval-valued subsets of U. A mapping $Y:S\to \mathbb{P}(U)$ is named a belief interval-valued soft set and indicated as (Y,S) on U, represented as

$$Y(s_j)=\left\{\left\langle l_i,Bl_{Y(s_j)}(l_i)\right\rangle | l_i\in U\right\}$$

where

$$Bl_{Y(s_j)}(l_i) = \left[Bel_{Y(s_j)}(l_i), Pl_{Y(s_j)}(l_i) \right], \quad \forall s_j \in S$$

Example 1. Let $U = \{l_1, l_2, l_3, l_4, l_5\}$ be a set of five candidates for a job and $E = \{s_1, s_2, s_3, s_4, s_5\}$ be a set of demanding capacities, where $s_j\,(j = 1,2,3,4,5)$ represent "experience", "computer knowledge", "skilled foreign language", "creativity", and "managerial skills", respectively. Let $S = \{s_1, s_2, s_3\} \subseteq E$.
According to Definition 3, a soft set is expressed as

$$(F,S) = \{(s_1, \{l_1, l_2, l_4\}), (s_2, \{l_1, l_3, l_4\}), (s_3, \{l_2, l_3, l_4\})\}$$

By Definition 4, a fuzzy soft set is expressed as

$$(G,S) = \left\{ \left(s_1, \left\{ \frac{l_1}{0.7}, \frac{l_2}{0.8}, \frac{l_3}{0.7} \right\} \right), \left(s_2, \left\{ \frac{l_1}{0.9}, \frac{l_2}{0.6}, \frac{l_4}{0.5} \right\} \right), \left(s_3, \left\{ \frac{l_1}{0.8}, \frac{l_2}{0.7}, \frac{l_3}{0.6} \right\} \right) \right\}$$

From Definition 5, an intuitionistic fuzzy soft set is expressed as

$$(I,S) = \left\{ \left(s_1, \left\{ \frac{l_1}{\langle 0,20,0.60 \rangle}, \frac{l_2}{\langle 0.60,0.30 \rangle}, \frac{l_4}{\langle 0.50,0.30 \rangle} \right\} \right) \right.$$

$$\left(s_2, \left\{ \frac{l_1}{\langle 0,70,0.20 \rangle}, \frac{l_3}{\langle 0.20,0.70 \rangle}, \frac{l_4}{\langle 0.30,0.60 \rangle} \right\} \right),$$

$$\left. \left(s_3, \left\{ \frac{l_2}{\langle 0,80,0.10 \rangle}, \frac{l_3}{\langle 0.40,0.50 \rangle}, \frac{l_4}{\langle 0.50,0.40 \rangle} \right\} \right) \right\}$$

By Definition 6, a belief interval-valued soft set can be expressed as

$$(Y,S) = \left\{ \left(s_1, \left\{ \frac{l_1}{\langle 0,20,0.40 \rangle}, \frac{l_2}{\langle 0.60,0.70 \rangle}, \frac{l_4}{\langle 0.50,0.70 \rangle} \right\} \right), \right.$$

$$\left(s_2, \left\{ \frac{l_1}{\langle 0,70,0.80 \rangle}, \frac{l_3}{\langle 0.20,0.30 \rangle}, \frac{l_4}{\langle 0.30,0.40 \rangle} \right\} \right),$$

$$\left. \left(s_3, \left\{ \frac{l_2}{\langle 0,80,0.90 \rangle}, \frac{l_3}{\langle 0.40,0.50 \rangle}, \frac{l_4}{\langle 0.50,0.60 \rangle} \right\} \right) \right\}$$

3. Soft belief value and soft belief degree

Assume $U = \{l_1, l_2, \ldots, l_n\}$ is the set of objects and $E = \{s_1, s_2, \ldots, s_m\}$ is the set of parameters. According to Dymova and Sevastjanov (Dymova and Sevastjanov, 2010) the belief interval is defined as

$$Bl_A(l_i) = \left[Bel_A(l_i), Pl_A(l_i) \right]$$

where $Bel_A(l_i) = \mu_A(l_i)$ and $Pl_A(l_i) = 1 - \upsilon_A(l_i)$ are the degrees of belief and plausibility, respectively. Vijayabalaji and Ramesh (Vijayabalajin and Ramesh, 2019) also considered the case $\mu_A(l_i) = \upsilon_A(l_i)$, then redefine $Bel_A(l_i) = \mu_A(l_i)$ and $Pl_A(l_i) = 1 - (\upsilon_A(l_i))^2$. We initiate some new concepts to measure the belief degree of one object to attributes which is different from Vijayabalaji and Ramesh (Vijayabalajin and Ramesh, 2019) below.

