Contents lists available at YXpublications

International Journal of Applied Mathematics in Control Engineering

Journal homepage: http://www.ijamce.com

Stability Analysis for Hybrid Systems with Impulse and Markovian Switching Effects

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ARTICLE INFO

Article history:
Received 25 February 2021
Accepted 28 April 2021
Available online 29 April 2021

Keywords:
Impulse
Markov switching
Hybrid systems
Mean stability
ITÔ formula
Stationary distribution

ABSTRACT

In this paper, the almost surely exponential stability is investigated of nonlinear hybrid stochastic systems with impulse and Markovian switching effects. Applying the stationary probability distribution of Markov process and the method of the average dwell-time approach, we obtained sufficient conditions for stable and unstable systems, respectively. The proposed results provide a method to set a suitable impulsive time sequences which depends not only on the continuous dynamics but also on the Markov switching signal. It is proved that when the continuous dynamics are stable, the destabilizing impulses should not occur frequently with a lower bound; Conversely, when the continuous dynamics are unstable, the stabilizing impulses must not be overly long with a upper bound, and the impulse can stabilize the continuous dynamics. The conclusion is illustrated by a numerical example.

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1. Introduction

Impulse and switching are regarded as the typical phenomena in the hybrid systems which have played a crucial role in many applications. Since systems in the real-world often need to run for a long period of time, some fundamental problems, such as stability and controllability, are still absorbing and challenging. Generally, systems are also subjected to unpredictable situations, such as structural changes, external environment disturbances, random failures of components, etc. In terms of application, impulse and switching provide a natural framework for mathematical modeling these situations.

Stochastic modelling has come to play an important role in science and industry, and the systems with Markovian switching constitute a classical branch of its research field. This kind of hybrid systems combine a part of the state that takes values continuously and another part of the state that takes discrete values. Developments on stochastic stability can be found in Henderson and Plaschko (2014), Mao (1990), Khasminskii (2012), Pardoux and Răşcanu (2014) among others. Due to its wide usages and applications, recently, there have been quite a lot of attentions paid to these systems. For example, Yuan, Mao and Lygeros (2009) investigated stochastic hybrid delay population dynamics; Sethi et al. (1994) presented a research on hierarchical control of stochastic manufacturing systems with linear production costs; Yin et al. (2008) studied communication power control problems for wireless communication networks; Boukas (2008) dealt with the stability and stabilization of continuous-time singular Markovian switching systems; Patrinos et al. (2014) considered stochastic optimal control problems for Markovian switching systems.

On the other hand, impulsive systems constitute a class of hybrid systems in which the states evolve according to continuous-time dynamics with instantaneous state jump or reset (also referred as impulses). Impulsive control theory has been extensively studied in past decades; see, e.g., Haddad (2014), Ambrosino et al. (2009), Guans et al. (2002), Lakshmikantham et al. (1989) and the references cited therein. In general, the study on such systems is divided into two classes which are impulsive perturbation problem (IPP) and impulsive control problem (ICP). Roughly speaking, IPP can be considered as a robustness analysis of the system subject to external continuous disturbances; meanwhile systems are regarded as ICP when they are under proper impulsive control to maintain certain performances, such as periodic solution, attractor, stability and so on. Because impulsive controller usually has a relatively simple structure, it has received much attentions recently in various applications, examples can be found in many fields such as neural networks (Chen et al., 2017), secure communication (Li et al., 2012), economics (Dykhta, 2014), biological models (Nundloll et al., 2010) and so on.

In this paper, we consider a class of hybrid systems that involve both impulse and Markovian switching. Impulse effects can be used to describe the sudden changes to system states, and Markovian

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switching can decide the current system operation mode. Based on the statistic property of Markov process and the method of the average dwell-time approach, sufficient conditions are derived for mean square exponential stable and unstable systems, respectively. The rest of this paper is organized as follows. In section 2, we give some necessary notations, and then formulate a hybrid stochastic system and some definitions. In section 3, the main results are given. In section 4, numerical examples are given to illustrate the results. And conclusions are given in section 5.

