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Composition Prediction of Mixed Steel Zone in Tundish Based On Difference Equation

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ABSTRACT

Continuous casting technology is the development emphases of iron and steel industry, during continuous casting, the mixing of different molten steel often leads to the mixing zone which does not meet the requirements of element content, which affects the quality of casting billet. Therefore, Simulation of molten steel mixing process and accurate prediction of mixed steel zone become an important topic. To solve this problem, according to the mass conservation principle, combined with the difference equation, the static prediction model and the dynamic prediction model are established to calculate the content of the outflow elements in the molten steel mixing process, and then accurately locate the mixed steel zone. The results show that the dynamic prediction model can accurately predict the mixing time of molten steel and the length of mixed steel zone.

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1. Introduction

Steel is a kind of widely used material, continuous casting technology greatly improves the production efficiency of steel industry. In the continuous casting technology, there is a technical difficulty in the simulation of molten steel mixing process. Part of the mixed steel zone can not meet the requirements of the element content of the previous ladle or the next ladle. Therefore, it is necessary to accurately locate and cut the zone to ensure the quality of casting billet and reduce the waste of casting billet.

Dan Sun et al. (2011) took the No.4 double current continuous caster of Shougang Jingtang company as the research object, divided the tundish into five zones, established a composition prediction model of molten steel in tundish with constant volume. However, the specific solving process of the model is not explained. Jilin Hou et al. (2000) based on the assumption that the molten steel flowing into the tundish can be completely mixed with the molten steel in the tundish, established a prediction model for the length of steel mixing zone, and the feasibility of the model was proved by water model experiment. Compared with the static model established by Jilin Hou et al. (2000) , Yanlong Wei et al. (2008) took the four current continuous caster as an example, according to the mass conservation principle, introduced the mixing rate of molten steel, predicted the content of elements in different parts of tundish. The model can change the prediction results by adjusting

the mixing rate of molten steel, which is a dynamic model. Qiulin, Yu et al. (2017) established a dynamic model with five parameters. However, the determination of some parameters depends on sensors, and the cost is relatively high. Aidong Wang et al. (2020) based on CFD simulation platform, simulated the flow field in tundish detailedly, but the process is complicated. Yafei Sui et al. (2019) analyzed the experimental data of continuous casting process, found that the composition of the casting billet changes in accordance with a logarithmic function. In addition, similar results are obtained in this article.

In addition to the experimental data simulation scheme, most of the above schemes are based on the same volume of molten steel in tundish, and the change of molten steel volume in tundish is not considered, or the process is complex and difficult to apply. Therefore, this article is based on the change of molten steel volume in tundish, the mixing process in tundish of single strand continuous caster is studied in this paper. The prediction equation of element content and the calculation method of mixing time of unqualified molten steel and length of unqualified mixing zone are given. This method is easy to understand and apply.

2. Introduction of mixing process of molten steel

The mixing process of molten steel is shown in the Fig.1. The top part is the steel ladle, which contains the molten steel to be cast. There is a cylindrical pipe below, which is used as the molten steel inlet pipe of tundish. The trapezoidal box under the molten steel inlet pipe is tundish. There is also a cylindrical pipe under the tundish as the molten steel outlet pipe of the tundish. The crystallizer is connected under the outlet pipe, and then the casting machine is responsible for pulling out the casting billet after crystallization.



Fig. 1. Mixing process of molten steel

Normally, when the next ladle of molten steel is injected into the tundish, in the tundish, there will be molten steel which has not been discharged from the last ladle. After mixing two kinds of molten steel with different element contents, the content of elements flowing out of tundish will change with time, and then produce unqualified steel mixing zone. The element content has a great influence on the quality of casting billet, therefore, it is necessary to accurately predict the starting position and length of the steel mixing zone, and then cut off the unqualified steel mixing zone.