Definition 7. The soft belief value of $l_i\,(i = 1,2,\ldots n)$ for $s_j\,(j = 1,2,\ldots m)$ on BIVSS(Y,S) is defined by

$$SBV_{Y(s_j)}(l_i) = Bel_{Y(s_j)}(l_i) + \frac{Bel_{Y(s_j)}(l_i)}{Bel_{Y(s_j)}(l_i) + \sqrt{1 - Pl_{Y(s_j)}(l_i)}} \times \left(Pl_{Y(s_j)}(l_i) - Bel_{Y(s_j)}(l_i) \right)$$

The soft belief degree of $l_i\,(i = 1,2,\ldots n)$ on BIVSS(Y,S) is

$$SBD_Y(l_i) = \frac{1}{n} \sum_{j=1}^{m} SBV_{Y(s_j)}(l_i)$$

Where $\left(Pl_{Y(s_j)}(l_i) - Bel_{Y(s_j)}(l_i) \right)$ represents the hesitation degree, $\frac{Bel_{Y(s_j)}(l_i)}{Bel_{Y(s_j)}(l_i) + \sqrt{1 - Pl_{Y(s_j)}(l_i)}} \times \left(Pl_{Y(s_j)}(l_i) - Bel_{Y(s_j)}(l_i) \right)$ represents the ratio of the belief value to the hesitation degree. Symmetrically, the non-plausibility part in the hesitation degree $\frac{\sqrt{1 - Pl_{Y(s_j)}(l_i)}}{Bel_{Y(s_j)}(l_i) + \sqrt{1 - Pl_{Y(s_j)}(l_i)}} \times \left(Pl_{Y(s_j)}(l_i) - Bel_{Y(s_j)}(l_i) \right)$ can be determined. $SBD_Y(l_i)$ is the general estimate of each object l_i for the all considered parameters.

Example 2. Let $U = \{l_1, l_2, l_3\}$ be the universe set and $S = \{s_1, s_2, s_3\}$ be the set of parameters, BIVSS(Y,S) can be expressed as,

$$(Y,S) = \left\{ \left(s_1, \left\{ \frac{l_1}{\langle 0,30,0.60 \rangle}, \frac{l_2}{\langle 0.40,0.50 \rangle}, \frac{l_3}{\langle 0.50,0.50 \rangle} \right\} \right), \right.$$

$$\left(s_2, \left\{ \frac{l_1}{\langle 0,60,0.80 \rangle}, \frac{l_2}{\langle 0.60,0.60 \rangle}, \frac{l_3}{\langle 0.50,0.70 \rangle} \right\} \right),$$

$$\left. \left(s_3, \left\{ \frac{l_1}{\langle 0,10,0.50 \rangle}, \frac{l_2}{\langle 0.40,0.80 \rangle}, \frac{l_3}{\langle 0.40,0.90 \rangle} \right\} \right) \right\}$$

Furthermore:

$$SBV_{Y(s_j)}(l_i) = Bel_{Y(s_j)}(l_i) + \frac{Bel_{Y(s_j)}(l_i)}{Bel_{Y(s_j)}(l_i) + \sqrt{1 - Pl_{Y(s_j)}(l_i)}} \times \left(Pl_{Y(s_j)}(l_i) - Bel_{Y(s_j)}(l_i) \right)$$

