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Vague *N*-soft Sets and Its Application in Multi-Attribute Decision Making

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ABSTRACT

In this article, we introduce a new hybrid model called vague *N*-soft sets by a suitable combination of vagueness with *N*-soft sets, a model that extends *N*-soft sets. In the definition of grade, the idea of probability is innovatively integrated into the treatment of vague interval. Some useful definitions and properties are given. Our novel concept is illustrated with real examples, which the membership degree of data provided by experts is vague and uncertain that can be correctly captured by this structure. What is more, its relationships with existing models are studied. Finally, extended grey relational analysis method based on (*V*, *N*)-soft sets is proposed.

1. Introduction

Many practical problems involve vagueness, imprecise or subjectivity. In the face of uncertain data, it is clear that traditional methods are no longer applicable. Therefore, scholars in related fields have put forward theories to solve imprecise problems, such as probability theory, interval mathematics (e.g., Von, 1975), fuzzy set theory (e.g., Zadeh, 1965) , rough sets (e.g., Pawlak, 1982) , vague sets (e.g., Gau and Buehrer, 1993) and so on. However, the most important problem of them is the inadequacy of parametric tools. The soft set theory was put forward and it overcome these difficulties (e.g., Molodtsov, 1999). The fuzzy soft sets and the intuitionistic fuzzy soft sets were proposed (Maji et al., 2001), and the multi-attribute decision model and the method based on the intuitionistic fuzzy sets were proposed (e.g., Li, 2005). Subsequently, a probabilistic soft set was proposed (Zhu and Wen, 2010; Fatimah et al., 2019). Vague soft sets and their properties we're proposed (e.g., Xu et al., 2010). The basic concepts of rough fuzzy sets, rough soft sets, soft rough sets and soft rough fuzzy sets, and corresponding basic properties were proposed (e.g., Feng, 2012). Hesitant fuzzy soft sets were proposed (e.g., Babitha et al., 2013). Interval valued intuitionistic hesitant fuzzy soft sets were proposed (e.g., Peng and Yang, 2015), and soft binary relation was proposed (e.g., Li et al., 2017).

From the hybrid model of soft sets, many researchers use binary evaluation inspired by soft sets. However, in our daily life, we often find non-binary and discrete structure data. Further n binary-value information system (Herawan and Deris, 2009) in soft sets where each of parameter has its own rankings, as compared to rating orders (e.g., Chen et al., 2013). Inspired by n binary-value information system, *N*-soft set (e.g., Fatimah et al., 2018), which describes the importance of order level in practical problems, breaks away from binary constraints and opens up thinking. However, the concept of *N*-soft set is not enough to provide information about rating occurrence, nor can it specifically describe the occurrence of uncertainty and vagueness in decision problems. For this purpose, new models of fuzzy *N*-soft sets (e.g., Akram et al., 2018), hesitant *N*-soft sets (e.g., Akram et al., 2019), interval-valued hesitant fuzzy *N*-soft sets (e.g., Akram and Adeel, 2019), intuitionistic fuzzy *N*-soft rough sets (e.g., Akram et al., 2019), Neutrosophic Vague *N*-soft sets (e.g., Jianbo et al, 2020) and generalized Vague *N*-soft sets (e.g., Yanan et al., 2020) were introduced. We can also combine these *N*-soft set models with other methods such as (e.g., Li et al., 2018; Hou et al., 2019; Yin et al., 2019) to better solve practical problems. In this paper, we have introduced a new hybrid model called (*V*, *N*)-soft sets. Our aim is to explain the problems when the membership degree of data is vague and uncertain. It is the new model that provides more accuracy and flexibility in multi-attribute decision making problems.

The organization of this article is as follows. Section 2 provides with the relevant theoretical background. In Section 3, we introduce our new hybrid model and its basic operations. In the definition of grade, the idea of probability is innovatively integrated into the treatment of vague interval, and then the positive membership degree is predicted reasonably by vague interval. Finally, the grade

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is determined according to different attributes. We studied the relationships between existing models and (V, N)-soft sets in Section 4. In Section 5, we show an application of (V, N)-soft sets in decision making mechanism in order to prove its accuracy and feasibility. Finally, we give our conclusion in Section 6.

2. Preliminaries

In this section, for completeness of presentation and convenience of subsequent discussions, we shall recall several definitions which are useful for our paper. They are stated as follows:

Definition 2.1. (Gau and Buehrer, 1993) A vague set A in the universe $U=\{x_1,\ x_2,\ \cdots,\ x_n\}$ can be expressed by the following notion, $A=\{x_i,\ \ [t_A(x_i),1-f_A(x_i)] \mid \ x_i\in U\}$,i.e. $A(x_i)=[t_A(x_i),\ 1-\ f_A(x_i)]$ and the condition $0\le t_A(x_i)\le 1-f_A(x_i)$ should hold for any $x_i\in U$, where $t_A(x_i)$ is called the membership degree (true membership) of element x_i to the vague set A, while $f_A(x_i)$ is the degree of non-membership (false membership) of the element x_i to the set A.