2. Preliminaries

Throughout this paper, unless otherwise specified, we use the following notations. Let R^n and $R^{n\times m}$ denote n-dimensional real space and $n\times m$-dimensional real matrix space, respectively. $|x|$ denotes the Euclidean norm for a vector $x\in R^n$. The superscript "T" denotes the transpose for vectors or matrices. For function $\varphi(t)$, the left-limit is denoted by $\varphi(t^-)=\lim_{s\to 0^-}\varphi(t+s)$. $PC([a,b];R^l)$ denotes the class of piece-wise continuous functions from $[a,b]$ to R^l. $E(\cdot)$ stands for the expectation. Let $(\Omega,\mathcal{F},P)$ denote a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\ge 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all p-null sets). $w(t)=(w_1(t),w_2(t),\cdots,w_m(t))$ stands for an d-dimensional-adapted Brownian motion defined on the probability space. Let $r(t)$, $t\ge 0$ be a right-continuous Markov chain on the probability space taking values in a discrete and finite state space $S=\{1,2,\cdots,M\}$. It is well known that almost every sample path of $r(t)$ is a right-continuous step function. The generator $\Gamma=(\pi_{ij})_{M\times M}$ is given by

$$\Pr\{r(t+\Delta t)=j|r(t)=i\}=\begin{cases}\pi_{ij}\Delta t+o(\Delta t), & if\ j\ne i\\ 1+\pi_{ii}\Delta t+o(\Delta t), & if\ j=i\end{cases}, \tag{1}$$

Where $\Delta t>0$ is a small time increment, $\pi_{ij}>0$ is the transition rate from mode i to mode j if $j\ne i$ while $\pi_{ii}=-\sum_{j=1,j\ne i}^{L}\pi_{ij}$ if $j=i$, and $\lim_{\Delta t\to 0^+}o(\Delta t)/\Delta t=0$. The mode switching time sequence $\{t_1,t_2,t_3,\cdots\}$ is strictly increasing without accumulation points and satisfies $\lim_{k\to\infty}t_k=\infty$. Furthermore, we assume that the Markov chain $r(t)$ is independent of the Brownian motion $w(t)$.

Consider the following hybrid system of stochastic differential equation with impulses and Markovian switching

$$\begin{cases}dx(t)=f(x(t),t,r(t))dt+g(x(t),t,r(t))dw(t), & t\ne\hat{t}_k\\ x(t)=I(x(t^-),r(t)), & t=\hat{t}_k\end{cases}, \tag{2}$$

Where $t\ge t_0=0$, $x(t)$ is the system state, $r(t)$ is a Markovian switching signal. The impulsive time sequence $\{\hat{t}_1,\hat{t}_2,\hat{t}_3,\cdots\}$ is strictly increasing without accumulation points while $\lim_{k\to\infty}\hat{t}_k=\infty$. Without losing generality, we assume that the impulsive time sequence $\{\hat{t}_1,\hat{t}_2,\hat{t}_3,\cdots\}$ is a subsequence of the switching time sequence $\{t_1,t_2,t_3,\cdots\}$. ξ is a $\mathcal{F}_0$-adapted random variable such that $E(\xi)<\infty$. The mappings $f:R^n\times R^+\times S\to R^n$, $g:R^n\times R^+\times S\to R^{n\times m}$, and $I:R^n\times S\to R^n$ are all Borel-measurable functions satisfying Lipschitz condition and linear growth condition (the precise growth condition will be given later). These conditions guarantee that E.q. (2) has a unique solution. Let $x(t;x_0)$ denote the solution of the equation, for simplicity, we write $x(t;x_0)=x(t)$ without confusion. For the purpose of stability study, we assume that $f(0,t,r(t))=0$ and $g(0,t,r(t))=0$, and $I(0,t,r(t))=0$ for $t\ge 0$.Thus E.q. (2) has a trivial solution $x(t;0)\equiv 0$.

Now some definitions and lemmas are given for deriving the main results.