3. Static prediction model

It is assumed that hot molten steel can mix with each other in a very short time, and hot casting billet almost does not mix (That is, the element content out of the tundish is approximately equal to the element content in the tundish). Based on this assumption, the molten steel in tundish can always be regarded as a whole, the billets obtained from the crystallization of molten steel can only be regarded as dispersed, and the element content of different parts is different. According to the assumption, the element content in the tundish can be calculated first, and the following model can be established:





In the figure, Q is flow per unit time (tons/s), C is elemental content, G_t is the total weight of molten steel in tundish at time t. According to the inflow and outflow relationship of tundish, it can be obtained as follows:

$$C_{t}G_{t} = C_{t-1}G_{t-1} + Q_{in}C_{in} - Q_{out}C_{out}$$
(1)

 C_t is the element content of tundish at the time t, C_{t-1} is the element content of tundish at the time t-1, G_t is the total weight (tons) of tundish steel water at time t, $C_{in}(C_{out})$ is the element content of inflow (outflow), $Q_{in}(Q_{out})$ is the weight of molten steel inflow (outflow) per unit time (tons/s). The calculation formula of Q_{out} is as follow:

$$Q_{out} = \rho v D \delta \tag{2}$$

 ρ is the specific gravity of hot casting billet (tons/m³), v is the drawing speed of casting machine (m/s), D is the width of casting billet (m), δ is the thickness of casting billet (m). The calculation formula of G_{i} is as follow:

$$G_t = G_0 + \Delta G \times t \tag{3}$$

 G_0 is the initial weight of molten steel in the tundish (tons), ΔG is the increment of the weight of molten steel in the tundish per unit time (tons/s). According to the above equation, it is assumed that the molten steel in the steel ladle is mixed with the molten steel in the tundish immediately after flowing into the tundish, and the $C_{out} = C_{t-1}$. Therefore, the difference equation for predicting element content can be obtained:

$$C_{t} = \frac{C_{t-1}[G_0 + \Delta G(t-1) - \rho v D\delta] + Q_{in}C_{in}}{G_0 + \Delta Gt}$$
(4)

In the continuous casting process, C_0 is the initial content of elements in the tundish, G_0 is the initial weight of molten steel in the tundish at the continuous casting process, Q_{in} is the weight of molten steel flowing in per unit time, Q_{out} is the weight of molten steel flowing out per unit time, ρ is the specific gravity of hot casting billet (tons/m³), v is the drawing speed of casting machine (m/s), D is the width of casting billet (m), δ is the thickness of casting billet (m). All are known process parameters. C_{in} is the content of a certain element flowing into molten steel, which can be expressed by the element content of the next ladle.

When calculating the element content of outflow, it should be noted that the element content of outflow is C_{t-1} .

The following model is established:



Fig. 3. Molten steel mixing model diagram of tundish

4. Dynamic prediction model

In the previous solution process, it is assumed that the molten steel enters the tundish and immediately mixes with the molten steel in the tundish. But in actual production, the mixing of molten steel takes time, and even part of the molten steel in the next ladle cannot be mixed with the current ladle molten steel. Therefore, consider improving the model and adding molten steel mixing rate η , one part of the in-flowing molten steel is fully mixed and the other part does not participate in the mixing, and only the mixed molten steel flows out of the tundish.

In the figure, η is molten steel mixing rate, according to the inflow and outflow relationship in the model, the following equation can be established:

$$\begin{bmatrix} G_0 + (\eta Q_{in} - Q_{out})t \end{bmatrix} C_t = \begin{bmatrix} G_0 + (\eta Q_{in} - Q_{out})(t-1) \end{bmatrix} C_{t-1} + \eta Q_{in} C_{in} - Q_{out} C_{out}$$
(5)

Focus only on the mixing part, since the mixing part can be fully mixed in a short time, it can be set $C_{out} = C_{t-1}$, available:

$$\begin{bmatrix} G_0 + (\eta Q_{in} - Q_{out})t \end{bmatrix} C_t = \begin{bmatrix} G_0 + (\eta Q_{in} - Q_{out})(t-1) \end{bmatrix} C_{t-1} + \eta Q_{in} C_{in} - Q_{out} C_{t-1}$$
(6)