Definition 2.2. (Gau and Buehrer, 1993) Let A, B be two vague sets in the universe $U=\{x_1,\ x_2,\ \cdots,\ x_n\}$, then the union, intersection and complement of vague sets are defined as follows:

$$A\cup B=\left\{\begin{matrix}(x_i,[\max(t_A(x_i),t_B(x_i)),\\ \max(1-f_A(x_i),1-f_B(x_i))] \mid x_i\in U\end{matrix}\right\}$$

$$A\cap B=\left\{\begin{matrix}(x_i,[\min(t_A(x_i),t_B(x_i))\\ ,\min(1-f_A(x_i),1-f_B(x_i))] \mid x_i\in U\end{matrix}\right\}$$

Definition 2.3. (Gau and Buehrer, 1993) Let A, B be two vague sets in the universe $U=\{x_1,\ x_2,\ \cdots,\ x_n\}$. If $\forall x_i\in U$, $t_A(x_i)\le t_B(x_i)$, $1-f_A(x_i)\le 1-f_B(x_i)$ then A is called a vague subset of B, denoted by $A\subseteq B$ where $1\ \le\ i\ \le\ n$.

Definition 2.4. (Gau and Buehrer, 1993) Let U be an initial universal set, V (U) the set of all vague subsets on U, E a set of parameters and $A\subseteq E$. A pair (F, A) is called a vague soft set over U, where F is a mapping given by F: $A\rightarrow V$ (U).

In other words, a vague soft set over U is a parameterized family of vague set of the universe U. For $\varepsilon\in A$, $\mu_{F(\varepsilon)}:U\rightarrow[0,1]^2$ is regarded as the set of ε-approximate elements of the vague soft set (F, A).

Definition 2.5. (Fatimah et al., 2018) Let U be a universe of objects under consideration and P is the set of attributes, $T\subseteq P$. Let $G=\{0,\ 1,\ 2,\cdots,\ N-1\}$ be the set of ordered grades where $N\in\{2,\ 3,\cdots\}$. A triple (F, T, N) is called an N-soft set on U if F is a mapping from T to $2^{U\times G}$, with the property that for each $t\in T$ and $x\in U$ there exists a unique $(x,g_t)\in U\times G$ such that $(x,g_t)\in F(t), g_t\in G$.

3. (V, N)-soft sets and their operations

This section presents our new model, which is based on the notion of N-soft sets (Fatimah et al., 2018) and vague sets. Afterwards, we explain its intuitive interpretation and suggest that a tabular representation simplifies its practical use.

3.1 The concept of (V, N)-soft sets

For notational convenience, we denote by $\upsilon(O)$ the set of all vague sets on O. When $I\in\upsilon(O)$ we also use $<o,I_O>\in I$ $(o\in O,I_O\subseteq[0,1])$ in order to mean $I_O=I(o)$. Similarly, if $O=\{o_1,\ o_2,\ \cdots,\ o_n\}$ is finite then we describe the mapping I as:

$$I=\{<o_1,I_{o_1}>,\cdots,<o_n,I_{o_n}>\}$$

$$=\{<o_1,[t_v(o_1),1-f_v(o_1)]>,\cdots,<o_n,[t_v(o_n),1-f_v(o_n)]>\}.$$

Definition 3.1. Let O be a universe of objects under consideration and E the set of attributes $T\subseteq E$. A pair (I, K) is called (V, N)-soft set when K = (V, T, N) is an N-soft set on O with $N\in\{2,\ 3,\cdots\}$, and I is a mapping , $I:T\rightarrow\bigcup_{t\in T}v(V(t))$ such that $I(t)\in v(V(t))$ for each $t\in T$.

According to Definition 3.1, with each attribute the mapping I assigns a vague set on the image of that attributes by the mapping v. Therefore for each $t\in T$ and there exists a unique $(o,g_t)\in O\times G$, such that $g_t\in$ G, and $<(o,g_t),I_t(o)>\ \in I(t)$, which is a notation that boils down to $I_t(o)=I(t)(o,g_t)$. This graded evaluation can easily identified by numbers. But when the membership degree of data is vague and uncertain, we need (V, N)-soft sets which provides us information to describe it, how these grades are given to candidates.