$$= 0.30 + \frac{0.30}{0.30+\sqrt{0.4}} \times (0.60-0.30)$$
$$\approx 0.3965$$

Therefore,

$$SBV_{Y(s_1)}(l_2) = 0.4361 \text{ , } SBV_{Y(s_1)}(l_3) = 0.5000 \text{ ,}$$

$$SBV_{Y(s_2)}(l_1) = 0.7146 \text{ , } SBV_{Y(s_2)}(l_2) = 0.6000 \text{ ,}$$

$$SBV_{Y(s_2)}(l_3) = 0.5954 \text{ , } SBV_{Y(s_3)}(l_1) = 0.1496 \text{ ,}$$

$$SBV_{Y(s_3)}(l_2) = 0.5889 \text{ , } SBV_{Y(s_3)}(l_3) = 0.6793 \text{ .}$$

For the arbitrary parameter, a decision can be made, such as s_1,

$$SBV_{Y(s_1)}(l_1) \le SBV_{Y(s_1)}(l_2) \le SBV_{Y(s_1)}(l_3)$$

so l_3 is the optimal choice, for the parameter s_2 ,

$$SBV_{Y(s_2)}(l_3) \le SBV_{Y(s_2)}(l_2) \le SBV_{Y(s_2)}(l_1),$$

so l_1 is the optimal choice, for the parameter s_3,

$$SBV_{Y(s_3)}(l_1) \le SBV_{Y(s_3)}(l_2) \le SBV_{Y(s_3)}(l_3) \quad ,$$

so l_3 is the optimal choice. Considering all parameters synthetically,

$$SBD_Y(l_1) = 0.4202$$
$$SBD_Y(l_2) = 0.5417$$
$$SBD_Y(l_3) = 0.5916$$

so

$$SBD_Y(l_1) \le SBD_Y(l_2) \le SBD_Y(l_3)$$

where l_3 is the best choice.

We propose an algorithm to make decisions as applications of the new concepts.

Algorithm 1: Decision making on BIVSS
BEGIN
1. Input BIVSS (Y,S)
2. Calculate the soft belief value of $l_i(1 \le i \le n)$ for $s_j(1 \le j \le m)$ and the soft belief degree of $l_i(1 \le i \le n)$.
3. Sort the options $l_i(1 \le i \le n)$ according to the soft belief degree of $l_i(1 \le i \le n)$.
END

The following is an example of utilizing the Algorithm 1.

Example 3. An investment company wants to invest in a project. Let $U = \{l_1, l_2, l_3, l_4, l_5\}$ be the set of five alternatives, where $l_i(i=1,2,3,4,5)$ describe "a pharmaceutical company", "a building materials company", "a software company", "a clothing company", "an electrical appliance company", respectively. The company evaluates the companies with respect to four aspects, which are: $S = \{s_1, s_2, s_3, s_4\}$. $s_j(j=1,2,3,4)$ describing "the investment risk", "the possible benefits", "the public influence", and "the environmental effect", respectively. There is a decision maker who considers the parameters above to evaluate the five candidates. The properties of the five candidates are indicated by BIVSS (Y,S).

Step 1. Input BIVSS matrix.

$$[(Y,S)] = \begin{bmatrix} [0.30,0.60] & [0.40,0.70] & [0.40,0.80] & [1.00,1.00] & [0.70,0.80] \\ [0.20,0.30] & [0.50,0.70] & [0.80,0.90] & [0.70,0.80] & [0.50,0.50] \\ [0.40,0.50] & [0.60,0.80] & [0.80,1.00] & [0.70,0.90] & [0.70,0.80] \\ [0.50,0.50] & [0.70,0.80] & [0.80,0.90] & [0.50,0.70] & [0.50,0.70] \end{bmatrix}$$

Step 2. Calculate the soft belief value of $l_i(i=1,2,3,4,5)$ for $s_j(j=1,2,3,4)$ and the soft belief degree of $l_i(i=1,2,3,4,5)$ indicated by S in Table 1.

Table 1. Soft belief degree of l_i in Example 3

	l_1	l_2	l_3	l_4	l_5
s_1	0.3965	0.5266	0.5889	1.0000	0 .7610
s_2	0.2193	0.5954	0.8717	0.7610	0.5000
s_3	0.4361	0.7146	1.0000	0.8378	0.7610
s_4	0.5000	0.7610	0.8717	0.5954	0.5954
$SBD_Y(l_i)$	0.3880	0.6494	0.8331	0.7986	0.6544

Step 3. Sort all the options $l_i(i=1,2,3,4,5)$ according to the soft belief degree of $l_i(i=1,2,3,4,5)$ Select the greatest soft belief degree of $l_k(k \in \{1,2,3,4,5\})$ as the optimal choice, So

$$l_3 > l_4 > l_5 > l_2 > l_1,$$

the optimal choice is l_3.

Vijayabalaji and Ramesh (Vijayabalajin and Ramesh, 2019) proposed two approaches to make multi-attribute decision on BIVSS and obtain the sequence of the options. We compare our approach with the one of Vijayabalaji and Ramesh (Vijayabalajin and Ramesh, 2019).