Definition 1 (Mao & Yuan 2006). Let $x(t;x_0)$ be the solution of Eq. (2), for any $x_0\in R^n$,

(i) E.q. (2) is said to be mean square Lyapunov exponential stable if $\limsup_{t\to\infty}\frac{1}{t}\log\left(E|x(t)|^2\right)<0$;

(ii) E.q. (2) is said to be mean square Lyapunov exponential unstable if $\liminf_{t\to\infty}\frac{1}{t}\log\left(E|x(t)|^2\right)>0$.

Definition 2 (Hespanha et al. 2008). Given an impulsive time sequence $\{\hat{t}_1,\hat{t}_2,\hat{t}_3,\cdots\}$, $N(t,s)$ denotes the number of instants $\hat{t}_k$ in the interval $(s,t]$. For a given $0<\varepsilon<1$, if there exist T_a such that

$$\frac{t-s}{(1+\varepsilon)T_a}\le N(t,s)\le\frac{t-s}{(1-\varepsilon)T_a},$$

then T_a is called the average impulsive interval (AII), and ε is called elasticity factor.

The following lemmas are necessary to get the main results.

Lemma 1 (Mao, 1999). Assume that the mappings $f:R^n\times R^+\times S\to R^n$ and $g:R^n\times R^+\times S\to R^{n\times m}$ are Lipschitz, and $f(0,t,r(t))=0$ and $g(0,t,r(t))=0$ for all $t\ge t_0$. Denote the solution of E.q. (4) by $x(t;x_0)$. Then there must be $\Pr\{x(t;x_0)\ne 0\ on\, t>t_0\}=1$ for all $x_0\ne 0$ in R^n. That is, almost any solution of E.q. (4) with non-zero initial states will not reach the origin.

Lemma 2. Assume that there exist a function $v(t)\in PC([t_0,+\infty],R)$, and constants α and β such that $\dot{v}(t)\le\alpha v(t)$, $t\ne\hat{t}_k$, $t>t_0$, and $v(t)\le\beta v(t^-)$, $t=\hat{t}_k$, then we have $v(t)\le\beta^{N(t,t_0)}e^{\alpha(t-t_0)}v(t_0)$.

Lemma 3. Assume that there exist a function

$v(t) \in PC([t_0, +\infty], R)$, and constants α and β such that $\dot{v}(t) \geq \alpha v(t)$, $t \neq \hat{t}_k$, $t > t_0$, and $v(t) \geq \beta v(t^-)$, $t = \hat{t}_k$, then we have $v(t) \geq \beta^{N(t,t_0)} e^{\alpha(t-t_0)} v(t_0)$.

3. Results

In order to investigate the stability of Eq. (2), we impose the following conditions to mappings f, g and I.

Assumption 1: For every mode $i \in S$, and all $(x,t) \in R^n \times R^+$, there exist constants α_i, β_i and σ_i such that

$$x^T f(x,t,i) \leq \alpha_i |x|^2, \; |g(x,t,i)|^2 \leq \beta_i |x|^2$$
$$|I(x(t),i)|^2 \geq \gamma_i |x|^2. \tag{5}$$

where $|g(x(t),t,i)|^2 = trace(g^T(x(t),t,i) g(x(t),t,i))$.

For simplicity, we denote $\alpha_{r(t)} = \alpha_{i_k}$, $\beta_{r(t)} = \beta_{i_k}$ and $\gamma_{r(t)} = \gamma_{i_k}$ when $r(t) = i \in S$ for $t \in [t_k, t_{k+1})$, $k \in N$. Unless otherwise specified, we take the same representations in the following.