For example, let A be a vague set with truth membership function t_A and false membership function f_A, respectively. If $[t_A(o_i),1-f_A(o_i)]=[0.6,0.9]$, then we can see that $t_A(o_i)=0.6$, $1-f_A(o_i)=0.9$, so $f_A(o_i)=0.1$. It can be interpreted as "the vote for resolution is 6 in favor, 1 against, and 3 abstentions". Feng's expectation score function is

$$\delta(A)=\frac{t_A-f_A+1}{2}=\frac{t_A+t_A+h_A}{2}=t_A+\frac{h_A}{2}.$$

It aims to divide the degree of hesitation into two, and distribute it equally to truth membership and false membership, respectively. However, abstaining is a kind of hesitancy. It is not necessarily the same degree of support and opposition in real life. If we want to make decisions based on the views of everyone, we need to predict the hesitant parts, for which δ_P plays an important role. δ_P contains the idea of probability that the division of truth membership and false membership by hesitation should be a variable based on the ratio of truth membership to false membership. It is similar to the problem of gambling bonus distribution in probability theory. Feng's expectation score function (Feng et al., 2018) is updated by us as follows.

Definition 3.2. The expectation score function based on probability is a mapping $\delta_P:L^*\rightarrow[0,1]$ such that

$$\delta_P(A) = t_A + \frac{t_A}{t_A + f_A} \cdot h_A = \frac{t_A}{t_A + f_A},$$

for all $A = (t_A, f_A) \in L^*$, $h_A = 1 - t_A - f_A$.

δ_P also satisfies some equivalent propositions about rating of δ. And the expectation score function based on probability δ_P satisfies many satisfactory properties shown as follows.

Proposition 3.3. Let $\delta_P : L^* \to [0,1]$ be the expectation score function based on probability and $A = (t_A, f_A) \in L^*$, $h_A = 1 - t_A - f_A$. Then, we have

(1) $\delta_P(0,1) = 0$;
(2) $\delta_P(1,0) = 1$;
(3) $\delta_P(t_A, f_A)$ is increasing with respect to t_A;
(4) $\delta_P(t_A, f_A)$ is decreasing with respect to f_A.

Definition 3.4. Let $A = (t_A, f_A)$, and $B = (t_B, f_B)$, be intuitionistic fuzzy vague sets in L^*. The binary relation $\leq_{(t,\delta_P)}$ on L^* is defined as

$$A \leq_{(t,\delta_P)} B \Leftrightarrow t_A \leq t_B \vee (t_A = t_B \wedge \delta_P(A) \leq \delta_P(B)).$$

Considering that the rating standard of each attribute is inconsistent, so we will sort and rate the objects under discussion based on each attribute. Suppose that $A \leq_{(t,\delta_P)} B$, we consider the following three cases:

(1) If $t_A \leq t_B$, then $A \leq_{(t,\delta_P)} B$;

(2) If $t_A = t_B$, and $\delta_P(A) \leq \delta_P(B)$, then $A \leq_{(t,\delta_P)} B$;

(3) If $t_A = t_B$, and $\delta_P(A) = \delta_P(B)$, then $A =_{(t,\delta_P)} B$.

Example 3.5. Let $O = \{o_1, o_2, o_3, o_4\}$ be the universe of candidates and E be the set of evaluation attributes. $T \subseteq E$, $T = \{t_1, t_2, t_3\}$. A $(V, 4)$-soft set can be obtained from Table 1.

Table 1: Tabular representation of $(V, 4)$-soft set (I, K)

(I, K)	t_1	t_2	t_3
o_1	< 2, [0.30,0.60] >	< 2, [0.40,0.52] >	< 2, [0.30,0.50] >
o_2	< 1, [0.10,0.36] >	< 1, [0.30,0.40] >	< 3, [0.50,0.65] >
o_3	< 3, [0.60,0.80] >	< 3, [0.60,0.70] >	< 1, [0.25,0.55] >
o_4	< 0, [0.10,0.30] >	< 0, [0.20,0.40] >	< 0, [0.14,0.20] >

Clearly, general representation such as the element $< g_{ij}, I_{ij} >$ in cell (i, j) means $<(o_i, g_{ij}), I_{ij} > \in I(t_j)$. From Table 1, we can get

$$I(t_1) = \{<(o_1, 2), [0.30,0.60]>, <(o_2, 1), [0.10,0.36]>, <(o_3, 3), [0.60,0.80]>, <(o_4, 0), [0.10,0.30]>\}.$$

3.2 Basic operations for (V, N)-soft sets

Now we proceed to define some basic algebraic operations in the new framework that we have introduced.

Definition 3.6. Let (I_1, K_1) and (I_2, K_2) be two (V, N)-soft sets on a universe O, where $K_1 = (V_1, T, N_1)$ and $K_2 = (V_2, S, N_2)$ are N-soft sets. Then (I_1, K_1) and (I_2, K_2) are said to be equal if and only if $K_1 = K_2$ and $I_1 = I_2$.