Example 4. Consider that $U = \{l_1, l_2, l_3, l_4, l_5, l_6\}$ is the set of sufferers, $E = \{N, P, H\}$ is the set of CAM therapies; $N = \{n_1, n_2, n_3, n_4, n_5, n_6\}$ denotes natural treatment items , where n_1 = no side effect, n_2 = non-toxic, n_3 = apply natural ingredients, n_4 = strengthen immune system, n_5 = cure the body on it own, and n_6 =enhance natural capacity; $P = \{p_1, p_2, p_3\}$ is priority of participation in health therapies projects, where p_1 = equal companions, p_2 = patients should be active, and p_3 = patients decide by themselves; $H = \{h_1, h_2, h_3, h_4\}$ is orientation toward overall health projects, where h_1 = coordinating your body, heart and soul, h_2 = concentrate on people' total happiness, h_3 = the body possesses a basic instinct, and h_4 = employ contemporary science and technique.

Step 1. Input belief interval-valued soft sets decision matrices $M_k = (Y_k, E_k)(k=1,2,3)$, where $E_1 = N$, $E_2 = P$, and $E_3 = H$.

$$M_1=\begin{bmatrix}[0.30,0.70] & [0.40,0.80] & [0.40,0.70] & [0.50,0.90] & [0.20,0.60] & [0.20,0.70]\\ [0.40,0.70] & [0.40,0.80] & [0.50,0.80] & [0.40,0.90] & [0.30,0.70] & [0.50,0.80]\\ [0.20,0.50] & [0.30,0.70] & [0.40,0.80] & [0.60,1.00] & [0.50,0.70] & [0.40,0.60]\\ [0.20,0.60] & [0.30,0.60] & [0.50,0.90] & [0.60,0.90] & [0.40,0.50] & [0.30,0.60]\\ [0.40,0.60] & [0.40,0.80] & [0.50,0.80] & [0.50,0.90] & [0.20,0.50] & [0.30,0.50]\\ [0.20,0.50] & [0.40,0.90] & [0.60,0.80] & [0.70,0.90] & [0.10,0.50] & [0.20,0.50]\end{bmatrix}$$

$$M_2=\begin{bmatrix}[0.40,0.70] & [0.30,0.60] & [0.50,0.70] & [0.20,0.60] & [0.30,0.50] & [0.40,0.70]\\ [0.20,0.50] & [0.40,0.80] & [0.40,0.60] & [0.50,0.90] & [0.40,0.70] & [0.30,0.60]\\ [0.30,0.60] & [0.40,0.70] & [0.30,0.60] & [0.60,1.00] & [0.20,0.50] & [0.30,0.60]\end{bmatrix}$$

$$M_3=\begin{bmatrix}[0.20,0.50] & [0.10,0.40] & [0.40,0.70] & [0.50,0.80] & [0.30,0.60] & [0.20,0.50]\\ [0.30,0.70] & [0.50,0.80] & [0.40,0.50] & [0.60,0.90] & [0.30,0.70] & [0.40,0.70]\\ [0.40,0.60] & [0.40,0.70] & [0.20,0.60] & [0.40,0.70] & [0.20,0.40] & [0.50,0.80]\\ [0.30,0.60] & [0.70,1.00] & [0.50,0.70] & [0.30,0.80] & [0.10,0.50] & [0.40,0.90]\end{bmatrix}$$

Step 2. Calculate the soft belief value of l_i for s_j and soft belief degree of l_i. Its tabular representation is expressed in Table 2-4.

Table 2. Soft belief degree of l_i on (Y_1,E_1)

	l_1	l_2	l_3	l_4	l_5	l_6
n_1	0.4416	0.5889	0.5266	0.7450	0.2721	0.3337
n_2	0.5267	0.5889	0.6584	0.6793	0.4416	0.6584
n_3	0.2662	0.4416	0.5889	1.0000	0.5954	0.4775
n_4	0.2962	0.3965	0.7450	0.7965	0.4361	0.3965
n_5	0.4775	0.5889	0.6584	0.7450	0.2661	0.3596
n_6	0.2661	0.6793	0.7146	0.8378	0.1496	0.2661
$SBD_Y(l_i)$	0.3791	0.5474	0.6487	0.8006	0.3602	0.4153