Theorem 1. Assume that E.q. (2) satisfies Assumption 1. For any initial value $x_0 = \xi \in R^n$, the solution of E.q. (2) follows

$$\limsup_{t\to\infty} \frac{1}{t} \log\left(E|x(t)|^2\right) \leq \sum_{1\leq i\leq M} \pi_i \begin{pmatrix} 2\alpha_i + \beta_i \\ + \dfrac{1}{(1+\sigma_i\varepsilon)T_a} \log \gamma_i \end{pmatrix} \tag{8}$$

where $\sigma_i = -\mathrm{sgn}(\log \gamma_i)$. In particular, the E.q. (2) is mean square Lyapunov exponential stable if

$$\sum_{1\leq i\leq M} \pi_i \left(2\alpha_i + \beta_i + \frac{1}{(1+\sigma_i\varepsilon)T_a} \log \gamma_i \right) < 0.$$

Proof. Clearly, the assertion (8) is true when $x_0 = \xi = 0$ because the solution $x(t;0) = 0$ in this case. For any initial value $x_0 = \xi \neq 0$, we write $x(t;x_0) = x(t)$ for simplicity. Let $V(t) = |x(t)|^2$, $t \geq t_0$. For $t \in [t_l, t_{l+1})$, $l \in N$. applying the ITÔ formula yields that

$$\begin{aligned} dV(t) &= 2x^T(t)\left[f(x(t),t,r(t))dt + g(x(t),t,r(t))dw(t) \right] \\ &\quad + trace\left[g^T(x(t),t,r(t)) g(x(t),t,r(t)) \right] dt \\ &= LV(t)dt + 2x^T(t) g(x(t),t,r(t)) dw(t) \end{aligned}, \tag{9}$$

Where

$$\begin{aligned} LV(t) &= 2x^T(t) f(x(t),t,r(t)) \\ &\quad + trace\left[g^T(x(t),t,r(t)) g(x(t),t,r(t)) \right] \end{aligned} \tag{10}$$

Let $\Delta t > 0$ be small enough such that $t + \Delta t \in [\hat{t}_l, \hat{t}_{l+1})$, making use of lemma 3.2 in Khasminskill (2012) and Fubini's theorem, we can obtain that

$$EV(x(t+\Delta t)) - EV(x(t)) = \int_t^{t+\Delta t} E(LV(t)) dt.$$

Noticing that $E(LV(t))$ is continuous in the interval $[\hat{t}_l, \hat{t}_{l+1})$, so we get that

$$(EV(x(t)))' = E(LV(t)). \tag{11}$$

Combing (7) with (11), it follows that

$$(EV(x(t)))' \leq (2\alpha_{r(t)} + \beta_{r(t)})(EV(x(t))). \tag{12}$$

For $t \in [t_0, t_1)$, Applying lemma 2, we have

$$EV(x(t)) \leq \gamma_{i_0}^{N(t,t_0)} e^{(2\alpha_{i_0} + \beta_{i_0})(t-t_0)} EV(x(t_0)).$$

From the above it follows that

$$EV(x(t_1)) \leq \gamma_{i_0}^{N(t_1,t_0)} e^{(2\alpha_{i_0} + \beta_{i_0})(t_1-t_0)} EV(x(t_0)). \tag{13}$$

Repeating the former process similarly, for $t \in [t_1, t_2)$, we can obtain that

$$EV(x(t)) \leq \gamma_{i_0}^{N(t,t_1)} e^{(2\alpha_{i_0} + \beta_{i_0})(t-t_1)} EV(x(t_1)). \tag{14}$$

Substituting (13) into (14) yields that

$$\begin{aligned} EV(x(t)) &\leq \gamma_{i_1}^{N(t,t_1)} e^{(2\alpha_{i_1} + \beta_{i_1})(t-t_1)} \\ &\quad \times \gamma_{i_0}^{N(t_1,t_0)} e^{(2\alpha_{i_0} + \beta_{i_0})(t_1-t_1)} EV(\xi) \end{aligned}$$

Since $r(t)$ is a right-continuous step function with a finite number jumps in any finite subinterval of R^+. For any $t \in [0, +\infty)$, there must exist an integer $n \in N$ such that $t \in [t_n, t_{n+1})$. By mathematical induction, we have

$$\begin{aligned} EV(x(t)) &\leq \gamma_{i_n}^{N(t,t_n)} e^{(2\alpha_{i_n} + \beta_{i_n})(t-t_n)} \\ &\quad \times \left(\prod_{0\leq k\leq n-1} \gamma_{i_k}^{N(t_{k+1},t_k)} e^{(2\alpha_{i_k} + \beta_{i_k})(t_{k+1}-t_k)} \right) EV(\xi) \end{aligned} \tag{15}$$