Definition 3.7. Let O be a universe of objects, and let (I_1, K_1) and (I_2, K_2) be two (V, N)-soft sets, where $K_1 = (V_1, T, N_1)$ and $K_2 = (V_2, S, N_2)$ are N-soft sets on O. Then their restricted intersection is denoted by $(I_1, K_1) \cap_R (I_2, K_2)$ and it is defined as $(\zeta, K_1 \cap_R K_2)$, where

$$K_1 \cap_R K_2 = (E, T \cap S, \min(N_1, N_2)),\ \forall t_j \in T \cap S \text{ and}$$

$$o_i \in O,\ (g_{ij}, I_{ij}) \in \zeta(t_j) \Leftrightarrow$$

$$g_{ij} = \min(g_{ij}^1, g_{ij}^2),$$

$$I_{ij} = [\min(t_{v_1}(o_i), t_{v_2}(o_i)), \min(1 - f_{v_1}(o_i), 1 - f_{v_2}(o_i))],$$

$$(g_{ij}^1, I_{ij}^1) \in I_1(t_j) \text{ and } (g_{ij}^2, I_{ij}^2) \in I_2(t_j), \text{ where } t_j^1 \in T, t_j^2 \in S.$$

Example 3.8. Consider (I_1, K_1) and (I_2, K_2), the two different $(V, 4)$-soft sets defined by Table 2 and Table 3 respectively, where $K_1 = (V_1, T, 4)$ and $K_2 = (V_2, S, 4)$ are N-soft sets on O. Their restricted intersection $(I_1, K_1) \cap_R (I_2, K_2) = (\zeta, K_1 \cap_R K_2)$ is given by Table 4.

Table 2: Tabular representation of $(V, 4)$-soft set (I_1, K_1)

(I_1, K_1)	t_1	t_2	t_3
o_1	< 2, [0.30,0.60] >	< 2, [0.40,0.52] >	< 2, [0.30,0.50] >
o_2	< 1, [0.10,0.36] >	< 1, [0.28,0.40] >	< 3, [0.45,0.65] >
o_3	< 3, [0.60,0.80] >	< 3, [0.70,0.86] >	< 1, [0.25,0.45] >
o_4	< 0, [0.10,0.30] >	< 0, [0.20,0.40] >	< 0, [0.04,0.10] >

Table 3: The $(V, 4)$-soft set (I_2, K_2)

(I_2, K_2)	t_1	t_2	S
o_1	< 1, [0.10,0.30] >	< 3, [0.40,0.52] >	< 2, [0.30,0.50] >
o_2	< 3, [1.00,1.00] >	< 2, [0.28,0.40] >	< 3, [0.60,0.80] >
o_3	< 2, [0.20,0.40] >	< 0, [0.20,0.34] >	< 1, [0.22,0.40] >
o_4	< 0, [0.08,0.10] >	< 1, [0.20,0.40] >	< 0, [0.15,0.30] >

Table 4: The restricted intersection $(I_1, K_1) \cap_R (I_2, K_2)$

$(I_1, K_1) \cap_R (I_2, K_2)$	t_1	t_2
o_1	< 1, [0.10,0.30] >	< 2, [0.40,0.52] >

o_2	< 1, [0.10,0.36] >	< 1, [0.28,0.40] >
o_3	< 2, [0.20,0.40] >	< 0, [0.20,0.34] >
o_4	< 0, [0.08,0.10] >	< 0, [0.20,0.40] >

Definition3.9. Let O be universe of objects, and let (I_1, K_1) and (I_2, K_2) be two different (V, N)-soft sets, where $K_1 = (V_1, T, N_1)$ and $K_2 = (V_2, S, N_2)$ are N-soft sets on O. Then their extended intersection is denoted by $(I_1, K_1) \cap_E (I_2, K_2)$ and it is defined as $(\rho, K_1 \cap_E K_2)$, where $K_1 \cap_E K_2 = (E, T \cup S, \max(N_1, N_2))$,

and $\forall t_j \in T \cup S$, $o_i \in O$ ρ_{tj} is given by

$$\rho_{tj} = \begin{cases} I_1 t_j = [t_{v_1}(o_i), 1 - f_{v_1}(o_i)], t_j \in T - S, \\ I_2 t_j = [t_{v_2}(o_i), 1 - f_{v_2}(o_i)], t_j \in S - T, \\ I_{ij} = [\min(t_{v_1}(o_i), t_{v_2}(o_i)), \min(1 - f_{v_1}(o_i), 1 - f_{v_2}(o_i))], \\ where \ \ (g_{ij}^1, g_{ij}^2) \in I_1(t_j), (g_{ij}^2, g_{ij}^2) \in I_1(t_j), t_j^1 \in T, t_j^2 \in S. \end{cases}$$

Example 3.10. In the situation of Example 3.8, the extended intersection $(I_1, K_1) \cap_E (I_2, K_2) = (\rho, K_1 \cap_E K_2)$ is given by Table 5.