Table 3. Soft belief degree of l_i on (Y_2,E_2)

	l_1	l_2	l_3	l_4	l_5	l_6
p_1	0.5266	0.3965	0.5954	0.2961	0.3596	0.5266
p_2	0.2661	0.5889	0.4775	0.7450	0.5266	0.3965
p_3	0.3965	0.5266	0.3965	1.0000	0.2661	0.3596
$SBD_Y(l_i)$	0.3964	0.5040	0.4898	0.6804	0.3841	0.4276

Table 4. Soft belief degree of l_i on (Y_3,E_3)

	l_1	l_2	l_3	l_4	l_5	l_6
h_1	0.2661	0.1343	0.5266	0.6584	0.3965	0.2661
h_2	0.4416	0.6584	0.4361	0.7965	0.4416	0.5266
h_3	0.4775	0.5266	0.2961	0.5266	0.2410	0.6584
h_4	0.3965	1.0000	0.5954	0.5007	0.1496	0.6793
$SBD_Y(l_i)$	0.3964	0.5798	0.4636	0.6206	0.3072	0.5326

Step 3. Utilize soft belief degree of l_i of each class parameters to aggregate alternative values $SBD_M(l_i)$,

$$SBD_M(l_1)=SBD_N(l_1)+SBD_P(l_1)+SBD_H(l_1)\approx 1.170872$$

so similarly,

$$SBD_M(l_2)\approx 1.631141,\quad SBD_M(l_3)\approx 1.602036,$$
$$SBD_M(l_4)\approx 2.101524,\quad SBD_M(l_5)\approx 1.05144,$$
$$SBD_M(l_6)\approx 1.375477.$$

Then, the ranking of the alternatives is shown as following.

$$SBD_M(l_5)<SBD_M(l_1)<SBD_M(l_6)<SBD_M(l_3)<SBD_M(l_2)<SBD_M(l_4),$$

hence $l_5<l_1<l_6<l_3<l_2<l_4$, the best choice is l_4 (max).

The comparison of our approach and the one of Vijayabalaji and Ramesh(Vijayabalajin and Ramesh, 2019) is given in Table 5.

Table 5. Contrast between the two methods

Method	sequence	Best choice(s)
Vijayabalaji(2019)	$l_4<l_2<l_3<l_6<l_1<l_5$	l_4
ours(max)	$l_5<l_1<l_6<l_3<l_2<l_4$	l_4

Compared with the approach of S. Vijayabalaji and A. Ramesh (Vijayabalajin and Ramesh, 2019), we can clearly see that our approach is easier to calculate and understand. Furthermore, the approach can be used to compare both horizontally and vertically among different parameters and different objects. The decision choices can be made based on an arbitrary parameter.

4. Parameter reduction of belief interval-valued soft set

Let $U=\{l_1,l_2,\ldots,l_n\}$ be the set of objects, $E=\{s_1,s_2,\ldots,s_m\}$ be the set of parameters and $S\subseteq E$. Based on the BIVSS (Y,S), we introduce related definitions about the constant sequence of choices and the algorithm of parameter reduction of BIVSS.

Definition 8. An indistinguishable relationship $INR(S)$ is

$$INR(S)=\{(l_i,l_j)\in U\times U: SBD_Y(l_i)=SBD_Y(l_j)\}$$

The decision partition is

$$R_S=\left\{(l_1',l_2',\ldots,l_i')_{SBD_Y(1)},(l_{i+1}',\ldots,l_j')_{SBD_Y(2)},\ldots(l_k',\ldots,l_n')_{SBD_Y(p)},\right\}$$

where for subclass

$$\{l_v',l_{v+1}',\ldots,l_{v+w}'\}_{SBD_Y(i)}$$

$$SBD_Y(l_v')=SBD_Y(l_{v+1}')=\cdots=SBD_Y(l_{v+w}')$$

recorded as $SBD_Y(i)$, may as well set up

$$SBD_Y(1)\ge SBD_Y(2)\ge\cdots\ge SBD_Y(p)$$

Generally speaking, objects in U are sorted in accordance with $SBD_Y(i)$.

Definition 9. If B is independent (B is the minimum subset of E that maintains sequence of decision choices constant) and $R_B=R_E$, B is a belief interval-valued soft set parameter reduction (BIVSSPR) of E.

We propose an algorithm that deletes superfluous parameters while maintaining the sequence constant. Example 5 is an application of Algorithm 2.

Algorithm 2: Parameter reduction on BIVSS

BEGIN

1. Input BIVSS .
2. Calculate the soft belief value and the soft belief degree.
3. . Check B if

$$R_B=R_E$$

and B is independent. Then B is a BIVSSPR.

