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Rough N-soft Set and Its Application in Decision Making

Xiaomin Wang*, Shengnan Cao, Piyu Li, Yang Liu

[a] *School of Mathematics and Statistics, Northeastern University at Qinhuangdao, Qinhuangdao, Hebei Province, China, 066004*

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ABSTRACT

Rough set theory is based on the equivalence relationship using the defined upper and lower approximate relationships to describe uncertain and incomplete information and it are a systematic approach for the classification of objects. As a multi-valued extension of soft sets, N-soft set theory is an effective model for binary and non-binary evaluation in a variety of decision problems, and it is an effective method to solve the problems of ranking system. In this paper, a new hybrid model called rough N-soft sets is introduced. It is a suitable combination of rough sets and N-soft sets. Basic properties and relevant definitions of rough N-soft sets are presented. Furthermore, an algorithm for decision-making problems is proposed. Finally, the validity of this model is proved by the application of the recruitment.

1. Introduction

The rough set theory, which was first proposed by Pawlak (Pawlak, 1982), and the soft set initiated by Russian scholar Molodtsov (Molodtsov, 1999), are two powerful mathematical tools to model various types of uncertainty. Soft set theory showed its potential applicability and significant uses in many different fields, such as medicine and probability theory (Molodtsov, 1999; Molodtsov, 2004). Until now, the research on soft sets is very active and progressing rapidly. Maji (Maji et al., 2003) defined and studied several calculation methods on soft sets. Chen (Chen et al., 2005) presented a new definition of soft set parametrization reduction, and compared it with attributes reduction in rough set theory. Kong (Kong et al., 2008) defined the parameter reduction of soft sets, by which they investigated the problem of suboptimal choice and added parameter set in soft set parametrization reduction. Ali (Ali et al., 2009) developed the idea of complement of soft set. For rough set theory, we discover that decision-making mechanism to solve practical problems is necessary. Yao (Yao, 2010) analyzed the three-way decision rule in the rough set model of decision theory. What's more, Cagman N and Maji (Cagman and Enginoglu , 2010; Cagman and Enginoglu , 2010; Maji et al., 2002) applied soft sets to decision-making. Maji (Maji et al., 2002) discussed the use of soft sets in decision-making problems, which was a first applied soft set to solve the decision-making problems based on the choice values.

Over the years, the theories of rough sets and soft sets have become closer and closer. Aktas H (Aktas and Cagman, 2007) related soft sets to fuzzy sets and rough sets. Further, Feng (Feng et al., 2011) introduced soft sets and soft rough sets. Inspired by Dubois and Prade's ideas on rough fuzzy sets, Feng F (Feng et al., 2010) considered the upper and lower approximations of soft sets in Pawlak approximation space, thus naturally generating the concept of rough soft sets. Ali M I (Ali, 2011) defined the approximate space of the rough soft set. Zhan, Jianming and Zhu, Kuanyun (Zhan and Zhu, 2015) constructed a new rough soft set decision method. Roy (Roy and Bera, 2015) established link between soft sets and rough sets in connection with an application in lattice. Liu Y (Liu et al., 2018) provided several proposals for decision making based on the hybrid soft sets of fuzzy soft sets and rough soft sets. Sun B (Sun and Ma, 2014) gave an approach to decision-making problems based on soft fuzzy rough set models.

For the most part, researchers in soft set theory models use binary evaluations (0 or 1), or use real numbers between 0 and 1. But in fact, we often find non-binary evaluations in many areas. For example, in the social judgment system, Alcantud (Alcantud and Laruelle, 2014) specified a ternary voting system. In addition, Herawan (Herawan and Deris, 2009) pointed out n binary-valued information systems in soft sets, where each parameter has its own ranking, compared to the ranking order described by Chen (Chen et al., 2013). Instead of ranking as an assessment, Ali (Ali et al., 2015) set the parameter elements in the soft set of the organized scoring system. To solve this problem, Fatima (Fatimah et al., 2018) designed an extended soft set model, N-soft set, which allows finer

* Corresponding author.
E-mail addresses: wangxiaomin@neuq.edu.cn (X. Wang)

granularity in parameterization. Some algebra definitions and properties are given and it is proved by example that N-soft set is a convincing model of binary and non-binary evaluation in many decision problems. However, the concept of N-soft sets is insufficient to provide the information about the occurrence of ratings or grades, and it is also unable to describe the occurrence of uncertainty and vagueness specially. For this purpose, Akram (Akram et al., 2019; Akram et al., 2018; Akram et al., 2018) introduced the novel models with applications called Fuzzy N-soft sets and hesitant N-soft sets as the generalization of N-soft sets. Alcantud (Alcantud et al., 2019) reveal a close connection between N-soft sets and rough structures of various types. There is also a combination of N-soft sets and other concepts (e.g., Riaz et al., 2019; Hüseyin et al., 2020; Zhang et al., 2020).