Table 5: The extended intersection $(I_1, K_1) \cap_E (I_2, K_2)$

(I_1, K_1) $\cap_E$ (I_2, K_2)	t_1	t_2	t_3	s
o_1	< 1, [0.10,0.30] >	< 2, [0.40,0.52] >	< 2, [0.30,0.50] >	< 2, [0.30,0.50] >
o_2	< 1, [0.10,0.36] >	< 1, [0.28,0.40] >	< 3, [0.45,0.65] >	< 3, [0.60,0.80] >
o_3	< 2, [0.20,0.40] >	< 0, [0.20,0.34] >	< 1, [0.25,0.45] >	< 1, [0.22,0.40] >
o_4	< 0, [0.08,0.10] >	< 0, [0.20,0.40] >	< 0, [0.04,0.10] >	< 0, [0.15,0.30] >

Definition 3.11. Let O be a universe of objects, and let (I_1, K_1) and (I_2, K_2) be two (V, N)-soft sets, where $K_1 = (V_1, T, N_1)$ and $K_2 = (V_2, S, N_2)$ are N-soft sets on O. Then their restricted union is denoted by $(I_1, K_1) \cup_R (I_2, K_2)$ and it is defined as $(\beta, K_1 \cup_R K_2)$, where

$$K_1 \cup_R K_2 = (E, T \cap S, \max(N_1, N_2)), \ \forall t_j \in T \cap S \text{ and}$$

$$o_i \in O, (g_{ij}, I_{ij}) \in \beta(t_j) \Leftrightarrow$$

$$g_{ij} = \max(g_{ij}^1, g_{ij}^2)$$

$$I_{ij} = [\max(t_{v_1}(o_i), t_{v_2}(o_i)), \max(1 - f_{v_1}(o_i), 1 - f_{v_2}(o_i))],$$

$$(g_{ij}^1, I_{ij}^1) \in I_1(t_j) \text{ and } (g_{ij}^2, I_{ij}^2) \in I_2(t_j), \text{ where } t_j^1 \in T, t_j^2 \in S.$$

Example 3.12. In the situation of Example 3.8, the restricted union $(I_1, K_1) \cup_R (I_2, K_2) = (\beta, K_1 \cup_R K_2)$ is given by Table 6.

Table 6: The restricted union $(I_1, K_1) \cup_R (I_2, K_2)$

$(I_1, K_1) \cup_R (I_2, K_2)$	t_1	t_2
o_1	< 2, [0.30,0.60] >	< 3, [0.40,0.52] >
o_2	< 3, [1.00,1.00] >	< 2, [0.28,0.40] >
o_3	< 3, [0.60,0.80] >	< 3, [0.70,0.86] >
o_4	< 0, [0.10,0.30] >	< 1, [0.20,0.40] >

Definition 3.13. Let O be universe of objects, and let (I_1, K_1) and (I_2, K_2) be two different (V, N)-soft sets, where $K_1 = (V_1, T, N_1)$ and $K_2 = (V_2, S, N_2)$ are N-soft sets on O. Then their extended union is denoted by $(I_1, K_1) \cup_E (I_2, K_2)$ and it is defined as $(\xi, K_1 \cup_E K_2)$, where $K_1 \cup_E K_2 = (E, \ \ T \cup S, \max(N_1, N_2))$,

and $\forall t_j \in T \cup S$, $o_i \in O$ ξ_{tj} is given by

$$\xi_{tj} = \begin{cases} I_1 t_j = [t_{v_1}(o_i), 1 - f_{v_1}(o_i)], t_j \in T - S, \\ I_2 t_j = [t_{v_2}(o_i), 1 - f_{v_2}(o_i)], t_j \in S - T, \\ I_{ij} = [\max(t_{v_1}(o_i), t_{v_2}(o_i)), \max(1 - f_{v_1}(o_i), 1 - f_{v_2}(o_i))], \\ where \ \ (g_{ij}^1, g_{ij}^2) \in I_1(t_i), (g_{ij}^2, g_{ij}^2) \in I_1(t_j), t_j^1 \in T, t_j^2 \in S. \end{cases}$$

Example 3.14. In the situation of Example 3.8, the extended union $(I_1, K_1) \cup_E (I_2, K_2) = (\xi, K_1 \cup_E K_2)$ is given by Table 7.

Table 7: The extended union $(I_1, K_1) \cup_E (I_2, K_2)$

(I_1, K_1) $\cup_E$ (I_2, K_2)	t_1	t_2	t_3	s
o_1	< 2, [0.30,0.60] >	< 3, [0.40,0.52] >	< 2, [0.30,0.50] >	< 2, [0.30,0.50] >
o_2	< 3, [1.00,1.00] >	< 2, [0.28,0.40] >	< 3, [0.45,0.65] >	< 3, [0.60,0.80] >
o_3	< 3, [0.60,0.80] >	< 3, [0.70,0.86] >	< 1, [0.25,0.45] >	< 1, [0.22,0.40] >
o_4	< 0, [0.10,0.30] >	< 1, [0.20,0.40] >	< 0, [0.04,0.10] >	< 0, [0.15,0.30] >

