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Stability Analysis of Nonlinear Hybrid Stochastic Systems with a Random Switching Signal

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ABSTRACT

In this paper, we investigate the almost surely exponential stability of nonlinear hybrid stochastic systems with a mixed switching signal. The switching signal is composed of fixed dwell time and random sojourn time, which is used to control the jump process of subsystems. By means of Lyapunov method and ITÔ formula, the sufficient conditions for almost surely exponential stability and unstable systems are established respectively, and the determination formulas are given. Meanwhile, we obtain the stationary distribution of the switching signal according to the property of Markovian chain. Numerical examples and simulations are given to verify our results.

1. Introduction

Along with the complexity of control problems, stochastic modelling has been widely adopted in scientific and industrial applications[1-6]. Hybrid stochastic system is one of the most important stochastic models which have been employed to describe control problems. In general terms, this kind of systems consists of several subsystems and a switching signal. Differential equations or stochastic differential equations are usually employed to dominate the dynamic characteristics of each subsystem. Switching signal dominates the subsystems switching behaviors between each other. However, some basic problems, such as stability and controllability, are still hot issues and still challenging. The research progress of hybrid stochastic systems can be referred to [7-12]. In the process of the system actual operation, the system is often affected by various unpredictable factors, such as the change of system structure, the interference of external environment, the damage of system components, etc. Hybrid stochastic system can provide a nature mathematical framework for modelling the above problems. Therefore, ow its wide range of usages and applications, this field has attracted quite a lot of attention, such as in Financial and economic areas[13-16], Biomedical Engineering[17-20], automatic control[21-24], communication network[25-27], etc., as well as references in the literature.

In general, the switching signals can be divided into deterministic and random switching signals. For deterministic switching signals, subsystems switch in a specific way, and the running time of each subsystem is fixed and has no randomness. For random switching signals, the switching between subsystems is uncertain, and the running time of each subsystem is stochastic. Among the random signals, Markovinan switching signal is a class of typical random signal, which is characterized by a Markov chain with finite or infinite states. Thus systems with Markovinan switching are widely studied in recent decades. For such system, quite a lot of problems have been studied. For example, in [28], Yuan Chenggui, and X. Mao studied the nonlinear stochastic delay differential equations with Markovian switching. They got some sufficient criteria for the system to be almost surely asymptotically stable, obtained a sufficient condition for linear system on controllability and robust stability, and gave a control law by solving a linear matrix inequality. Kao Yonggui et al studied Markovian jump neutral-type systems with partly unknown transition probabilities in [29]. Based on Lyapunov-Krasovskii functional method, they obtained a sufficient condition for exponential stability for the system. In [30], Patrinos et al studied constrained stochastic optimal control problems for a kind of discrete-time markovian switching systems, obtained terminal conditions for such problem, and proposed an off-line control law finally. In [31], Wang et al studied the sliding mode control problem for continuous-time Markovian switching systems. They proposed a sliding surface which is dependent on the

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Markovian chain, and obtained a sufficient condition on the stability of such system by means of linear matrix inequality.

In practical terms, due to computation, network transmission and mechanical inertia etc, time-delay is a very common phenomenon in the actual running of systems. In [32], Xiong, Lam, Shun and Mao first proposed another random switching signal which consists of fixed dwell time and random sojourn time. The fixed part is used to describe the switching time-delay caused by the inertia of subsystems, which is analogue to the dwell time in the deterministic switching system. The random part is used to describe the uncertainty as mentioned in former, which is analogue to the sojourn time in Markov switching system. Compared with the Markovian switching signals, this switching signal is a more general view of switching. Thus it is of significance and necessary to study systems with such switching signal. Xiong etc studied the stability of autonomous linear hybrid systems. They proposed the necessary and sufficient conditions for the stability of the system and analyzed the influence of fixed dwell time on system stability.

In the real world, more systems are nonlinear. However, Xiong et al did not study nonlinear stochastic systems. so it is necessary to study the nonlinear stochastic systems with such switching signal. Motivated by such a reason, we studied the nonlinear stochastic hybrid system with the switching signal in [32]. The main contributions of the study are as follows: (1) Making use of ITô formula and Lyapunov method, sufficient conditions for exponential stability and instability are obtained for nonlinear hybrid stochastic system with the switching signal; (2) we deduced the stationary probability distribution of the switching signal; (3) For linear hybrid stochastic system, we gave sufficient conditions for stability.