Similarly, by the hybridization of two well-known concepts called rough sets and N-soft sets, we introduce a new hybrid model called rough N-soft sets, record as (R,N)-soft set. This model provides more accuracy and flexibility as compared to previously existing approaches, because it contains more information and it is more reasonable and comprehensive. Proposed model provides complete information about occurrence of ratings, uncertainty. It is also valuable because we extend the method of N-soft sets. In this review, we try to describe rough N-soft sets in decision-making in more details. The rest of this paper is organized as follows. In section 2, we first recall the basic concepts and properties of rough sets, N-soft sets and decision information system. In section 3, we give related definitions and properties of rough N-soft sets. In section 4, we show decision-making based on rough N-soft sets and put forward a revised algorithm. In section 5, we present conclusion.

2. Preliminaries

In the section, we recall some basic notions, such as rough set, N-soft set and decision information system, that are useful for discussion in the next section.

2.1 Rough sets

Definition 1 (Pawlak, 1982) Let R be an equivalence relation on the universe U, $K=(U,R)$ be a Pawlak approximation space. A subset $X\subseteq U$ is called definable if $\underline{Apr}_R(X)=\overline{Apr}_R(X)$; in the opposite case, i.e., if $\underline{Apr}_R(X)-\overline{Apr}_R(X)\neq\varnothing$, X is said to be a rough set, where the two operations are defined as:

$$\underline{Apr}_R(X)=\{x\in U:[x]_R\subseteq X\}, \tag{1}$$

$$\overline{Apr}_R(X)=\{x\in U:[x]_R\cap X\neq\varnothing\} \tag{2}$$

assigning to every subset $X\subseteq U$, two sets $\overline{Apr}_R(X)$ and $\underline{Apr}_R(X)$ are called the lower and upper approximations of X with respect to (U,R). In addition,

$$Pos_R\{X\}=\underline{Apr}_R(X),$$
$$Neg_R\{X\}=U-\overline{Apr}_R(X),$$
$$Bnd_R\{X\}=\overline{Apr}_R(X)-\underline{Apr}_R(X)$$

are called the positive, negative, and boundary regions of X, respectively.

2.2 N-soft sets

Definition 2 (Fatimah et al., 2018) Let U be a universe set of objects and E be attributes, $A\subseteq E$. Let $G_N=\{0,1,\ldots,N-1\}$ be a set of ordered grades where $N\in\{2,3,\ldots\}$. We say that (F,A,N) is an N-soft set on U if $F:A\to 2^{U\times G_N}$. With the property that for each $a\in A$ and there exists a unique $(u,r_a)\in U\times G_N$ such that $(u,r_a)\in F(a)$, $u\in U$, $r_a\in G_N$.

Given attribute a, every object u in U receives exactly one evaluation from the assessments space G_N, namely the unique r_a for which $(u,r_a)\in F(a)$. We also write $F(a)(u)=r_a$ as a shorthand for $(u,r_a)\in F(a)$. Henceforth, we assume that both $U=u_i$, $i=1,2,\ldots,p$ and $A=a_j$, $j=1,2,\ldots,q$ are finite unless otherwise stated. Clearly, in that case the N-soft set can be presented by a tabular form as well where r_{ij} means $(u_i,r_{ij})\in F(a_j)$ or $F(a_j)(u_i)=r_{ij}$. As shown in Tab. 1.

Tab. 1. Tabular form of N-soft set

(F,A,N)	a_1	a_2	$\cdots$	a_q
u_1	r_{11}	r_{12}	$\cdots$	r_{1q}
u_2	r_{21}	r_{22}	$\cdots$	r_{2q}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
u_p	r_{p1}	r_{p2}	$\cdots$	r_{pq}

Definition 3 (Fatimah et al., 2018) Let U be a fixed universe of objects. The restricted intersection of (F,A,N_1) and (G,B,N_2) is denoted by $(F,A,N_1)\cap_{\Re}(G,B,N_2)$. It is defined as $(H,A\cap B,\min(N-1,N-2))$ where for all $a\in A\cap B$ and $u\in U$, $(u,r_a)\in H(a)\Leftrightarrow r_a\min(r_a^1,r_a^2)$ if $(u,r_a^1)\in F(a)$ and $(u,r_a^2)\in G(a)$. The restricted union of (F,A,N_1), and (G,B,N_2) is denoted by $(F,A,N_1)\cup_{\Re}(G,B,N_2)$. It is defined as $(K,A\cap B,\max(N-1,N-2))$ where for all $a\in A\cap B$ and $u\in U$. $(u,r_a)\in K(a)\Leftrightarrow r_a\max(r_a^1,r_a^2)$, if $(u,r_a^1)\in F(a)$ and $(u,r_a^2)\in G(a)$. The extended intersection of (F,A,N_1) and (G,B,N_2) is denoted by $(F,A,N_1)\cap_{\varepsilon}(G,B,N_2)$. It is $(J,A\cup B,\max(N_1,N_2))$ which is defined by

$$J(a)=\begin{cases} F(a) & if\ a\in A\setminus B \\ G(a) & if\ a\in B\setminus A \\ (u,r_a)\text{ such that } r_a=\min(r_a^1,r_a^2) \\ (u,r_a^1)\in F(a)\ and\ (u,r_a^2)\in G(a) \end{cases}$$