Proposition 3.15. Let (I_1, K_1), (I_2, K_2) and (I_3, K_3) be three (V, N)-soft sets. Supposed that they have the same objects of study, we can obtain the following properties from the above definitions:

(1) $(I_1, K_1) \cap_R (I_2, K_2) = (I_2, K_2) \cap_R (I_1, K_1)$;

(2) $(I_1, K_1) \cap_E (I_2, K_2) = (I_2, K_2) \cap_E (I_1, K_1)$;

(3) $(I_1, K_1) \cup_R (I_2, K_2) = (I_2, K_2) \cup_R (I_1, K_1)$;

(4) $(I_1, K_1) \cup_E (I_2, K_2) = (I_2, K_2) \cup_E (I_1, K_1)$;

(5) $((I_1, K_1) \cap_R (I_2, K_2)) \cap_R (I_3, K_3)$
$= (I_2, K_2) \cap_R ((I_1, K_1) \cap_R (I_3, K_3))$;

(6) $((I_1, K_1) \cup_R (I_2, K_2)) \cup_R (I_3, K_3)$

$= (I_2, K_2) \cup_R ((I_1, K_1) \cup_R (I_3, K_3)).$

4. Some relationships

The concept of (*V*, *N*)-soft sets can be related to both *N*-soft sets, soft sets and vague soft sets. We explain these relationships in this section. *O* denotes a universe of objects and (*I*, *K*) is an (*V*, *N*)-soft set, where *K* = (*V*, *T*, *N*) is an *N*-soft set. We say that the *N*-soft set associated with (*I*, *K*) is *K*. From another perspective, this simple assignment shows that (*V*, *N*)-soft sets generalize *N*-soft sets, therefore soft sets as well. In order to derive vague soft sets and soft sets from (*I*, *K*) we use the following definition:

Definition 4.1. Let 0 < *R* < *N* be a threshold. The vague soft set over *O* associated with (*I*, *K*) and *R* is and (v^R, T) defined by: for each $t \in T$, $(v^R, T) \in v(O)$ is such that

$$v^R(t)(o) = \begin{cases} I(t)(o, g_t), & if \quad (o, g_t) \in V(t), g_t \geq R, \\ [0,0], & otherwise. \end{cases}$$

The computations from the tabular form of (*I*, *K*) are very simple: for every cell where the grade is at least as good as R, we associate its vague interval; otherwise we associate [0,0]. In this way we obtain the tabular representation of the associated vague soft set over *O*. Now we rely on two thresholds to associate soft sets with (*V*, *N*)-soft sets.

Definition 4.2. Let 0 < *R* < *N* and $I_\tau \subseteq [0,1]$ be thresholds. The vague soft set over *O* associated with (*I*, *K*) and (R, I_τ) is and $(f^{(R,I_\tau)}, T)$ defined by the assignment: for each $t \in T$,

$$(f^{(R,I_\tau)}, T) = \{o \in O : I^R(t)(o) \supseteq I_\tau\}.$$

Example 4.3. Consider the (*V*, 4)-soft set (I_p, K_p) where $K_p = (V_p, T, N_p)$ is a 4-soft set, represented by Table 8. From Definition 4.1, we have 0 < *R* < 4. The possible vague soft sets associated with feasible thresholds 1-3 and (I_p, K_p) is given by Tables 9-11.

Table 8: Tabular representation of (*V*, 4)-soft set (I_p, K_p)

(I_p, K_p)	t_1	t_2	t_3
o_1	< 2, [0.50,0.60] >	< 0, [0.30,0.40] >	< 2, [0.30,0.50] >
o_2	< 1, [0.20,0.36] >	< 1, [0.38,0.40] >	< 3, [0.55,0.65] >
o_3	< 3, [0.70,0.80] >	< 3, [0.80,0.86] >	< 0, [0.24,0.30] >
o_4	< 0, [0.10,0.40] >	< 2, [0.40,0.60] >	< 1, [0.25,0.45] >

Table 9: Vague soft set associated with (I_p, K_p) and threshold 1

(I_p, K_p)	t_1	t_2	t_3
o_1	[0.50,0.60]	[0.00,0.00]	[0.30,0.50]
o_2	[0.20,0.36]	[0.38,0.40]	[0.55,0.65]
o_3	[0.70,0.80]	[0.80,0.86]	[0.00,0.00]
o_4	[0.00,0.00]	[0.40,0.60]	[0.25,0.45]

Table 10: Vague soft set associated with (I_p, K_p) and threshold 2

(I_p, K_p)	t_1	t_2	t_3
o_1	[0.50,0.60]	[0.00,0.00]	[0.30,0.50]
o_2	[0.00,0.00]	[0.00,0.00]	[0.55,0.65]
o_3	[0.70,0.80]	[0.70,0.86]	[0.00,0.00]
o_4	[0.00,0.00]	[0.40,0.60]	[0.00,0.00]