The paper is arranged as follows. In section 2, we present some notations and definitions, and formulate a hybrid stochastic system. In section 3, we deduced the main results. In section 4, numerical examples and simulations are carried out to illustrate our results. And conclusions are given in section 5.

2. Preliminaries

In this paper, we use the following notations. Let R^n denote n-dimensional real vector space and $|x|$ denotes the Euclidean norm for $x \in R^n$. Let $R^{n\times m}$ denote $n\times m$-dimensional real matrix space and $|A| = \sqrt{trace(A^T A)}$ denotes the trace norm for a matrix $A \in R^{n\times m}$. The superscript "T" represents the transposition of a vector or matrix. We denote $A^* = A + A^T$. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the smallest and largest eigenvalue for symmetric matrix A, respectively.

Let $(\Omega, \mathcal{F}, P)$ denotes a complete probability space. $\{\mathcal{F}_t\}_{t\ge 0}$ is an increasing and right continuous filtration of $(\Omega, \mathcal{F}, P)$ and $\mathcal{F}_0$ contains all p-null sets. $w(t) = (w_1(t), w_2(t), \cdots, w_m(t))$ is an m-dimensional Brownian motion which is $\mathcal{F}_t$-adapted and defined on the probability space $(\Omega, \mathcal{F}, P)$. Let $r(t) \in S$ be a switching signal dominating current mode, and $S = \{1, 2, \cdots, L\}$ containing a finite number of states. Suppose that the mode switching time sequence $\{t_0, t_1, t_2, \cdots\}$ is strictly increasing without accumulation points while $\lim_{k\to\infty} t_k = \infty$ and t_0. As described in [32], $r(t)$ can be described as follows. Assume the system is in mode i at time t_k (i.e., $r(t_k) = i$). The parameter $d_i \ge 0$ is a fixed constant, which is similar to the fixed dwell time in a deterministic switching system. For $t \in [t_k + d_i)$, the switching of subsystems does not occur with probability 1, that is

$$\Pr\{r(t+\Delta t) = j \mid r(t) = i\} = \begin{cases} 0, & if\ \ j \ne i \\ 1, & if\ \ j = i \end{cases}, \tag{1}$$

Where $\Delta t > 0$ is a small time increment, and it satisfies $\lim_{\Delta t \to 0^+} o(\Delta t)/\Delta t = 0$. For $t > t_k + d_i$, switching is allowed and complies with transition probabilities, that is

$$\Pr\{r(t+\Delta t) = j \mid r(t) = i\} = \begin{cases} \gamma_{ij}\Delta t + o(\Delta t), & if\ \ j \ne i \\ 1 + \gamma_{ii}\Delta t + o(\Delta t), & if\ \ j = i \end{cases}, \tag{2}$$

Where $\gamma_{ij} \ge 0$ is the transition rate from mode i to j if $j \ne i$, meanwhile $\gamma_{ii} = -\sum_{j=1, j\ne i}^{L} \gamma_{ij}$. For convenience of description, let $\eta_{ik} := t_{k+1} - (t_k + d_i)$, and η_{ik} is similar to the random sojourn time in Markovian switching systems. Obviously, η_{ik} is a random variable, and it obeys the exponential distribution of parameter $-\gamma_{ii}$. Without confusion, let $\tilde{t}_k = \sum_{l=0}^{k} \eta_{il}$, and we define $\tau(t) := r(t_k)$, $t \in [\tilde{t}_k, \tilde{t}_{k+1})$, then $\tau(t)$ is a Markovian chain. Meanwhile it satisfies $\tau(t) \in S$ and

$$\Pr\{\tau(t+\Delta t) = j \mid \tau(t) = i\} = \begin{cases} \gamma_{ij}\Delta t + o(\Delta t), & if\ \ j \ne i \\ 1 + \gamma_{ii}\Delta t + o(\Delta t), & if\ \ j = i \end{cases}. \tag{3}$$