The extended intersection of (F,A,N_1) and (G,B,N_2) is denoted by $(F,A,N_1)\cup_{\varepsilon}(G,B,N_2)$. It is

$\left(L, A\bigcup B, \max\left(N_1, N_2\right)\right)$, and it is defined by

$$L(a)=\begin{cases} F(a) \; if \;\; a\in A\setminus B \\ G(a) \;\; if \;\; a\in B\setminus A \\ (u,r_a) \; such \; that \; r_a=\max\left(r_a^1,r_a^2\right) \\ \left(u,r_a^1\right)\in F(a) \; and \; \left(u,r_a^2\right)\in G(a) \end{cases}$$

Definition 4 (Pawlak et al., 2007) A decision information system can be defined as $I=\left(U, At=C\bigcup D, V_a\middle| a\in At,\, f_a\middle| a\in At\right)$. where U be a nonempty set of objects, At be a nonempty set of attributes, C be a set of condition attributes, D be a decision attribute, V_a be a nonempty set of values for each attribute $a\in At$, and $f_a: U\to V_a$ be an information function for each attribute $a\in At$.

3. Rough N-soft set

Our next definition introduces a novel model that emerges from the hybridization of N-soft sets and rough sets. Naturally we consider the lower and upper approximations of a N-soft set in a Pawlak approximation space, which gives rise to the following notions in a natural way.

Definition 5 Let $K=(U,R)$ be a Pawlak approximations space, U be a universe of objects under consideration, R be the equivalence relation on domain U, and E be attributes, $A\subseteq E$, $S=(F,A,N)$ is an N-soft set on U. Based on K, the exact solutions of two approximation are defined, which are called the lower, upper rough N-soft approximations of S with respect to K, can be expressed as

$$\left(\overline{F}_{t\geq}, A\right)=\underline{Apr}_R(S),\quad \left(\underline{F}_{t\geq}, A\right)=\overline{Apr}_R(S)$$

For every $e\in A$,

$$F_t(e)=\left\{u\in U\middle|(u,t)\in F(e)\right\} \tag{3}$$

$$\underline{F}_{t\geq}(e)=\underline{Apr}_R(S)=\left\{u\in U:[u]_R\subseteq F_t(e)\right\}. \tag{4}$$

$$\overline{F}_{t\geq}(e)=\overline{Apr}_R(S)=\left\{u\in U:[u]_R\bigcap F_t(e)\neq\varnothing\right\}. \tag{5}$$

$$F_{t\geq}\left(a_j\right)(u_i)=\begin{cases} r_{ij} & if \;\; r_{ij}\geq t, 0\leq t\leq N \\ 0 & otherwise \end{cases}. \tag{6}$$

If $\underline{Apr}_R(S)=\overline{Apr}_R(S)$, S is called definable.

If $\underline{Apr}_R(S)\neq\overline{Apr}_R(S)$, then the order pair $\left(\underline{Apr}_R(S), \overline{Apr}_R(S)\right)$ is a rough N-soft set. It can be recorded as a (R,N)-soft set.

Afterwards, we explain its intuitive interpretation and suggest that a tabular representation simplifies its practical use.

Example 1 Suppose there is a company recruiting for the position of algorithm engineer and there are multiple candidates competing. The company wants to choose the most suitable candidate. Through a written test and the first round of interviews, each candidate has at least two kinds of competence that is what company value. Let $U=\{u_1,u_2,u_3,u_4,u_5\}$ be the universe of alternatives, there are five kinds of ability inspections for this application (referred to as u_1, u_2, u_3, u_4, u_5). Considering various factors such as academic qualifications and internship experience, we try to find one or more candidates for employment. There are four candidates, denoted by $\mathrm{A}=\{a_1,a_2,a_3,a_4\}$. In terms of the essence of these five capabilities, we regard as $(u_1,u_2)\in R$, $(u_3,u_4,u_5)\in R$, R be an equivalence relation on U, A 6-soft set can be obtained from Tab. 2. The graded evaluation by check marks can easily identified with numbers as $G_N=\{0,1,2,3,4,5\}$, where

- 0 serves as "□", represents "poor",
- 1 serves as "√", represents "normal",
- 2 serves as "√√", represents "good",
- 3 serves as "√√√", represents "very good",
- 4 serves as "√√√√", represents "exceptional",
- 5 serves as "√√√√√", represents "excellent".

