Contents lists available at YXpublications

International Journal of Applied Mathematics in Control Engineering

Journal homepage: http://www.ijamce.com

Stability Analysis of Switching Multiple Model Adaptive Control

Weicun Zhang[a,*]

[a] School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing, MI 100083, CHINA

ARTICLE INFO

Article history:
Received 5 March 2021
Accepted 8 May 2021
Available online 10 May 2021

Keywords:
Stability analysis
Switching
Multiple model
Adaptive control
Virtual equivalent system

ABSTRACT

Despite the fundamental progress achieved so far, a general theory on stability and convergence of switching multiple model adaptive control system is still absent. In the bulk of literature, most of the existing works have been specific to particular switching function or particular control scheme, while only a few attempts have been made towards a unified analysis of the subject from a general perspective, and the results are far from satisfaction. Based on the new concept—Virtual Equivalent System, a unified theoretical framework for switching multiple model adaptive control system is established in this paper.

1. Introduction

Adaptive control technology was originated from gain-scheduling control scheme of high-performance autopilot in 1950s. As a main component of adaptive control, model reference adaptive control (MRAC) was proposed by Professor Whitaker to solve the autopilot control problem. From the viewpoint of theory research of optimal control of stochastic system with unknown or time varying parameters, self-tuning control (STC) was proposed by Professor Kalman, and then connected with applications through the pioneering work of Professor Astrom and Professor Wittenmark. It has been well-known that MRAC is actually a special class of STC. The only difference is that MRAC was first developed for continuous-time plants for model following, whereas STC was initially developed for stochastic discrete-time plant.

Multiple model adaptive control (MMAC) strategy was suggested in 1990s to improve the transient performance of classical adaptive control systems. Up to now, many switching MMAC algorithms have been put forward. Generally speaking, there are mainly two types of switching MMAC: indirect switching[1-5] and direct switching[6-10]. Indirect switching control can also be viewed as supervisory control because a supervisory function is used to decide when and which controller should be switched. Narendra, K. S. and Autenrieth, T. have used this method to improve the transient response of adaptive control systems[11-13]. As for direct switching control, the choice of when to switch to the next controller in a predetermined sequence is based directly on the output of the system. Since mid 1980's, papers about switching MMAC have covered continuous time system , discrete time system[14-15], nonlinear system[16], stochastic system[17], etc. and there are also some successful practical applications in this field.

Despite the fundamental progress achieved so far, there is still no a unified theory on adaptive control (conventional adaptive control and multiple model adaptive control)；Here we list some remarks to support the viewpoint.

In spite of 40 years of research, several books and hundreds of articles we still lack, in our view, a universally accepted design methodology for adaptive control which is based on sound theoretical issues and suitable for engineering implementations in real-life control systems[18].

A good theory should give also good clues to the construction of new algorithms. Unfortunately, there is no collection of results that can be called a theory of adaptive control in the sense specified[19].

Despite a significant number of practical applications and significant supporting theory, we are still a long way from having a full understanding of this important class of control strategies[20].

Despite the vast literature on the subject, there is still a general feeling that adaptive control is a collection of unrelated technical tools and tricks[21].

With the help of virtual equivalent system concept[22], we have developed two criteria to judge the stability and convergence of different switching MMAC algorithms. To a certain extent, these criteria are independent of specific control law and parameter estimation algorithm, and can thus provide a unified theoretical framework for understanding and evaluating different kinds of

* Corresponding author.
E-mail addresses: weicunzhang@ustb.edu.cn (W. Zhang)

switching MMAC schemes.

2. Description of switching MMAC

The basic architectures of switching MMAC systems are shown in Figure 1, which was concerned with continuous time plant in state space[12]. Generally speaking, there are three components of a switching MMAC system: model set, controller set and switching logic or mechanism. With model set $\mathbb{M}=\{M_i, i=1,\ldots,N\}$, we want to cover the uncertainty of plant P to be controlled. There is an estimated model in $\mathbb{M}$. According to each M_i, C_i is designed to satisfy some performance index. $\mathbb{C}=\{C_i,\ i=1,\ \ldots,\ N\}$ is the controller set. There is an adaptive controller in $\mathbb{C}$ according to the estimated model. Switching mechanism is used to decide when and which controller should be switched.