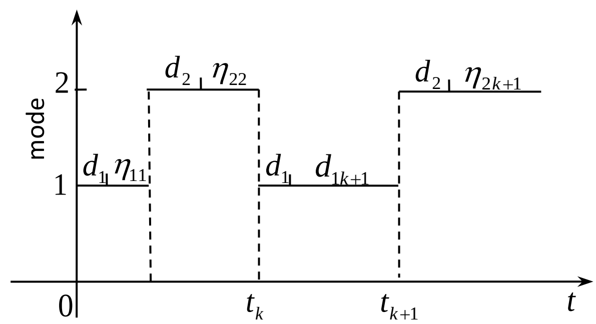

Fig. 1 sample path of $r(t)$

For illustration, figure 1 gives an example of $r(t)$ with two modes. According to the property of Markov chain, we obtain that there are at most finite number of switching in any finite subinterval of $R^+ = [0, +\infty)$ for every sample path of $\tau(t)$ with probability 1. And so is $r(t)$. Assuming that $\tau(t)$ is irreducible, we can get its stationary probability distribution $\pi = (\pi_1, \pi_2, \cdots, \pi_L)$ by solving the equation $\pi\Gamma = 0$, which is subject to $\sum_{i=1}^{L} \pi_i = 1$, $\pi_i > 0$, $\forall i \in S$ and

$\Gamma = (\gamma_{ij})_{L\times L}$. In addition, we assume that $r(t)$ is independent of the Brownian motion $w(t)$.

Consider the hybrid stochastic system of the form

$$dx(t) = f(x(t),t,r(t))dt + g(x(t),t,r(t))dw(t), \quad t \geq 0 \tag{4}$$

where $x(t) \in R^n$ is the system state with initial value $x(0) = x_0 \in R^n$, and the mappings $f: R^n \times R^+ \times S \to R^n$, $g: R^n \times R^+ \times S \to R^{n\times m}$ are all Borel-measurable functions satisfying Lipschitz condition and at most linear growth condition which guarantee that E.q. (4) has a unique solution. Let $x(t;x_0)$ denote the solution of the equation, for simplicity, we write $x(t;x_0) = x(t)$ without confusion. Meanwhile we assume that $f(0,t,r(t)) = 0$ and $g(0,t,r(t)) = 0$ for $t \geq 0$. Thus E.q. (4) has a trivial solution $x(t;0) \equiv 0$.

Here, we are interested in the almost surely exponential stability of E.q. (4). To this end, it is necessary to give the following definition.

Definition 1[33]: Let $x(t;x_0) = x(t)$ be the solution of E.q. (4), for any $x_0 \in R^n$,

(i) E.q. (4) is said to be almost surely exponential stable if

$$\limsup_{t\to\infty} \frac{1}{t}\log(|x(t)|) < 0 \quad \text{a.s.}$$

(ii) E.q. (4) is said to be almost surely exponential unstable if

$$\liminf_{t\to\infty} \frac{1}{t}\log(|x(t)|) > 0 \quad \text{a.s.}$$

3. Results

In this paper, we investigate the almost surely exponential stability of E.q. (4) . The following lemmas are needed.

Since stochastic process $\tau(t)$ is a continuous-time markovian chain, it is well-known that the random sojourn time in each mode before switching obeys exponential distribution. According to the memoryless of Markov chain and the theorem of large numbers, it is easily to get lemma 1. The proof is left to readers.

Lemma 1: Let $\overline{\eta}_i$ denote the expectation of stochastic process $\tau(t)$ when $\tau(t) = i$, $i \in S$. For any $\varepsilon > 0$, there must be $\Pr\left\{\left|\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}(\eta_{ij} - \overline{\eta}_i)\right| < \varepsilon\right\} = 1$, where $\overline{\eta}_i = -(\gamma_{ii})^{-1}$.

Lemma 2: Assume that the mappings $f: R^n \times R^+ \times S \to R^n$ and $g: R^n \times R^+ \times S \to R^{n\times m}$ are Lipschitz, and $f(0,t,r(t)) = 0$ and $g(0,t,r(t)) = 0$ for all $t \geq t_0$. Denote the solution of E.q. (4) by $x(t;x_0)$. Then there must be $\Pr\{x(t;x_0) \neq 0 \ on\, t > t_0\} = 1$ for all $x_0 \neq 0$ in R^n. That is, almost any solution of E.q. (4) with non-zero initial states will not reach the origin.

To investigate the almost surely exponential stability of E.q. (4), we impose the following assumption to mappings f and g as imposed in [33].