Therefore, a 6-soft set $(F,A,6)$ may be considered as follows,

$$F(a_1)=\{(u_1,3),(u_2,3),(u_3,0),(u_4,4),(u_5,0)\}$$
$$F(a_2)=\{(u_1,4),(u_2,0),(u_3,2),(u_4,5),(u_5,0)\}$$
$$F(a_3)=\{(u_1,0),(u_2,3),(u_3,0),(u_4,3),(u_5,2)\}$$
$$F(a_4)=\{(u_1,0),(u_2,0),(u_3,4),(u_4,0),(u_5,1)\}$$

The information extracted from related data is described in Tab. 2., and the table representation of its associated 6-soft set is given in Tab. 3..

Tab. 2. Information extracted from the real data

(U/A)	a_1	a_2	a_3	a_4
u_1	√√√	√√√√	□	□
u_2	√√√	□	√√√	□
u_3	□	√√	□	√√√√
u_4	√√√√	√√√√√	√√√	□
u_5	□	□	√√	√

Tab. 3. Tabular representation of the corresponding 6-soft set

(F,A,N)	a_1	a_2	a_3	a_4
u_1	3	4	0	0
u_2	3	0	3	0
u_3	0	2	0	4
u_4	4	5	3	0
u_5	0	0	2	1

It is enough when the information is extracted from real data, but when the data is vague and uncertain, we need rough N-soft sets, which provide us information how these grades are given to alternatives.

Tab. 4.Table for 6-soft set $F_{2\geq}(A,2)$

$(F_{2\geq}(A,2))$	a_1	a_2	a_3	a_4
u_1	3	4	0	0
u_2	3	0	3	0
u_3	0	2	0	4
u_4	4	5	3	0
u_5	0	0	2	0

Tab. 5. Table for (R,6)-soft set $\underline{F}_{2\geq}(A,2)$

$(\underline{F}_{2\geq}(A,2))$	a_1	a_2	a_3	a_4
u_1	3	0	0	0
u_2	3	0	0	0
u_3	0	0	0	0
u_4	0	0	0	0
u_5	0	0	0	0

Tab. 6. Table for (R,6)-soft set $\overline{F}_{2\geq}(A,2)$

$(\overline{F}_{2\geq}(A,2))$	a_1	a_2	a_3	a_4
u_1	3	4	3	0
u_2	3	4	3	0
u_3	4	2	2	4
u_4	4	2	2	4
u_5	4	2	2	4

Tab. 7. Table for 6-soft set $F_{3\geq}(A,3)$

$(F_{3\geq}(A,3))$	a_1	a_2	a_3	a_4
u_1	3	4	0	0
u_2	3	0	3	0
u_3	0	0	0	4
u_4	4	5	3	0
u_5	0	0	0	0

Let $0 \leq t \leq 6$ be a threshold, then $t=\{2,3,4\}$. If $r_{ij} \geq t$, let $F_{t\geq}(a_j)(u_i)=r_{ij}$, otherwise, $F_{t\geq}(a_j)(u_i)=0$. Therefore, when $t=2$, based on Pawlak rough sets and (R,N)-soft sets, the 6-soft set can be given in Tab. 4. and the lower, upper (R,6)-soft approximations can be given in Tab. 5., Tab. 6., respectively. When $t=3$, the 6-soft set can be given in Tab. 7. and the lower, upper (R,6)-soft approximations can be given in Tab. 8., Tab. 9., respectively.... when $t=4$, the 6-soft set can be given in Tab. 10. and the lower, upper (R,6)-soft approximations can be given in Tab. 11., Tab. 12., respectively.

Tab. 8. Table for (R,6)-soft set $\underline{F}_{3\geq}(A,3)$

$(\underline{F}_{3\geq}(A,3))$	a_1	a_2	a_3	a_4
u_1	3	0	0	0
u_2	3	0	0	0
u_3	0	0	0	0
u_4	0	0	0	0
u_5	0	0	0	0

Tab. 9. Table for (R,6)-soft set $\overline{F}_{3\geq}(A,3)$

$(\overline{F}_{3\geq}(A,3))$	a_1	a_2	a_3	a_4
u_1	3	4	3	0
u_2	3	4	3	0
u_3	4	5	3	4
u_4	4	5	3	4
u_5	4	5	3	4

Tab. 10. Table for 6-soft set $F_{4\geq}(A,4)$

$(F_{4\geq}(A,4))$	a_1	a_2	a_3	a_4
u_1	0	4	0	0
u_2	0	0	0	0
u_3	0	0	0	4
u_4	4	5	0	0
u_5	0	0	0	0