Move items to get:

$$C_{t} = \frac{\left[G_{0} + (\eta Q_{in} - Q_{out})(t-1)\right]C_{t-1} + \eta Q_{in}C_{in} - Q_{out}C_{t-1}}{\left[G_{0} + (\eta Q_{in} - Q_{out})t\right]}$$
(7)

 η is the mixing ratio of molten steel, C_0 is the initial content of

elements in the tundish, G_0 is the initial weight of molten steel in the tundish at the continuous casting process, Q_{in} is the weight of molten steel flowing in per unit time, Q_{out} is the weight of molten steel flowing out per unit time. All are known process parameters. C_{in} is the content of a certain element flowing into molten steel, which can be expressed by the element content of the next ladle. The unknown variable on the right side of the equation increases the mixing rate of molten steel η .

Again, when calculating the element content of outflow, it should be noted that the element content of outflow is C_{t-1} .

5. Experiment and results

The element contents of molten steel in ahead and later ladle are as follows:

Tab. 1. Element content table of molten steel in the ahead ladle

Ahead ladle	Percentage composition (%)		
Element	Lower limit	Upper limit	Actual
С	0.018	0.220	0.200
Si	0.200	0.350	0.290
Mn	0.350	0.600	0.510
Р	0.000	0.025	0.015
S	0.000	0.012	0.006

The upper and lower limits in the table indicate the requirements for the element content of the casting billet, and different ladles correspond to different requirements. For element C, it can be concluded from the above table that C_0 is 0.002, C_{in} is 0.00455, the weight of molten steel in tundish is 20 tons, the weight of molten steel in later ladle is 30 tons, the flow rate is 6 tons/min, and the duration of the process is 300 seconds.

Tab. 2. Element content table of molten steel in the later ladle

Later ladle	Percentage composition (%)		
Element	Lower limit	Upper limit	Actual
С	0.28	0.480	0.455
Si	0.20	0.350	0.265
Mn	0.55	0.750	0.650
Р	0.00	0.025	0.019
S	0.00	0.012	0.007

The size of tundish is shown in the following figure:





Fig. 4. Schematic diagram of static prediction results of C element content

As can be seen from the figure, the bottom surface of tundish is 2500mm * 960mm, the size of upper surface is 5000mm * 1260mm, and the height is 1100mm. The tundish of this size can hold about 35 tons of molten steel.

The parameters of continuous casting are as follows:

Tab. 3. Continuous casting parameters			
Parameter name	Parameter value		
Specific gravity of hot casting billet $~ ho~$	7.6 tons/m ³		
Casting speed v	0.075m/s		
Casting billet width D	1.55m		
Casting billet thickness δ	0.072m		
Weight of molten steel	0.1 tons/s		
in-flowing per unit time $Q_{_{in}}$			

The following equation can be obtained by substituting the parameters into the static model:

$$C_{t} = C_{t-1} + \frac{0.1C_{in} - 0.095 C_{t-1}}{20 + 0.0364 t}$$
(8)

For element C, the static prediction model of element C can be obtained by substituting $C_{in} = 0.455\%$, $C_0 = 0.2\%$ into equation

(8):

$$\begin{cases} C_t = C_{t-1} + \frac{4.55 \times 10^{-4} - 0.095 C_{t-1}}{20 + 0.0364 t} \\ C_0 = 0.002 \end{cases}$$
(9)

Change process of C element content was simulated by the equation (9), the result is as follow:



Fig. 5. Schematic diagram of static prediction results of C element content

In the figure, the horizontal axis represents time and the vertical axis represents the content of C element. According to the intersection point of the C content curve and the middle two lines, the start time and end time of the unqualified steel mixing zone can be determined as 15s and 72s respectively, and the duration is 57s.