Table 11: Vague soft set associated with (I_p, K_p) and threshold 3

(I_p, K_p)	t_1	t_2	t_3
o_1	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
o_2	[0.00,0.00]	[0.00,0.00]	[0.55,0.65]
o_3	[0.70,0.80]	[0.70,0.86]	[0.00,0.00]
o_4	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]

5. Extended grey relational analysis method

Grey relational analysis is a multi-factor statistical method, which is based on the sample data of each factor and uses grey correlation degree to describe the strength, size and order of the relationship between factors. If the trend of the two factors reflected by the sample data is basically the same, then the correlation degree between them is larger; on the contrary, the correlation degree is small. The advantages of this method are that the train of thought is clear, the loss caused by information asymmetry can be reduced to a great extent, and the data requirements are low and the workload is less. The main disadvantage of this method is that the optimal value of each index needs to be determined at present, the subjectivity is too strong, and the optimal value of some indexes is difficult to determine. However, the grey relational analysis method based on (*V*, *N*)-soft sets can make up for this disadvantage. It is easy to determine the optimal value by truth membership function t and the expectation score function δ_p. At the same time, it has objectivity. Next, we extend this method to deal with (*V*, *N*)-soft sets.

Let $O = \{o_1, \cdots, o_i, \cdots, o_p\}$ be the universe and $T = \{t_1, \cdots, t_j, \cdots, t_q\}$ be the set of parameters considered by the decision makers. The decision makers have an authority to assign weights to each parameter according to their choice.

At the same time, each attribute has its own rankings. We suppose that the correlation coefficient is $r_{o_i}(j)$. Our formula is as follows:

$$r_{o_i}(j) = \frac{\Delta(\min) + \rho\Delta(\max)}{\Delta i(j) + \rho\Delta(\max)}, \rho \in (0,1). \tag{1}$$

$$\begin{aligned}\Delta(\max) &= \max_i \max_j \Delta i(j) \\ &= <\max(g_{ij}),[\max(t_{ij}),\max(1-f_{ij})]> \\ &= <\max(g_{ij}),\max(\delta_{p_{ij}})> \\ &= <3,[0.70,0.86]> \\ &= <3,0.83>.\end{aligned} \tag{2}$$

$$\begin{aligned}\Delta(\min) &= \min_i \min_j \Delta i(j) \\ &= <\min(g_{ij}),[\min(t_{ij}),\min(1-f_{ij})]> \\ &= <\min(g_{ij}),\min(\delta_{p_{ij}})> \\ &= <0,[0.04,0.10]> \\ &= <0,0.04>.\end{aligned} \tag{3}$$

$$\begin{aligned}X_0 &= \{x_0(1),x_0(2),\cdots,x_0(q)\} \\ &= \{<\max(g_{i1}),\max(\delta_{p_{i1}})>,\cdots,<\max(g_{iq}),\max(\delta_{p_{iq}})>\} \\ &= \{<3,0.75>,<3,0.83>,<3,0.56>\}.\end{aligned} \tag{4}$$

$$\begin{aligned}X_i &= \{x_i(1),x_i(2),\cdots,x_i(q)\} \\ &= \{<g_{i1},[t_{i1},1-f_{i1}]>,\cdots,<g_{iq},[t_{iq},1-f_{iq}]>\} \\ &= \{<g_{i1},\delta_{p_{i1}}>,\cdots,<g_{iq},\delta_{p_{iq}}>\}.\end{aligned} \tag{5}$$

$$\Delta i(j) = \left|x_0(j)-x_i(j)\right|, i=1,2,\cdots,p;\ j=1,2,\cdots,q. \tag{6}$$

Among them, ρ is the resolution coefficient used to weaken the influence of Δ(max) and distort the correlation coefficient. At the same time, it can improve the significant difference between the correlation coefficients. Its value is between 0 and 1. ρ usually takes a value of 0.5. $\overline{o}_i$ is the mean value of the object o_i after synthesizing each attribute.

Example 5.1. In a university, the champion of the annual speech contest final was generated by 100 mass-reviewed votes. Let $O=\{o_1,\ o_2,\cdots,\ o_p\}$ be the universe of the 4 contestants entering the final and $T=\{t_1,\ t_2,\ t_3\}$ be the set of parameters considered by the reviewers. So, we can see the real date from Table 12.

Table 12: Real data of actual voting results

(Support, Against)	t_1	t_2	t_3
o_1	(30, 40)	(15, 70)	(25, 55)
o_2	(10, 64)	(70, 14)	(4, 90)
o_3	(60, 20)	(40, 48)	(30, 50)
o_4	(10, 70)	(22, 60)	(45, 35)

This information is enough, because we usually make a final choice based on the degree of support. However, in this selection, some of the reviewers did not have time to show their opinions at the specified time, that is, they were hesitant. In many cases, if these hesitant factors are not taken into account, it will have an impact on the performance of contestants. So, in this complex situation, we may need to use (*V*, *N*)-soft sets, which provide us with more flexible information about how these grades are given to contestants. We use $\delta_{p_{ij}}$ to predict the views of reviewers who do not have time to vote, which improves the reliability of the results.