Assumption 1: For every mode $i \in S$, and all $(x,t) \in R^n \times R^+$, there exist constants α_i, β_i and σ_i such that

$$\begin{aligned} x^T f(x,t,i) &\leq \alpha_i |x|^2, \\ |g(x,t,i)| &\leq \beta_i |x|, \\ |x^T g(x,t,i)| &\geq \sigma_i |x|^2. \end{aligned} \tag{5}$$

Theorem 1: Let $x(t;x_0)$ be the solution of E.q. (4). Under Assumption 1, the solution $x(t;x_0)$ satisfies

$$\limsup_{t\to\infty}\frac{1}{t}\log(|x(t;x_0)|) \leq \sum_{i=1}^{L}\overline{\pi}_i(\alpha_i + 0.5\beta_i^2 - \sigma_i^2) \quad \text{a.s.} \tag{6}$$

for all $x_0 \in R^n$, where $\overline{\pi}_i = \pi_i(1 + d_i/\overline{\eta}_i) \Big/ \sum_{i=1}^{L}\pi_i(1 + d_i/\overline{\eta}_i)$.

In particularly, If

$$\sum_{i=1}^{L}\overline{\pi}_i(\alpha_i + 0.5\beta_i - \sigma_i^2) < 0, \tag{7}$$

then E.q. (4) is almost surely exponential stable.

Proof : It is evident that assertion (6) is true when $x_0 = 0$ because in this case $x(t;x_0) \equiv 0$. For simplicity, we denote $x(t;x_0) \equiv x(t)$ in this section without confusion. According to Lemma 1, $x(t)$ will never reach origin with probability one for $x_0 \neq 0$. Making use of ITÔ formula, we can get that

$$\begin{aligned} d\left[|x(t)|^2\right] &= \frac{2x^T(t)}{|x(t)|^2}\left[f(x(t),t,r(t))dt + g(x(t),t,r(t))dw(t)\right] \\ &+ \frac{1}{2}\left[\frac{2|g(x(t),t,r(t))|^2}{|x(t)|^2} - \frac{4|x^T(t)g(x(t),t,r(t))|^2}{|x(t)|^4}\right]dt \\ &= h(t)dt + v(t)dw(t) \end{aligned} \tag{8}$$

Where

$$h(t) = \frac{2x^T(t)}{|x(t)|^2}f(x(t),t,r(t)) + \frac{|g(x(t),t,r(t))|^2}{|x(t)|^2} - \frac{2|x^T(t)g(x(t),t,r(t))|^2}{|x(t)|^4}, \quad v(t) = \frac{2x^T(t)}{|x(t)|^2}g(x(t),t,r(t)).$$

Further more, we obtain that

$$\log(|x(t)|^2) = \log(|x(0)|^2) + \int_0^t h(s)ds + M(t), \tag{9}$$

Where $M(t)=\int_0^t v(s)dw(s)$ is a continuous martingale and it vanishes at $t=0$. By Assumption 1, the quadratic variation of $M(t)$ satisfies

$$\langle M(t),M(t)\rangle=\int_0^t |v(s)|^2 ds \le 4t \max_{1\le i\le L}\beta_i^2 . \tag{10}$$

According to the strong law of large numbers for local martingale, it follows that

$$\lim_{t\to\infty}\frac{M(t)}{t}=0 \text{ a.s.} \tag{11}$$

In addition, applying Assumption 1 again, we obtain that

$$\int_0^t h(s)ds \le \int_0^t \left(2\alpha_{r(s)}+\beta_{r(s)}^2-2\sigma_{r(s)}^2\right)ds . \tag{12}$$

According to the properties of Markovian chain, it is easy to get that for any subinterval of R^+, there are at most a finite number of switchings for almost every sampling path of $r(t)$. Thus for any $t\in R^+$, there must exist an integer $n\in N$ such that $t\in[t_n,t_{n+1})$. Let $\bar{t}=\sum_{k=0}^n \eta_{r(t_k),k}$ and $\tilde{t}=\sum_{k=0}^n d_{r(t_k),k}$, then there must be $t=\bar{t}+\tilde{t}$. Since $\tau(t)$ is Markovian chain with a stationary probability distribution $\pi=(\pi_1,\pi_2,\cdots,\pi_L)$, we get $\bar{t}=\sum_{i=1}^L \pi_i\bar{t}$. By virtue of the law of large numbers and lemma 1, for a sufficiently large t, we can easily get that

$$\Pr\left\{\left|\tilde{t}-\sum_{i=1}^L\left(\pi_i\bar{t}d_i/\overline{\eta}_i\right)\right|<\varepsilon\right\}=1, \tag{13}$$

for any $\varepsilon>0$. that is $\tilde{t}=\sum_{i=1}^L\left(\pi_i\bar{t}d_i/\overline{\eta}_i\right)$ with probability 1 as $t\to\infty$.