Tab. 11.Table for (R,6)-soft set $\underline{F}_{4\geq}(A,4)$

$(\underline{F}_{4\geq}(A,4))$	a_1	a_2	a_3	a_4
u_1	0	0	0	0
u_2	0	0	0	0
u_3	0	0	0	0
u_4	0	0	0	0
u_5	0	0	0	0

Definition 6 Let (F,A,N_1) and (G,B,N_2) be two N-soft sets over U. Then (G,B,N_2) is called an N-soft subset of (F,A,N_1), denoted by $(F,A,N_1) \subseteq (G,B,N_2)$, if $B \subset A$ and $G(b) \subseteq F(b)$ for all $b \in B$. Two N-soft sets over U are said to be equal, denoted by $(F,A,N_1)=(G,B,N_2)$, if $(F,A,N_1) \subseteq (G,B,N_2)$ and $(G,B,N_2) \subseteq (F,A,N_1)$ .Then two (R,N)-soft sets are said to be equal, if

$(F_{t\geq}, A, N_1) \subseteq (G_{t\geq}, B, N_2)$ and $(G_{t\geq}, B, N_2) \subseteq (F_{t\geq}, A, N_1)$.

Tab. 12. Table for (R,6)-soft set $\overline{F}_{4\geq}(A,4)$

$\left(\overline{F}_{4\geq}(A,4)\right)$	a_1	a_2	a_3	a_4
u_1	0	4	0	0
u_2	0	4	0	0
u_3	4	5	0	4
u_4	4	5	0	4
u_5	4	5	0	4

Definition 7 Let $(F_{t\geq}, S)$ be a (R,N)-soft set on U, where $S=(F,A,N)$ is an N-soft set. Then we say that $(F_{t\geq}, S^c)$ is a weak complement if $S^c=(F^c,A,N)$ is a weak complement of S. By this we mean $F^c_{t\geq}(a) \bigcap F_{t\geq} = \varnothing$ for all $a \in A$.

Example 2 Two weak complements of the (R,6)-soft set in Example 1, namely $\left(\underline{F}^c_{2\geq}, A, 6\right)$ and $\left(\overline{F}^c_{2\geq}, A, 6\right)$ are defined in Tab. 13. and Tab. 14., respectively. Obviously, the collection of weak complements of the (R,6)-soft set in Example 1 is much larger. Observe that $\left(\underline{F}^c_{2\geq}, A, 6\right)$ and $\left(\overline{F}^c_{2\geq}, A, 6\right)$ are efficient. In that same way, $\left(\underline{F}^c_{3\geq}, A, 6\right), \left(\overline{F}^c_{3\geq}, A, 6\right), \left(\underline{F}^c_{4\geq}, A, 6\right)$ and $\left(\overline{F}^c_{4\geq}, A, 6\right)$ also can be defined.

Tab. 13. Table for weak complements (R,6)-soft set $\left(\underline{F}^c_{2\geq}, A, 6\right)$

$\left(\underline{F}^c_{2\geq}, (A,6)\right)$	a_1	a_2	a_3	a_4
u_1	4	3	1	1
u_2	4	1	2	1
u_3	1	3	2	2
u_4	1	4	1	2
u_5	1	2	1	2

Tab. 14. Table for weak complements (R,6)-soft set $\left(\overline{F}^c_{2\geq}, A, 6\right)$

$\left(\overline{F}^c_{2\geq}, (A,6)\right)$	a_1	a_2	a_3	a_4
u_1	2	5	1	1
u_2	5	1	2	1
u_3	2	3	1	3
u_4	3	2	4	1
u_5	1	2	3	2

We can get the following properties，which are easily obtained from the definitions.

Let (U,R) be a Pawlak approximation space, R is an equivalence relation, $S=(F,A,N_1)$, $Y=(G,B,N_2)$, are N-soft sets over U. Then we have

(1) $\underline{F}_{t\geq}(S) \subseteq S \subseteq \overline{F}_{t\geq}(S)$

(2) $\underline{F}_{t\geq}(S^c) = \overline{F}_{t\geq}(S)^c$

(3) $\overline{F}_{t\geq}(S^c) = \left(\underline{F}_{t\geq}(S)\right)^c$

(4) $\underline{F}_{t\geq}(S \bigcap_{\mathfrak{R}} Y) = \underline{F}_{t\geq}(S) \bigcap_{\mathfrak{R}} \underline{F}_{t\geq}(Y)$

(5) $\underline{F}_{t\geq}(S \bigcap_{\varepsilon} Y) = \underline{F}_{t\geq}(S) \bigcap_{\varepsilon} \underline{F}_{t\geq}(Y)$