A vague soft set extracted from real data can be described in Table 13. For example, we find 30 supported and 40 opposed for o_1, others are hesitating. Thus,

$$t_1(x_1)=\frac{30}{100}=0.30,$$

$$1-f_1(x_1)=1-\frac{40}{100}=0.60,$$

$$\delta_{p_{11}}=\frac{t_1(x_1)}{t_1(x_1)+f_1(x_1)}=0.43.$$

Table 13: Real data of actual voting results

($t_j(x_i)$, 1- $f_j(x_i)$)	t_1	t_2	t_3
o_1	[0.30,0.60]	[0.15,0.30]	[0.25,0.45]
o_2	[0.10,0.36]	[0.70,0.86]	[0.04,0.10]
o_3	[0.60,0.80]	[0.40,0.52]	[0.30,0.50]
o_4	[0.10,0.30]	[0.22,0.40]	[0.45,0.65]

Then, the grade of objects under each attribute is given by definition 3.4, as shown in Table 14. It is easy to obtain tabular representation of (*V*, 4)-soft set (*I*o, *K*o) in Table 15. Next, we can get Table 16 by formula (6). Finally, the correlation coefficient and their means are shown in Table 17. It is easy to see $o_3>o_2>o_4>o_1$. So, o_3 is the champion according to the opinion of all reviewers.

Table 14: Tabular representation of X_i

(g_{pij} δ_{pij})	t_1	t_2	t_3
o_1	< 2, 0.43 >	< 0, 0.18 >	< 1, 0.31 >
o_2	< 1, 0.14 >	< 3, 0.83 >	< 0, 0.04 >
o_3	< 3, 0.75 >	< 2, 0.45 >	< 2, 0.38 >
o_4	< 0, 0.13 >	< 1, 0.27 >	< 3, 0.56 >

Table 15: Tabular representation of (*V*, 4)-soft set (*I*o, *K*o)

(*I*o, *K*o)	t_1	t_2	t_3
o_1	< 2, [0.30,0.60] >	< 0, [0.15,0.30] >	< 1, [0.25,0.45] >
o_2	< 1, [0.10,0.36] >	< 3, [0.70,0.86] >	< 0, [0.04,0.10] >
o_3	< 3, [0.60,0.80] >	< 2, [0.40,0.52] >	< 2, [0.30,0.50] >
o_4	< 0, [0.10,0.30] >	< 1, [0.22,0.40] >	< 3, [0.45,0.65] >

Table 16: Absolute difference of X_i and X_0

(g_{pij} δ_{pij})	t_1	t_2	t_3
o_1	< 1, 0.32 >	< 3, 0.65 >	< 2, 0.25 >
o_2	< 2, 0.61 >	< 0, 0.00 >	< 3, 0.52 >

o_3	< 0, 0.00 >	< 1, 0.38 >	< 1, 0.18 >
o_4	< 3, 0.62 >	< 2, 0.56 >	< 0, 0.00 >

Table 17: Correlation coefficient of X_i and X_0

(g_{pij} δ_{pij})	t_1	t_2	t_3	$\overline{o}_i$
o_1	< 0.60,0.61 >	< 0.33,0.42 >	< 0.43,0.68 >	< 0.45,0.57 >
o_2	< 0.43,0.44 >	< 1.00,1.10 >	< 0.33,0.48 >	< 0.59,0.67 >
o_3	< 1.00,1.10 >	< 0.60,0.56 >	< 0.60,0.76 >	< 0.73,0.81 >
o_4	< 0.33,0.43 >	< 0.43,0.46 >	< 1.00,1.10 >	< 0.59,0.66 >

6. Summary

In the present paper, we have introduced a new hybrid model called (V, N)-soft sets, by combining vague set theory with N-soft sets. We have put forward real life examples that adopt the format of (V, N)-soft sets. In the definition of grade, the idea of probability is innovatively integrated into the treatment of vague interval, and then the positive membership degree is predicted reasonably by vague interval. The grade is determined according to different attributes. Moreover, we have investigated the basic operations of (V, N)-soft sets. Finally, we have explored grey relational analysis method based on (V, N)-soft sets.

The model provides more flexibility in multi-attribute decision making problems. In the era of big data, there will be more and more cases to prove the effectiveness of this approach. In the future, we can try to build richer relationships between soft sets and other soft computing models.

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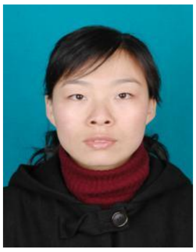

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