By the ergodic property of the switching signal $r(t)$, we can obtain its stationary probability distribution

$$\overline{\pi}=(\overline{\pi}_1,\overline{\pi}_2,\cdots,\overline{\pi}_L),$$

where $\overline{\pi}_i=\pi_i(1+d_i/\overline{\eta}_i)\Big/\sum_{i=1}^L\pi_i(1+d_i/\overline{\eta}_i)$.

Furthermore, we get that

$$\begin{aligned}&\lim_{t\to\infty}\frac{1}{t}\int_0^t\left(2\alpha_{r(s)}+\beta_{r(s)}^2-2\sigma_{r(s)}^2\right)ds\\&\qquad=\sum_{i=1}^L\overline{\pi}_i\left(2\alpha_i+\beta_i^2-2\sigma_i^2\right)\end{aligned} \text{ a.s.} \tag{14}$$

From (9)、(12) and (14) , we can get that

$$\begin{aligned}&\limsup_{t\to\infty}\frac{1}{t}\log(|x(t;x_0)|)\\&\qquad\le\sum_{i=1}^L\overline{\pi}_i\left(\alpha_i+0.5\beta_i^2-\sigma_i^2\right)\end{aligned} \text{ a.s.}$$

Therefore, the assertion (6) is true. Evidently，if

$$\sum_{i=1}^L\overline{\pi}_i\left(\alpha_i+0.5\beta_i^2-\sigma_i^2\right)<0,$$

then E.q. (4) is almost surely exponential stable. The proof is accomplished.

Remark 1: In [32], the switching signal $r(t)$ was first proposed by Xiong etc as a special random switching signal with a fixed dwell time. It provides a better mathematical framework to model the actual system switching behavior. Although the signal was cited in other papers, they did not give stationary probability distribution of $r(t)$. In this paper, we deduced that it has a stationary probability distribution on $\overline{\pi}=(\overline{\pi}_1,\overline{\pi}_2,\cdots,\overline{\pi}_L)$, where $\overline{\pi}_i=\pi_i(1+d_i/\overline{\eta}_i)\Big/\sum_{i=1}^L\pi_i(1+d_i/\overline{\eta}_i)$.

Evidently, $\overline{\pi}$ is equal to π when all d_i ($i\in S$) is equal to 0.

Corollary 1: Under Assumption 1, the E.q. (4) is almost surely exponential stable if $\alpha_i+0.5\beta_i^2-\sigma_i^2<0$ for all $i\in S$. In such case, the stability of E.q. (4) is unrelated to the switching signal.

In the following, we discuss the instability of E.q. (4) , and impose the assumption to mappings f and g below as imposed in [33].

Assumption 2: For every mode $i\in S$, and all $(x,t)\in R^n\times R^+$, there exist constants α_i, β_i and σ_i such that

$$\begin{aligned}x^T f(x,t,i)&\ge\alpha_i|x|^2,\\|f(x,t,i)|&\ge\beta_i|x|,\\|x^T g(x,t,i)|&\le\sigma_i|x|^2.\end{aligned} \tag{15}$$

Theorem 2: Let $x(t;x_0)$ be the solution of E.q. (4). Under Assumption 2, the solution $x(t;x_0)$ satisfies

$$\begin{aligned}&\liminf_{t\to\infty}\frac{1}{t}\log(|x(t;x_0)|)\\&\qquad\ge\sum_{i=1}^L\overline{\pi}_i\left(\alpha_i+0.5\beta_i^2-\sigma_i^2\right)\end{aligned} \text{ a.s.} \tag{16}$$

for all $x_0\in R^n$. In particularly, If

$$\sum_{i=1}^L\overline{\pi}_i\left(\alpha_i+0.5\beta_i^2-\sigma_i^2\right)>0, \tag{17}$$

then E.q. (4) is almost surely exponential unstable.