Therefore, we obtain that

$$\begin{aligned} \log(EV(x(t))) &\leq (2\alpha_{i_n} + \beta_{i_n})(t - t_n) \\ &\quad + \sum_{0\leq k\leq n-1} (2\alpha_{i_k} + \beta_{i_k})(t_{k+1} - t_k) \\ &\quad + N(t,t_n) \log \gamma_{i_n} \\ &\quad + \sum_{0\leq k\leq n-1} N(t_{k+1},t_k) \log \gamma_{i_k} + \log EV(\xi) \end{aligned} \tag{16}$$

Moreover, by the ergodic property of the Markov chain and the definition of $N(t,s)$, we get

$$\lim_{t\to\infty} \frac{1}{t} \begin{pmatrix} (2\alpha_{i_n} + \beta_{i_n})(t - t_n) \\ + \sum_{0\leq k\leq n-1} (2\alpha_{i_k} + \beta_{i_k})(t_{k+1} - t_k) \end{pmatrix} \leq \sum_{0\leq k\leq M} \pi_i (2\alpha_i + \beta_i) \tag{17}$$

$$\lim_{t\to\infty} \frac{1}{t} \begin{pmatrix} N(t,t_n) \log \gamma_{i_n} \\ + \sum_{0\leq k\leq n-1} N(t_{k+1},t_k) \log \gamma_{i_k} \end{pmatrix} \leq \sum_{0\leq i\leq M} \frac{\pi_i}{(1+\sigma_i\varepsilon)T_a} \log \gamma_{i_k} \tag{18}$$

where $\sigma_i = -\mathrm{sgn}(\log \gamma_i)$.

Since $E(\xi) < \infty$, we yield that

$$\lim_{t\to\infty} \frac{1}{t} \log(EV(\xi)) = 0. \tag{19}$$

It follows from (15) to (19) that

$$\limsup_{t\to\infty}\frac{1}{t}\log\left(E|x(t)|^2\right)\le\sum_{1\le i\le M}\pi_i\begin{pmatrix}2\alpha_i+\beta_i\\+\frac{1}{(1+\sigma_i\varepsilon)T_a}\log\gamma_i\end{pmatrix}.$$

Thus the assertion (8) is obtained. Evidently, the E.q. (2) is mean square Lyapunov exponential stable if

$$\sum_{1\le i\le M}\pi_i\left(2\alpha_i+\beta_i+\frac{1}{(1+\sigma_i\varepsilon)T_a}\log\gamma_i\right)<0.$$

The proof is completed.

Remark 1. After proving theorem 1, let us provide a deep insight into the condition (8). when $2\alpha_i+\beta_i>0$ is true for everry $i\in S$, that is every mode is exponential unstable in the sense of mean square, this condition can not be hold for any impulse time sequence if $\gamma_i>1$ for everry $i\in S$. All other combinations of α_i, β_i and σ_i lead to interesting results.

We now explore some cases as examples. Suppose that a hybrid system satisfies the assumption 1 as in (7). when $2\alpha_i+\beta_i>0$ is true for every $i\in S$, we can choose $\gamma_i<e^{-(2\alpha_i+\beta_i)(1+\sigma_i\varepsilon)T_a}$ for every $i\in S$ that makes condition (8) to hold. In this case, the inequality $2\alpha_i+\beta_i>0$ implies the continuous dynamics of mode i is exponential unstable in the sense of mean square. Since $2\alpha_i+\beta_i>0$, the impulses must stabilize the system efficiently. Thus we must have $\gamma_i<1$ and the stabilizing impulses must not be overly long with a upper bound $-\log\gamma_i/[(2\alpha_i+\beta_i)(1+\sigma_i\varepsilon)T_a]$, that is $T_a<-\log\gamma_i/[(2\alpha_i+\beta_i)(1+\sigma_i\varepsilon)]$. Conversely, when $2\alpha_i+\beta_i<0$ is true for every $i\in S$ and $\gamma_i>1$, the impulses can potentially destroy the stable and should not occur frequently with a lower bound $-\log\gamma_i/[(2\alpha_i+\beta_i)(1+\sigma_i\varepsilon)T_a]$, that is $T_a>-\log\gamma_i/[(2\alpha_i+\beta_i)(1+\sigma_i\varepsilon)]$. The other combinations are left to readers.