END

The following is an example utilizing algorithm for the parameter reduction of the belief interval-valued soft set.

Example 5. Let $U=\{l_1,l_2,l_3,l_4\}$ be a set of four candidates who want to get the post that a company wants to fill.

$S=\{s_1,s_2,s_3,s_4,s_5,s_6,s_7,s_8\}$ be a set of candidates, where $s_j(j=1,2,3,4,5,6,7,8)$ represents "young age", "experience", "higher education", "computer knowledge", "creativity", "training", "managerial skills", and "skilled in a foreign language", respectively.

The features of four candidates are indicated by BIVSS (Y,S).

There is a decision maker who considers the parameters above to evaluate the four candidates.

Step 1. Input BIVSS of . The BIVSS matrix is as follows:

$$[(Y,S)]=\begin{bmatrix} [0.50,0.70] & [0.10,0.30] & [0.20,0.40] & [0.60,0.80] \\ [0.20,0.50] & [0.50,0.80] & [0.10,0.40] & [0.40,0.60] \\ [0.80,1.00] & [0.70,0.90] & [0.60,0.80] & [0.30,0.50] \\ [0.70,0.90] & [0.30,0.50] & [0.20,0.40] & [0.50,0.70] \\ [0.30,0.60] & [0.60,0.70] & [0.50,0.80] & [0.80,1.00] \\ [0.10,0.30] & [0.70,0.70] & [0.80,0.90] & [0.60,0.80] \\ [0.40,0.50] & [0.60,0.80] & [0.90,1.00] & [0.70,0.90] \\ [0.50,0.70] & [0.60,0.90] & [0.80,0.90] & [0.60,0.70] \end{bmatrix}$$

Step 2. Calculate soft belief value of $l_i(1\le i\le 4)$ for $s_j(1\le j\le 8)$ and the soft belief degree of $l_i(1\le i\le 4)$ according to BIVSS in Table 6.

Table 6. Soft belief degree of l_i in Example 5

	l_1	l_2	l_3	l_4
s_1	0.5954	0.1214	0.2410	0.7146
s_2	0.2662	0.6584	0.1343	0.4775
s_3	1.0000	0.8378	0.7146	0.3596
s_4	0.8378	0.3596	0.2410	0.5954
s_5	0.3965	0.6523	0.6584	1.0000
s_6	0.1214	0.7000	0.8717	0.7146
s_6	0.4361	0.7146	1.0000	0.8378
s_7	0.5954	0.7965	0.8717	0.6523
$SBD_Y(l_i)$	0.5311	0.6051	0.5916	0.6690

Step 3. Check B, if $R_B=R_E$ and B is independent. It is easily to obtain that

$$R_E=\{\{l_4\}_{0.6690},\{l_2\}_{0.6051},\{l_3\}_{0.5916},\{l_1\}_{0.5311}\}$$

from Table 6. It turns out that for

$$S=\{s_1,s_2,s_3,s_4,s_5,s_6,s_7,s_8\},$$

$$R_{\{s_2,s_5\}}=\{\{l_4\}_{0.7388},\{l_2\}_{0.6553},\{l_3\}_{0.3963},\{l_1\}_{0.3313}\}$$

Thus, the parameters reduction of the BIVSS above is $\{s_2,s_5\}$.

In Example 4, $E=\{N,P,H\}$ is the set of CAM therapies, according to the approach of parameter reduction above, $\{n_1,n_2,p_1,h_2\}$ (not all) is the parameter reduction.

5. Group decision-making and application on BIVSSs

In the preceding section, we make decision via the newly proposed concepts based on only one BIVSS. Then we performed a group decision-making(GDM) based on two or more BIVSSs. This section introduces the group decision making as a procedure in which some experts apply their knowledge to judge optimal alternative. In the following, we propose weight method that gives each expert a weight depending on the importance of the expert based on two or more BIVSSs.

Definition 10. Let (Y_1,S) (Y_2,S) ,…,(Y_p,S) be BIVSSs over U. Let $U=\{l_1,l_2,\ldots,l_n\}$, $S=\{s_1,s_2,\ldots,s_m\}$. $SBD_{Y_j}(l_i)$ is the soft belief degree of $l_i(i=1,2,\ldots,n)$ over the j^{th} BIVSS $(j=1,2,\ldots,p)$, $0\le w_j\le 1$, such that

$$\sum_{j=1}^{p} w_j=1 ,$$

and the weight score for p BIVSSs is defined by

$$S_w(l_i)=\sum_{j=1}^{p}\left(w_j\times SBD_{Y_j}(l_i)\right)(i=1,2,\ldots,n)$$

$S_w(l_i)$ is the weight score of the object calculated by the i^{th} weights of all experts.