Proof : When $x_0=0$, assertion (16) is true apparently. Therefore we assume that the initial value $x_0\ne 0$. For simplicity, we denote $x(t;x_0)=x(t)$ again in this section. Similar to the process of theorem 1, we still have

$$\log\left(|x(t)|^2\right)=\log\left(|x(0)|^2\right)+\int_0^t h(s)ds+M(t), \tag{18}$$

Where $h(t)$ and $M(t)$ are the same as in Theorem 1. By Assumption 2, the estimation of the quadratic variation of $M(t)$ is following that

$$\langle M(t),M(t)\rangle=\int_0^t|v(s)|^2ds\le 4t\max_{1\le i\le L}\sigma_i^2 .$$

Employing the strong law of large numbers for local martingale, we can get

$$\lim_{t\to\infty}\frac{M(t)}{t}=0 \text{ a.s.} \tag{19}$$

Furthermore, according to Assumption 2 , we obtain that

$$\int_0^t h(s)ds\ge\int_0^t\left(2\alpha_{r(s)}+\beta_{r(s)}^2-2\sigma_{r(s)}^2\right)ds .$$

Since $r(t)$ has a stationary probability distribution on $\overline{\pi}$,we can obtain that

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t\left(2\alpha_{r(s)}+\beta_{r(s)}^2-2\sigma_{r(s)}^2\right)ds = \sum_{i=1}^{L}\bar{\pi}_i\left(2\alpha_i+\beta_i^2-2\sigma_i^2\right) \tag{20}$$

From (18), (19) and (20) , we get that

$$\liminf_{t\to\infty}\frac{1}{t}\log\left(|x(t)|\right) \ge \sum_{i=1}^{L}\bar{\pi}_i\left(\alpha_i+0.5\beta_i^2-\sigma_i^2\right) \quad \text{a.s.}$$

Thus, the assertion (16) is correct. Evidently, if

$$\sum_{i=1}^{L}\bar{\pi}_i\left(\alpha_i+0.5\beta_i^2-\sigma_i^2\right)>0,$$

then E.q. (4) is almost surely exponential unstable. The proof is accomplished.

Corollary 2: Under Assumption 2, the E.q. (4) is almost surely exponential unstable if $\alpha_i+0.5\beta_i^2-\sigma_i^2>0$ for all $i\in S$. In such case, the stability of E.q. (4) is unrelated to the switching signal.

In the following, we extend the conclusion of Theorem 1 to the linear hybrid stochastic systems.

Consider the linear hybrid system of the form

$$dx(t)=A_{r(t)}x(t)dt+C_{r(t)}x(t)dw(t). \tag{21}$$

For simplicity in derivation, we assume that $w(t)$ is a 1-dimensional Brownian motion here. The switching signal $r(t)$ is the same as in Theorem 1. If we denote

$$\alpha_i=\lambda_{\max}\left(A_i^*\right)/2,\ \beta_i=\|C_i\|,\ \sigma_i=\lambda_{\min}\left(C_i^*\right)/2$$

for all $i\in S$. Then Assumption 1 is satisfied. Thus we obtain the following corollary.

Corollary 3: The linear hybrid stochastic system of E.Q. (21) is almost surely exponential stable if

$$\sum_{i=1}^{L}\bar{\pi}_i\left(\alpha_i+0.5\beta_i^2-\sigma_i^2\right)<0.$$

In particular, if we add a control input $u(t)$ to E.Q. (21), then we get the systems in the following form

$$dx(t)=\left[A_{r(t)}x(t)+B_{r(t)}u(t)\right]dt +\left[C_{r(t)}x(t)+D_{r(t)}u(t)\right]dw(t). \tag{22}$$

Generally speaking, state feedback is a classical control method. If we take a state feedback control law $u(t)=K_{r(t)}x(t)$, then the Assumption 1 is satisfied with

$$\alpha_i=\lambda_{\max}\left(\bar{A}_i^*\right)/2,\ \beta_i=\|\bar{C}_i\|,\ \sigma_i=\lambda_{\min}\left(\bar{C}_i^*\right)/2$$

for all $i\in S$, where $\bar{A}_i=A_i+B_iK_i$, $\bar{C}_i=C_i+D_iK_i$. Thus we obtain the following corollary.