The following result obtains from the above observations.

Corollary 1. Suppose E.q. (2) satisfies Assumption 1.

(i) When $2\alpha_i+\beta_i>0$ and $0<\gamma_i<1$ for all $i\in S$, E.q. (2) is exponential stable in the sense of mean square if

$$T_a<\min_{1\le i\le M}-\log\gamma_i/[(2\alpha_i+\beta_i)(1+\sigma_i\varepsilon)];$$

(ii) When $2\alpha_i+\beta_i<0$ and $\gamma_i>1$ for all $i\in S$, E.q. (2) is exponential stable in the sense of mean square if

$$T_a>\max_{1\le i\le M}-\log\gamma_i/[(2\alpha_i+\beta_i)(1+\sigma_i\varepsilon)].$$

Assumption 2. For every mode $i\in S$, and all $(x,t)\in R^n\times R^+$, there exist constants α_i, β_i and σ_i such that

$$x^T f(x,t,i)\ge\alpha_i|x|^2,\ |g(x,t,i)|^2\ge\beta_i|x|^2,$$
$$|I(x(t),i)|^2\ge\gamma_i|x|^2, \tag{20}$$

where $|g(x(t),t,i)|^2=trace(g^T(x(t),t,i)g(x(t),t,i))$.

Theorem 2. Assume that E.q. (2) satisfies Assumption 1. For any initial value $x_0=\xi\in R^n$, the solution of E.q. (2) follows

$$\liminf_{t\to\infty}\frac{1}{t}\log\left(E|x(t)|^2\right)\ge\sum_{1\le i\le M}\pi_i\begin{pmatrix}2\alpha_i+\beta_i\\+\frac{1}{(1+\sigma_i\varepsilon)T_a}\log\gamma_i\end{pmatrix}, \tag{21}$$

where $\sigma_i=-\mathrm{sgn}(\log\gamma_i)$. In particular, the Eq. (2) is mean square Lyapunov exponential unstable if

$$\sum_{1\le i\le M}\pi_i\left(2\alpha_i+\beta_i+\frac{1}{(1+\sigma_i\varepsilon)T_a}\log\gamma_i\right)>0.$$

Proof. The process is similar to the theorem 1. For any initial value $x_0=\xi\in R^n$, we write $x(t,\xi)=x(t)$ for simplicity again. Let $V(t)=|x(t)|^2$, $t\ge t_0$. According to the property of Markov chain, for any $t\in[0,+\infty)$, there must exist an integer $n\in N$ such that $t\in[t_n,t_{n+1})$. By (20) and lemma 3, we can get from (11) that

$$\begin{aligned}EV(x(t))&\ge\gamma_{i_n}^{N(t,t_n)}e^{(2\alpha_{i_n}+\beta_{i_n})(t-t_n)}\\&\times\left(\prod_{0\le k\le n-1}\gamma_{i_k}^{N(t_{k+1},t_k)}e^{(2\alpha_{i_k}+\beta_{i_k})(t_{k+1}-t_k)}\right)EV(\xi)\end{aligned} \tag{22}$$

Therefore, we obtain that

$$\begin{aligned}\log(EV(x(t)))\ge&(2\alpha_{i_n}+\beta_{i_n})(t-t_n)\\&+\sum_{0\le k\le n-1}(2\alpha_{i_k}+\beta_{i_k})(t_{k+1}-t_k)\\&+N(t,t_n)\log\gamma_{i_n}\\&+\sum_{0\le k\le n-1}N(t_{k+1},t_k)\log\gamma_{i_k}\\&+\log EV(\xi)\end{aligned} \tag{23}$$