In the current context, we design an algorithm as follows to calculate the final group decision.

Algorithm 3: Group decision-making based on BIVSSs

BEGIN

1. Input BIVSSs of (Y_1,S) (Y_2,S) ,…,(Y_p,S).
2. Calculate the soft belief degree of $l_i(i=1,2,\ldots,k)$ for each BIVSS, respectively.
3. Give different BIVSSs different weights such that
$$\sum_{j=1}^{p} w_j=1 \quad (0\le w_j\le 1).$$
4. Compute weight score
$$S_w(l_i)=\sum_{j=1}^{p}\left(w_j\times SBD_{Y_j}(l_i)\right).$$
5.Sort the options $l_i(i=1,2,\ldots k)$ in accordance with $S_w(l_i)$.

END

The following is an example of utilizing Algorithm 3.

Example 6. Dengue virus is an acute insect-borne infectious disease, which can not only involve blood, nerve, circulation and other systems, but also cause damage to liver function, and severe cases can endanger life. It is hard for doctors to diagnose if a patient is suffering from the Dengue fever. Under these circumstances, we apply the group decision-making of BIVSS by the weight score to check and diagnose Dengue fever. Let $U=\{l_1,l_2,l_3,l_4\}$ be the set of four patients and let $S=\{s_1,s_2,s_3,s_4\}$ be the set of four symptoms for Dengue fever, where s_1 = intense joint and muscle ache, s_2 = severe headache, s_3 = hyperpyrexia, and s_4 = erythra. There are three experts to check the patients to give the three BIVSSs (Y_1,S), (Y_2,S), (Y_3,S) represented by the following matrixes.

Step 1. Input BIVSSs of (Y_1,S), (Y_2,S) and (Y_3,S),

$$\left[(Y_1,S)\right]=\begin{bmatrix}[0.40,0.70] & [0.30,0.80] & [0.60,0.70] & [0.70,1.00]\\ [0.20,0.40] & [0.60,0.70] & [0.50,0.70] & [0.70,0.90]\\ [0.40,0.60] & [0.50,0.80] & [0.80,0.90] & [0.60,0.90]\\ [0.50,0.70] & [0.70,0.90] & [0.80,0.90] & [0.60,0.70]\end{bmatrix}$$

$$\left[(Y_2,S)\right]=\begin{bmatrix}[0.40,0.60] & [0.40,0.70] & [0.40,0.80] & [1.00,1.00]\\ [0.20,0.30] & [0.80,0.90] & [0.80,0.90] & [0.50,0.80]\\ [0.30,0.50] & [0.60,0.80] & [0.80,1.00] & [0.70,0.90]\\ [0.80,0.90] & [0.70,0.80] & [0.60,0.90] & [0.50,0.70]\end{bmatrix}$$

$$\left[(Y_3,S)\right]=\begin{bmatrix}[0.30,0.60] & [0.60,0.70] & [0.50,0.80] & [0.80,1.00]\\ [0.10,0.30] & [0.70,0.70] & [0.80,0.90] & [0.60,0.80]\\ [0.40,0.50] & [0.60,0.80] & [0.90,1.00] & [0.70,0.90]\\ [0.50,0.70] & [0.60,0.90] & [0.80,0.90] & [0.60,0.70]\end{bmatrix}$$

Step 2. Calculate soft belief degree of $l_i\left(i=1,2,3,4\right)$ for each BIVSS, respectively. We can obtain the soft belief value and soft belief degree that we described in previous sections in table 7-9.