For element Si, the static prediction model of element Si can be obtained by substituting $C_{in} = 0.265\%$, $C_0 = 0.29\%$ into equation (8):

$$\begin{cases} C_t = C_{t-1} + \frac{2.65 \times 10^{-4} - 0.095C_{t-1}}{20 + 0.0364t} \\ C_0 = 0.0029 \end{cases}$$
(10)

Change process of Si element content was simulated by the equation (10), the result is as follow:



Fig. 6. Schematic diagram of static prediction results of Si element content It can be seen from the above figure that the content of Si is

always in the qualified range.

For element Mn, the static prediction model of element Mn can be obtained by substituting $C_{in} = 0.65\%$, $C_0 = 0.51\%$ into equation (8):

$$\begin{cases} C_t = C_{t-1} + \frac{6.5 \times 10^{-4} - 0.095C_{t-1}}{20 + 0.0364t} \\ C_0 = 0.0051 \end{cases}$$
(11)

Change process of Mn element content was simulated by the equation (11), the result is as follow:



Fig. 7. Schematic diagram of static prediction results of Mn element content



Fig. 8. Schematic diagram of static prediction results of S element content It can be seen from the above figure that the qualified range of manganese content in the last package is 0-150s, and that of the next package is 54-300s.

For element S, the static prediction model of element S can be obtained by substituting $C_{in} = 0.007$ %, $C_0 = 0.006$ % into equation (8).

$$\begin{cases} C_t = C_{t-1} + \frac{7 \times 10^{-6} - 0.095 C_{t-1}}{20 + 0.0364 t} \\ C_0 = 0.00006 \end{cases}$$
(12)

Change process of S element content was simulated by the

equation (12), the result is as follow:

It can be seen from the above figure that the content of S is always in the qualified range.

For element P, the static prediction model of element P can be obtained by substituting $C_{in} = 0.019\%$, $C_0 = 0.015\%$ into equation (8):

$$\begin{cases} C_{t} = C_{t-1} + \frac{1.9 \times 10^{-5} - 0.095 C_{t-1}}{20 + 0.0364 t} \\ C_{0} = 0.00015 \end{cases}$$
(13)

Change process of P element content was simulated by the equation (13), the result is as follow:



Fig. 9. Schematic diagram of static prediction results of P element content

It can be seen from the above figure that the content of P is always in the qualified range.

The dynamic model was used to simulate the change of C element content. In reference (Yanlong Wei et al., 2008; Dehui Li et al., 2006), the mixing rate of molten steel η obtained from the real data of the composition change of each casting billet cutting, which is taken as 0.95, and the following equation is obtained:

$$C_{t} = C_{t-1} + \frac{0.095 \left(C_{in} - C_{t-1}\right)}{20 + 0.0314 t}$$
(14)

For element C, the dynamic prediction model of element C can be obtained by substituting $C_{in} = 0.455\%$, $C_0 = 0.2\%$ into equation (14):

$$\begin{cases} C_t = C_{t-1} + \frac{4.3225 \times 10^{-4} - 0.095 C_{t-1}}{20 + 0.0314 t} \\ C_0 = 0.002 \end{cases}$$
(15)

Change process of C element content was simulated by the equation (15), the result is as follow:



Fig. 10. Schematic diagram of dynamic prediction results of C element content

In the figure, the horizontal axis represents time and the vertical axis represents the content of C element. According to the intersection point of the C content curve and the middle two lines, the start time and end time of the unqualified steel mixing zone can be determined as 17s and 79s respectively, and the duration is 62s.