Meanwhile, by the ergodic property of the Markov chain we still have (17), and by the definition of $N(t,s)$, we get

$$\lim_{t\to\infty}\frac{1}{t}\begin{pmatrix}N(t,t_n)\log\gamma_{i_n}\\+\sum_{0\le k\le n-1}N(t_{k+1},t_k)\log\gamma_{i_k}\end{pmatrix}\ge\sum_{0\le i\le M}\frac{\pi_i}{(1+\sigma_i\varepsilon)T_a}\log\gamma_{i_k} \tag{24}$$

where $\sigma_i=\mathrm{sgn}(\log\gamma_i)$.

Combining (17), (19), (23), (24) together, it follows that

$$\liminf_{t\to\infty}\frac{1}{t}\log\left(E|x(t)|^2\right)\ge\sum_{1\le i\le M}\pi_i\begin{pmatrix}2\alpha_i+\beta_i\\+\frac{1}{(1+\sigma_i\varepsilon)T_a}\log\gamma_i\end{pmatrix}.$$

Thus the assertion (21) is obtained. Clearly,the Eq. (2) is mean square Lyapunov exponential unstable if

$$\sum_{1\le i\le M}\pi_i\left(2\alpha_i+\beta_i+\frac{1}{(1+\sigma_i\varepsilon)T_a}\log\gamma_i\right)>0.$$

The proof is completed.

4. Simulation examples

In this section, two examples are given to illustrate the results in the previous section.

Example 1. Consider a 1-dinmetinal hybrid system with two subsystems, the parameters are given as following:

$f(x,t,1)=x(2+\cos^2 x)$, $f(x,t,2)=x\sin 2x$, $g(x,t,1)=x$, $g(x,t,2)=3x$, $r(t)\in S=\{1,2\}$,

$d_1 = 0.1$, $d_2 = 0.2$, $\Gamma = \begin{pmatrix} -1.2 & 1.2 \\ 0.4 & -0.4 \end{pmatrix}$.

Then the stationary distribution of switching signal $r(t)$ is $\bar{\pi} = (0.2642, 0.7358)$. we take constants $\alpha_1 = 3$, $\alpha_2 = 1$, $\beta_1 = 1$, $\beta_2 = 3$, $\sigma_1 = 1$, $\sigma_2 = 3$, then Assumption 1 is satisfied. We obtain that

$$\sum_{i=1}^{L} \bar{\pi}_i \left(\alpha_i + 0.5\beta_i^2 - \sigma_i^2\right) = -1.033 < 0.$$

$r(t) \in S = \{1,2\}$, $\Gamma = \begin{pmatrix} -0.4 & 0.4 \\ 0.7 & -0.7 \end{pmatrix}$, $f(x,t,1) = x(2+\cos^2 x)$, $f(x,t,2) = x \sin 2x$, $g(x,t,1) = x$, $g(x,t,2) = 3x$, $I(x,1) = 0.7x$, $I(x,1) = 1.2x$. Then the stationary distribution of Markov chain $r(t)$ is $(\pi_1, \ \pi_2) = (0.6363, 0.3637)$. we take constants $\alpha_1 = 3$, $\alpha_2 = 1$, $\beta_1 = 1$, $\beta_2 = 3$, $\gamma_1 = 0.7$, $\gamma_2 = 1.2$, then Assumption 1 is satisfied. We let $\varepsilon = 0.05$ in the definition of $N(t,s)$. If we take $T_a \le 0.0322$, then we have $\sum_{1 \le i \le 2} \pi_i \left(2\alpha_i + \beta_i + \frac{1}{(1+\sigma_i \varepsilon) T_a} \log \gamma_i \right) < 0$. By theorem 1, we obtain that E.q.(2) is mean square Lyapunov exponential stable.