Table 7. Soft belief degree of l_i in Example 6

	l_1	l_2	l_3	l_4
s_1	0.5266	0.5007	0.6523	1.0000
s_2	0.2410	0.6523	0.5954	0.8378
s_3	0.4775	0.6584	0.8717	0.7965
s_4	0.5954	0.8378	0.8717	0.6523
$SBD_{Y_1}\left(l_i\right)$	0.4601	0.6623	0.7478	0.8216

Table 8. Soft belief degree of l_i in Example 6

	l_1	l_2	l_3	l_4
s_1	0.4775	0.5266	0.5889	1.0000
s_2	0.2193	0.8717	0.8717	0.6584
s_3	0.3596	0.7146	1.0000	0.8378
s_4	0.8717	0.7610	0.7965	0.5954
$SBD_{Y_2}\left(l_i\right)$	0.4820	0.7185	0.8143	0.7729

Table 9. Soft belief degree of l_i in Example 6

	l_1	l_2	l_3	l_4
s_1	0.3965	0.6523	0.6584	1.0000
s_2	0.1214	0.7000	0.8717	0.7146
s_3	0.4361	0.7146	1.0000	0.8378
s_4	0.5954	0.7965	0.8717	0.6523
$SBD_{Y_3}\left(l_i\right)$	0.3874	0.7159	0.8505	0.8012

Step 3. Give different BIVSSs different weights such that

$$\sum_{j=1}^{3} w_j = 1\ \left(0 \le w_j \le 1\right)$$

Suppose the weights $w_1=0.3$, $w_2=0.5$, $w_3=0.2$ are assigned to the three doctors.

Step 4. Compute weight score

$$S_w\left(l_i\right)=\sum_{j=1}^{3}\left(w_j \times SBD_{Y_j}\left(l_i\right)\right)\left(i=1,2,3,4\right)$$

$$S_w\left(l_1\right)=0.3\times SBD_{Y_1}\left(l_1\right)+0.5\times SBD_{Y_2}\left(l_1\right)+0.2\times SBD_{Y_3}\left(l_1\right)$$

$$S_w\left(l_2\right)=0.3\times SBD_{Y_1}\left(l_2\right)+0.5\times SBD_{Y_2}\left(l_2\right)+0.2\times SBD_{Y_3}\left(l_2\right)$$

$$S_w\left(l_3\right)=0.3\times SBD_{Y_1}\left(l_3\right)+0.5\times SBD_{Y_2}\left(l_3\right)+0.2\times SBD_{Y_3}\left(l_3\right)$$

The weight scores can be as expressed in table 10.

Table 10. Weight score of l_i in Example 6

	l_1	l_2	l_3	l_4
$SBD_{Y_1}\left(l_i\right)\cdot w_1$	0.1380	0.1987	0.2243	0.2465
$SBD_{Y_2}\left(l_i\right)\cdot w_2$	0.2410	0.3592	0.4071	0.3865
$SBD_{Y_3}\left(l_i\right)\cdot w_3$	0.0775	0.1432	0.1701	0.1602
$S_w\left(l_i\right)$	0.4565	0.7011	0.8015	0.7932

Step 5. Sort the options $l_i\left(i=1,2,3,4\right)$ in accordance with $S_w\left(l_i\right)$. We can obtain the decision $l_3 > l_4 > l_2 > l_1$ in Table 10.

6. Conclusions

We introduced the soft belief value and the soft belief degree on BIVSS as a more available and simpler approach to solve decision making problems. For investment issues and recruitment issues, we calculated the soft belief degree, which only involved the ratio of the belief and plausibility, and obtained the best choice by sorting the soft belief degree of different objects. Compared with the previous approach to solving the CAM issue, our approach was easier to calculate and understand, while obtaining the same correct result. Furthermore, our approach could be used to compare both horizontally and vertically among different parameters and different objects. The decision choices could be made based on an arbitrary parameter. Thus, parameter reduction, which is different from traditional soft set reduction, could be proposed. It considered the minimal subset of parameters that kept the sequence of decision choices constant. Some examples such as recruitment issues and CAM problems were used to illustrate the method of parameter reduction. We also presented a weight score method of group decision making problems and a corresponding algorithm in accordance with the soft belief degree. For instance, medical diagnosis, which includes multiple experts, was one of the group decision making problems. It explained the weight score method, and we could see that it was easy to obtain the final diagnostic results.

The limitation was that the parameter reduction was complicated when there were too many parameters. Thus, we will continue to study the parameter reduction on BIVSS. For the new method of decision making, each interval was finally processed into a decimal number, so the robustness and the sensitivity were better than previous methods. For the parameter reduction, the robustness was better.

Acknowledgements

Xiaomin Wang would like to acknowledge financial support provided by Nature Science Foundation of China under Grant No.61602100.

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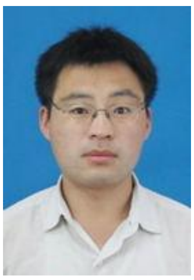

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