(6) $\overline{F}_{t\geq}(S \bigcap_{\mathfrak{R}} Y) = \overline{F}_{t\geq}(S) \bigcap_{\mathfrak{R}} \overline{F}_{t\geq}(Y)$

(7) $\overline{F}_{t\geq}(S \bigcap_{\varepsilon} Y) = \overline{F}_{t\geq}(S) \bigcap_{\varepsilon} \overline{F}_{t\geq}(Y)$

(8) $\underline{F}_{t\geq}(S \bigcup_{\mathfrak{R}} Y) \supseteq \underline{F}_{t\geq}(S) \bigcup_{\mathfrak{R}} \underline{F}_{t\geq}(Y)$

(9) $\underline{F}_{t\geq}(S \bigcup_{\varepsilon} Y) \supseteq \underline{F}_{t\geq}(S) \bigcup_{\varepsilon} \underline{F}_{t\geq}(Y)$

(10) $\overline{F}_{t\geq}(S \bigcup_{\mathfrak{R}} Y) \supseteq \overline{F}_{t\geq}(S) \bigcup_{\mathfrak{R}} \overline{F}_{t\geq}(Y)$

(11) $\overline{F}_{t\geq}(S \bigcup_{\varepsilon} Y) \supseteq \overline{F}_{t\geq}(S) \bigcup_{\varepsilon} \overline{F}_{t\geq}(Y)$

(12) $S \subseteq Y \Rightarrow \underline{F}_{t\geq}(S) \subseteq \underline{F}_{t\geq}(Y), \overline{F}_{t\geq}(S) \subseteq \overline{F}_{t\geq}(Y)$

Proof N_3 is the value after t filtering. This is easily obtained from definition 1, definition 5. The proof process of the theorem (1), (2), (4)-(12) are given as follows.

(1) For every $x \in \underline{F}_{t\geq}(e)$, now by definition 5, $[x]_R \subseteq F(e)$, and $x \in [x]_R$, then $x \in F(e)$, so we deduce that $\underline{F}_{t\geq}(S) \subseteq S$. For every $x \in F(e)$, there exists $x \in [x]_R$ based on equivalence relation R, $[x]_R \bigcap \underline{F}_{t\geq}(e) \neq \varnothing$, then $x \in \overline{F}_{t\geq}(e)$, so we deduce that $S \subseteq \overline{F}_{t\geq}(S)$. Hence, it follows that $\underline{F}_{t\geq}(S) \subseteq S \subseteq \overline{F}_{t\geq}(S)$.

(2) Let $\underline{F}_{t\geq}(S^c) = \left(\underline{F}^c_{t\geq}, A\right)$ then $\underline{F}^c_{t\geq}(e) = \{x \in U : [x]_R \subseteq F^c(e)\} = \{x \in U : [x]_R \not\subset F(e)\} = \{x \in U : [x]_R \bigcap F(e) \neq \varnothing\} = \left(\overline{F}_{t\geq}(S)\right)^c$. Hence, $\underline{F}_{t\geq}(S^c) = \overline{F}_{t\geq}(S)^c$

(4) Let $S \bigcap_{\mathfrak{R}} Y = \mathfrak{I} = (H, C, \min(N_1, N_2))$. Then, $C = A \bigcap B$ and $H(x) = F(x) \bigcap G(x), \forall x \in C.$ Using

definition (Maji et al.2002), $\underline{F}_{t\geq}(\Im)=(H_{t\geq},C,N_3)$, where

$\underline{H}_{t\geq}(x)=\underline{F}_{t\geq}(H(x))=\underline{F}_{t\geq}(F(x)\cap G(x))$, for all $x\in C$.

Now $\underline{F}_{t\geq}(F(x)\cap G(x))=\underline{F}_{t\geq}(F(x))\cap\underline{F}_{t\geq}(G(x))$, and so

we deduce that $\underline{H}_{t\geq}(x)=\underline{F}_{t\geq}(F(x)\cap G(x))$ for all $x\in C$.

Hence, $\underline{F}_{t\geq}(S\cap_{\Re}Y)=\underline{F}_{t\geq}(S)\cap_{\Re}\underline{F}_{t\geq}(Y)$.

(5) Let $S\cap_{\varepsilon}Y=\Im=(H,C,\max(N_1,N_2))$. Then,

$C=A\cup B$, for all $e\in C$.

$$H(e)=\begin{cases}F(e), & if\ e\in A-B\\ G(e), & if\ e\in B-A\\ F(e)\cap G(e), & if\ e\in B\cap A\end{cases}$$

for all $e\in C$. Now $\underline{F}_{t\geq}(F(e)\cap G(e))=\underline{F}_{t\geq}(F(e))\cap$

$\underline{F}_{t\geq}(G(e))$ and so we deduce that

$$\underline{F}_{t\geq}(H(e))=\begin{cases}\underline{F}_{t\geq}(F(e)), & if\ e\in A-B\\ \underline{F}_{t\geq}(G(e)), & if\ e\in B-A\\ \underline{F}_{t\geq}(F(e)\cap G(e)), & if\ e\in B\cap A\end{cases}$$

for all $x\in C$ Hence, $\underline{F}_{t\geq}(S\cap_{\varepsilon}Y)=\underline{F}_{t\geq}(S)\cap_{\varepsilon}\underline{F}_{t\geq}(Y)$.