Example 2. Consider the hybrid system with two subsystems as following

$$\begin{cases} dx(t) = A_{r(t)} x(t) dt + C_{r(t)} x(t) dw(t), & t \ne \hat{t}_k \\ x(t) = B_{r(t)} x(t^-), & t = \hat{t}_k \end{cases}.$$

The data is as below:

$S = \{1,2\}$ $A_1 = \begin{pmatrix} -1.5 & 5 \\ -0.8 & -1.1 \end{pmatrix}$, $A_2 = \begin{pmatrix} -1.2 & 4.15 \\ 1.1 & -0.8 \end{pmatrix}$, $B_1 = \begin{pmatrix} -0.17 & -0.5 \\ 0.3 & 0.51 \end{pmatrix}$, $B_2 = \begin{pmatrix} -0.31 & 0 \\ 0.5 & -0.5 \end{pmatrix}$, $C_1 = \begin{pmatrix} 0 & 0.25 \\ -0.6 & -0.8 \end{pmatrix}$, $C_2 = \begin{pmatrix} 0.1 & 0.2 \\ -0.4 & 0.3 \end{pmatrix}$, $\Gamma = \begin{pmatrix} -0.4 & 0.4 \\ 0.7 & -0.7 \end{pmatrix}$.

Then the stationary distribution of Markov chain $r(t)$ is $(\pi_1, \ \pi_2) = (0.6363, 0.3637)$. we take constants $\alpha_1 = 0.8095$, $\alpha_2 = 1.3350$, $\beta_1 = 1.0409$, $\beta_2 = 0.2520$, $\gamma_1 = 0.6226$, $\gamma_2 = 0.5530$, then Assumption 1 is satisfied. We let $\varepsilon = 0.05$ in the definition of $N(t,s)$. If we take $T_a \le 0.1965$, then we have $\sum_{1 \le i \le 2} \pi_i \left(2\alpha_i + \beta_i + \frac{1}{(1+\sigma_i \varepsilon) T_a} \log \gamma_i \right) < 0$. By theorem 1, we obtain that E.q.(2) is mean square Lyapunov exponential stable.

Numerical simulation is executed. Fig. 1 shows the switching signal with two modes, and Fig. 2 shows the system state trajectory.

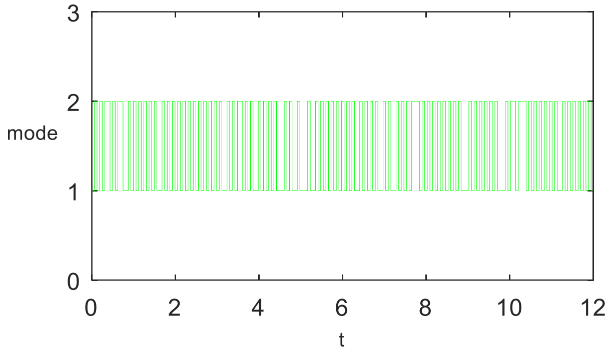

Fig. 1 switching signal with two modes

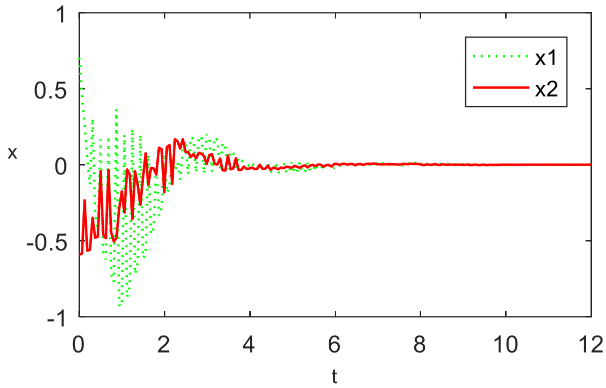

Fig. 2 the system state trajectory

5. Conclusion

In this paper, the stability property of hybrid stochastic systems is investigated with impulse and Marlovian switching effects. We derived the sufficient conditions for exponential stable and unstable in the sense of mean square. And then a few numerical simulations are given to illustrate the results.

Acknowledgements

Yang Li would like to acknowledge financial support provided by Maine Space Grant Consortium and Hsu would like to acknowledge National Science Founation (OCE-0644497; CBET-0426811).

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