Corollary 4: If there are matrixes K_i, $i\in S$, such that $\sum_{i=1}^{L}\bar{\pi}_i\left(\alpha_i+0.5\beta_i^2-\sigma_i^2\right)<0$ holds, then the linear hybrid stochastic system of E.Q. (22) is almost surely exponential stable.

4. Simulation analysis

Here, we give three examples to illustrate our results obtained in section 3.

Example 1. Consider a 1-dinmetinal nonlinear hybrid stochastic system with two modes in the form of E.Q.(4), the parameters are given as following

$$f(x,t,1)=x\left(2+\cos^2 x\right),\ f(x,t,2)=x\sin 2x,$$
$$g(x,t,1)=x,\ g(x,t,2)=3x,\ r(t)\in S=\{1,2\},$$
$$d_1=0.1,\ d_2=0.2,\ \Gamma=\begin{pmatrix}-1.2 & 1.2\\ 0.4 & -0.4\end{pmatrix}.$$

Then the stationary distribution of switching signal $r(t)$ is $\bar{\pi}=(0.2642,0.7358)$. we take constants $\alpha_1=3$, $\alpha_2=1$, $\beta_1=1$, $\beta_2=3$, $\sigma_1=1$, $\sigma_2=3$, then Assumption 1 is satisfied. We obtain that

$$\sum_{i=1}^{L}\bar{\pi}_i\left(\alpha_i+0.5\beta_i^2-\sigma_i^2\right)=-1.033<0.$$

Thus the system is almost surely exponential stable.

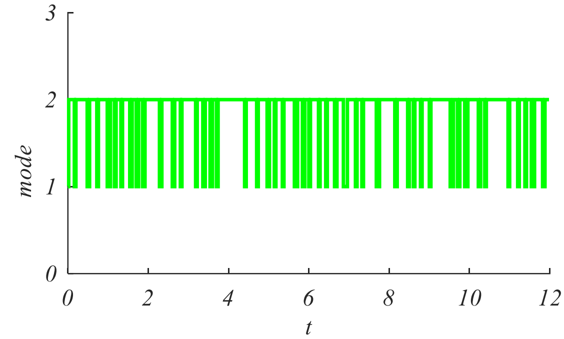

Fig. 2 Sample path of $r(t)$

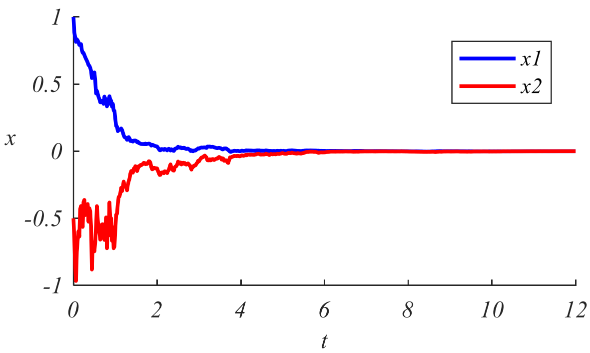

Fig. 3 Solution behavior of x_1 and x_2

Example 2. Let us consider a 2-dimensional hybrid system with two modes as given in (21). The data is as below:

$$A_1=\begin{pmatrix}-1 & 0.1\\ -0.8 & -1.1\end{pmatrix},\ A_2=\begin{pmatrix}-1 & 0.15\\ -0.1 & 0.25\end{pmatrix},$$
$$C_1=\begin{pmatrix}0 & 0.3\\ -0.2 & -0.6\end{pmatrix},\quad C_2=\begin{pmatrix}0.14 & -0.1\\ -0.004 & -0.11\end{pmatrix},$$
$$\Gamma=\begin{pmatrix}-1 & 1\\ 2 & -2\end{pmatrix},\quad S=\{1,2\},\quad d_1=0.1,\quad d_1=0.2,$$

$x_0=(1,-0.5)^T$. Then we get $\alpha_1=-0.6964$, $\alpha_2=-0.2492$, $\beta_1=1.0334$, $\beta_2=0.5020$, $\sigma_1=-0.8328$, $\sigma_2=0.0586$. Furthermore, we obtain $\sum_{i=1}^{2}\bar{\pi}_i\left(\alpha_i+0.5\beta_i^2-\sigma_i^2\right)=-1.0038<0$. According to Corollary 3, the system is almost surely exponential stable. Simulation is executed for this system. Fig. 2 shows the sample path of $r(t)$ with two modes, and Fig. 3 shows the system state trajectory.