(6) Prove the same as (4).

(7) Prove the same as (5).

(8) Let $S\cup_{\Re}Y=\Im=(H,C,max(N_1,N_2))$. Then $C=A\cap B$ and $H(x)=F(x)\cup G(x)$, $\forall x\in C$. Using definition (Maji et al.2002), $\underline{F}_{t\geq}(\Im)=(\underline{H}_{t\geq},C,N_3)$, where $\underline{H}_{t\geq}(x)=\underline{F}_{t\geq}(H(x))$ $=\underline{F}_{t\geq}(F(x)\cup G(x))$, for all $x\in C$. Now $\underline{F}_{t\geq}(F(x)\cap G(x))$ $\supseteq\underline{F}_{t\geq}(F(x))\cap\underline{F}_{t\geq}(G(x))$, and so we deduce that $\underline{H}_{t\geq}(x)\supseteq$ $\underline{F}_{t\geq}(F(x))\cap\underline{F}_{t\geq}(G(x))$ for all $x\in C$. Hence, $\underline{F}_{t\geq}(S\cup_{\Re}Y)$ $\supseteq\underline{F}_{t\geq}(S)\cup_{\Re}\underline{F}_{t\geq}(Y)$.

(9) Let $S\cup_{\Re}Y=\Im=(H,C,max(N_1,N_2))$. Then $C=A\cup B$, for all $e\in C$.

$$H(e)=\begin{cases}F(e) & if\ e\in A-B\\ G(e) & if\ e\in B-A\\ F(e)\cap G(e), & if\ e\in B\cap A\end{cases}$$

Using definition (Maji et al.2002),

$\underline{F}_{t\geq}(\Im)=(\underline{H}_{t\geq},C,N_3)$,where,

$$\underline{F}_{t\geq}(H(e))=\begin{cases}\underline{F}_{t\geq}(F(e)) & if\ e\in A-B\\ \underline{F}_{t\geq}(G(e)) & if\ e\in B-A\\ \underline{F}_{t\geq}(F(e)\cap G(e)), & if\ e\in B\cap A\end{cases}$$

for all $e\in C$. Now $\underline{F}_{t\geq}(F(e)\cup G(e))\supseteq\underline{F}_{t\geq}(F(e))\cap$ $\underline{F}_{t\geq}(G(e))$, and so we deduce that, $\underline{F}_{t\geq}(S\cup_{\varepsilon}Y)\supseteq\underline{F}_{t\geq}(S)\cup_{\varepsilon}\underline{F}_{t\geq}(Y)$.

(10) Prove the same as (8).

(11) Prove the same as (9).

(12) Assume that $S\subseteq Y$. Then we have $A\subseteq B$ and $F(x)\subseteq G(x)$, $\forall x\in A$. It can be obtained that $\underline{F}_{t\geq}(F(x))\subseteq$ $\underline{F}_{t\geq}(G(x))$ and $\overline{F}_{t\geq}(F(x))\cap\overline{F}_{t\geq}(G(x))$ for all $x\in C$. So we deduce that, $S\subseteq Y\Rightarrow\underline{F}_{t\geq}(S)\subseteq\underline{F}_{t\geq}(Y)$, $\overline{F}_{t\geq}(S)\subseteq$ $\overline{F}_{t\geq}(Y)$.□

4. Application of rough N-soft set in decision making

In this section, we study the application of rough soft sets in decision making. We illustrate a kind of new decision making method for rough N-soft sets. In order to practical application, we give an algorithm about rough N-soft set.

We will put forward the new method to find which is best parameter a of a given N-soft set $(F_{t\geq},A,N)$. In other words, $F(a)$ is the most expected material, with respect to an equivalence relation on the universe U.

Let $A=\{a_1,a_2,\ldots,a_m\}\subseteq E$ and $S=(F_{t\geq},A,N)$ be an original description N-soft set over U. Then we present the decision algorithm for rough N-soft sets as follows:

Step1 Input the original description universe U, N-soft set (F,A,N) and Pawlak approximation space (U,R).

Step2 Compute the lower and upper rough N-soft approximation operators $\overline{Apr_R}F_{t\geq}(e)$ and $\underline{Apr_R}F_{t\geq}(e)$, respectively.