The dynamic prediction model takes into account the in-homogeneity of molten steel mixing process, and can give the mixing rate of molten steel under certain production conditions by experimental data, which has greater flexibility and practical application value. Therefore, the dynamic prediction model is used to predict the content of other elements. Firstly, the influence of the mixing rate of molten steel on the prediction results is discussed, set the mixing rate of molten steel as 0.7, 0.75, 0.80, 0.85, 0.90, 0.95 and 1 respectively, and the results are as follows:



Fig. 11. Schematic diagram of influence of molten steel mixing rate on results

In the figure, the curve from top to bottom corresponds to the situation that the mixing rate of molten steel is 1, 0.95, 0.9, 0.85, 0.80, 0.75 and 0.7 respectively. When the mixing rate of molten steel is 0.70, the corresponding unqualified time period is 17-78s, and the duration is 61s, but when the mixing ratio of molten steel is 1, the corresponding unqualified time period is 24-108s, the duration is 84s, and the difference of the duration is 23s. Therefore,

the mixing rate of molten steel has a great influence on the prediction results of the model, and according to the actual situation, the ideal results can be obtained by correcting the mixing rate of molten steel. Using the dynamic model to predict the content of other elements, the mixing ratio of molten steel is still 0.95.

For element Si, the dynamic prediction model of element Si can be obtained by substituting $C_{in} = 0.265\%$, $C_0 = 0.29\%$ into equation (14):

$$\begin{cases} C_{t} = C_{t-1} + \frac{2.5175 \times 10^{-4} - 0.095 C_{t-1}}{20 + 0.0314 t} \\ C_{0} = 0.0029 \end{cases}$$
(16)

Change process of Si element content was simulated by the equation (16), the result is as follow:



Fig. 12. Schematic diagram of dynamic prediction results of Si element content

It can be seen from the above figure that the content of Si is always in the qualified range.

Change process of Mn element content was simulated by the equation (17), the result is as follow:



Fig. 13. Schematic diagram of dynamic prediction results of Mn element content

For element Mn, the dynamic prediction model of element Mn can be obtained by substituting $C_{in} = 0.65\%$, $C_0 = 0.51\%$ into equation (14):

$$\begin{cases} C_t = C_{t-1} + \frac{6.175 \times 10^{-4} - 0.095 C_{t-1}}{20 + 0.0314 t} \\ C_0 = 0.0051 \end{cases}$$
(17)

It can be seen from the above figure that the qualified range of manganese content in the ahead ladle is 0-198s, and that of the later ladle is 72-300s.

For element S, the dynamic prediction model of element S can be obtained by substituting $C_{in} = 0.007 \%$, $C_0 = 0.006 \%$ into equation (14):

$$\begin{cases} C_t = C_{t-1} + \frac{6.65 \times 10^{-6} - 0.095 C_{t-1}}{20 + 0.0314 t} \\ C_0 = 0.00006 \end{cases}$$
(18)

Change process of S element content was simulated by the equation (18), the result is as follow:



Fig. 14. Schematic diagram of dynamic prediction results of S element content



Fig. 15. Schematic diagram of dynamic prediction results of P element content

It can be seen from the above figure that the content of S is always in the qualified range.

For element P, the dynamic prediction model of element P can be obtained by substituting $C_{in} = 0.019\%$, $C_0 = 0.015\%$ into equation (14):

$$\begin{cases} C_t = C_{t-1} + \frac{1.805 \times 10^{-5} - 0.095 C_{t-1}}{20 + 0.0314 t} \\ C_0 = 0.00015 \end{cases}$$
(19)

Change process of P element content was simulated by the equation

(19), the result is as follow.

It can be seen from the above figure that the content of P is always in the qualified range. Ensure that each element is in the qualified range, the corresponding unqualified time period is 17-79s, and the duration is 62s. Because the casting speed is 4.5m/min, the length of unqualified mixing zone can be calculated: $l = vt = 4.5/60 \times 62m = 4.65m$.

6. Summary

The static prediction model is only used to understand the process of model establishment. The mixing rate of molten steel in the dynamic model can be obtained by fitting the experimental data, and then the accurate model reflecting the actual production situation can be obtained.

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