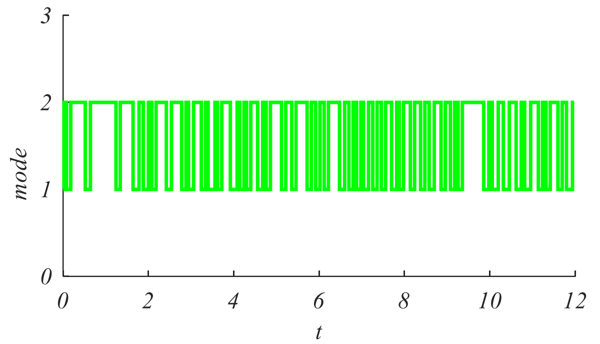

Fig. 4 Switching signal of $r(t)$

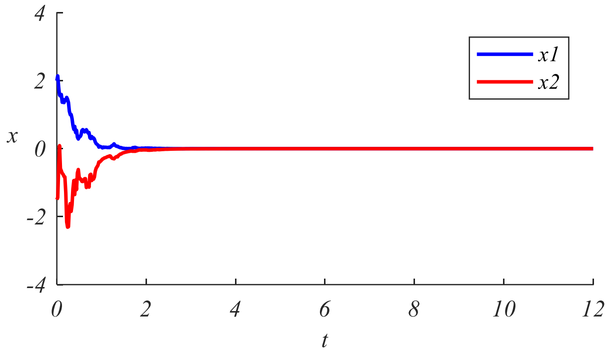

Fig. 5 the system state trajectory of x_1 and x_2

Example 3. Consider a 2-dimention hybrid system with two modes in the form of E.Q. (22) with

$$A_1 = \begin{pmatrix} 1 & 1.5 \\ 1.5 & -2 \end{pmatrix},\ A_2 = \begin{pmatrix} -1.2 & 0.95 \\ 1.1 & -0.8 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} -1 & 0.3 \\ 0.8 & -1.2 \end{pmatrix},\ B_2 = \begin{pmatrix} 0.3 & 0.5 \\ -2 & 1 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} 0.5 & 0.25 \\ -0.6 & -0.8 \end{pmatrix},\ C_2 = \begin{pmatrix} -0.5 & 0.2 \\ -0.8 & 0.6 \end{pmatrix},$$

$$D_1 = \begin{pmatrix} 0 & 0.5 \\ -0.3 & 0.4 \end{pmatrix},\ D_2 = \begin{pmatrix} 0.5 & 1 \\ 0.6 & 0.7 \end{pmatrix},$$

$$K_1 = \begin{pmatrix} -1 & -0.3 \\ 0.5 & 1 \end{pmatrix},\ K_2 = \begin{pmatrix} -0.7 & 0.6 \\ -0.5 & -0.6 \end{pmatrix},$$

$$\Gamma = \begin{pmatrix} -8 & 8 \\ 2 & -2 \end{pmatrix},\ S = \{1,2\},\ d_1 = 0.25,\ d_2 = 0.15,$$

$$x_0 = (1,-0.5)^T.$$

Then we get $\alpha_1 = 0.4453$, $\alpha_2 = -2.7876$, $\beta_1 = 1.1003$, $\beta_2 = 1.8483$, $\sigma_1 = -0.4017$, $\sigma_2 = -9245$.

Furthermore we obtain

$$\sum_{i=1}^{2} \bar{\pi}_i \left(\alpha_i + 0.5\beta_i^2 - \sigma_i^2\right) = -1.2283 < 0.$$

According to Corollary 4, the system is almost surely exponential stable. Simulation is carried outed for this system. Fig. 4 shows the sample path of $r(t)$ with two modes, and Fig. 5 shows the system state trajectory of x_1 and x_2 respectively.

Conclusion

In this paper, the stability property of hybrid stochastic systems is investigated with a random switching signal in which the dwell time consists of a fixed time and a random sojourn time. We derived the sufficient conditions for almost surely exponential stable and unstable of such systems. And then a few numerical simulations are given to illustrate the results of the theorems.

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