Step3 Compute the value of $\|F(a_i)\|$, where $\|F(a_i)\|=\dfrac{|\overline{F}_{t\geq}(a_i)|-|\underline{F}_{t\geq}(a_i)|}{|F(a_i)|}$.

Step4 Find the minimum value $\|F(a_k)\|$ of $\|F(a_i)\|$, where $\|F(a_k)\|=\min\|F(a_i)\|$.

Step5 The decision is $F(a_k)$.

Now, use the example to illustrate the application of this method.

Example 3 (Choice of company employees) Consider the rough 6-soft set $(F_{t\geq},A,6)$ described in example 1. Then compute the value of $\left\|F\left(a_i\right)\right\|=\dfrac{\left|\overline{F}_{t\geq}\left(a_i\right)\right|-\left|\underline{F}_{t\geq}\left(a_i\right)\right|}{\left|F\left(a_i\right)\right|}$.

When $t=2$, $F\left(a_1\right)=\{u_1,u_2,u_4\}$, $F\left(a_2\right)=\{u_1,u_3,u_4\}$, $F\left(a_3\right)=\{u_2,u_4,u_5\}$, $F\left(a_4\right)=\{u_3\}$. $\left\|F\left(a_1\right)\right\|=1$, $\left\|F\left(a_2\right)\right\|=1.667$, $\left\|F\left(a_3\right)\right\|=1.667$, $\left\|F\left(a_4\right)\right\|=3.$ Find the minimum value $\left\|F\left(a_k\right)\right\|$ of $\left\|F\left(a_i\right)\right\|$, where $\left\|F\left(a_k\right)\right\|=\left\|F\left(a_1\right)\right\|=1,$ the decision is $F\left(a_1\right)$.

When $t=3$, $F\left(a_1\right)=\{u_1,u_2,u_4\}$, $F\left(a_2\right)=\{u_1,u_4\}$, $F\left(a_3\right)=\{u_2,u_4\}$, $F\left(a_4\right)=\{u_3\}$. $\left\|F\left(a_1\right)\right\|=1,$ $\left\|F\left(a_2\right)\right\|=2.5,$ $\left\|F\left(a_3\right)\right\|=2.5,$ $\left\|F\left(a_4\right)\right\|=3$. Where $\left\|F\left(a_k\right)\right\|=\left\|F\left(a_1\right)\right\|=1$, the decision is $F\left(a_1\right)$.

When $t=4$, $F\left(a_1\right)=\{u_4\}$, $F\left(a_2\right)=\{u_1,u_4\}$, $F\left(a_4\right)=\{u_3\}$. $\left\|F\left(a_1\right)\right\|=3$, $\left\|F\left(a_2\right)\right\|=2.5$, $\left\|F\left(a_3\right)\right\|=0$, $\left\|F\left(a_4\right)\right\|=3$. Where $\left\|F\left(a_k\right)\right\|=\left\|F\left(a_3\right)\right\|=0$. In this case, there is no corresponding candidate, which does not meet the actual situation.

So $F\left(a_1\right)$ is the expected decision, a_1 is the most suitable candidate.

5. Conclusion

In this article, we have a presented novel model called rough N-soft set, which is the hybridization of N-soft sets and rough sets, providing more flexibility in decision-making problems. In addition, we have investigated some basic properties of the new hybridization. And we also propose a rough N-soft set decision algorithm and illustrate the effectiveness of the algorithm. In further research, the generation model of rough N-soft set theory is an interesting issue to be addressed.

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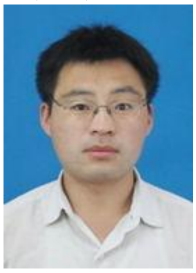
Xiaomin Wang has received the D.Sc. degree from Harbin Institute of Technology, Harbin, China. His main research interests are in the areas of big data analysis method, mathematical method of knowledge representation and functional analysis.



Piyu LI has received the D.Sc. degree in basic mathematics from Capital Normal University, Beijing, China, in 2012. He obtained his B.S. degree from Liaocheng University, China in 2003 and his M.S. degree from Nanjing Normal University, China in 2007. His main research interests are in the areas of topology algebra, quantum information and big data visualization. He is also a Commentator of Mathematical Reviews.

Shengnan Cao is currently pursuing her M.S. study at the Institute of Mathematics and Statistics, Northeastern University at Qinhuangdao, Qinhuangdao, China. She obtained her B.S. degree from Zhengzhou University of Light Industry, China in 2018. Her main research interests are in the areas of soft set and rough set.

Yang Liu is currently pursuing her M.S. study at the Institute of Mathematics and Statistics, Northeastern University at Qinhuangdao, Qinhuangdao, China. She obtained her B.S. degree from Jilin Normal University, China in 2018. Her main research interests are in the areas of soft set and rough set.