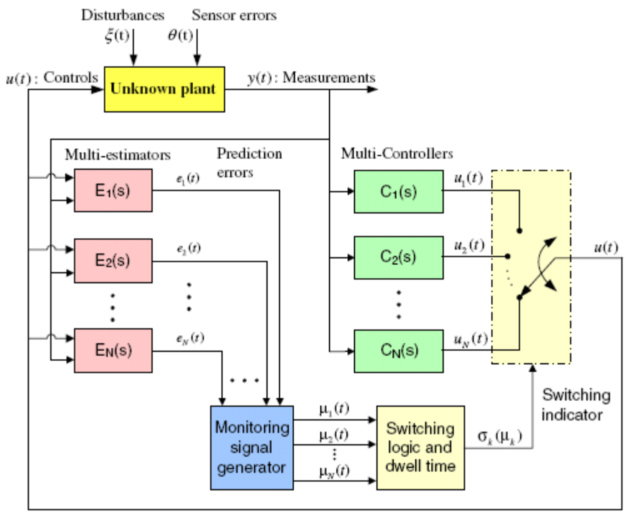

Fig. 1. Switching MMAC[18]

From analysis point of view, we could use a very simple block diagram to represent a switching MMAC system, see Figure 2. The essential characteristic of switching MMAC is that the controller is time-varying. And the details of controller switching is minor for stability analysis.

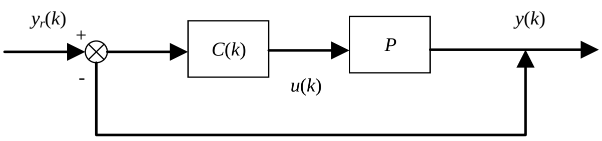

Fig. 2. Simplified Block Diagram of Switching MMAC

In Figure 2, $y_r(k)<\infty$ is the reference input of the closed-loop control system. $C(k)$ denotes the time-varying controller of switching MMAC. P is the plant to be controlled, which takes the form

$$
\begin{aligned}
A(q^{-1})y(k) &= q^{-d}B(q^{-1})u(k) \\
A(q^{-1}) &= 1+a_1q^{-1}+\ \ldots\ +a_nq^{-n} \qquad (1) \\
B(q^{-1}) &= b_0+b_1q^{-1}+\ \ldots\ +b_mq^{-m}
\end{aligned}
$$

3. Virtual equivalent system of switching MMAC

In this section we give three kinds of virtual equivalent systems of switching MMAC according to the situations of parameter estimates.

3.1Parameter estimates converge to its real values

If parameter estimates converge to its real values, i.e. the parameters of P, and the switching mechanism switch to adaptive controller finally, the time-varying controller $C(k)$ in Figure 2 will converge to a certain time-invariant controller $C = f(P)$, if only the mapping is continuous.

$$f:\ M\rightarrow \qquad (2)$$

Then we can construct a virtual equivalent system of switching MMAC in the input-output sense; see Figure 3, where $\Delta u(k)$ is a complementary signal and it will play an very important role in the analysis.

$$\Delta u(k)=u(k)-u_0(k)=\phi_c^T(k)\theta_c(k)\text{-}\phi_c^T\theta_c \qquad (3)$$

$\phi_c^T(k)$is the regression vector

of control signal, generally speaking we have

$$\phi_c^T(k)=[y(k),y(k\text{-}1),\ \ldots,u(k\text{-}1),\ \ldots] \qquad (4)$$

of course , the number of elements of $\phi_c^T(k)$ is limited.

$\theta_c(k)$ and θ_c are the parameter vectors of time-varying controller $C(k)$ and time-invariant controller C respectively.

Then we have

$$\Delta u(k)\ =\ o(||\phi_c(k)||) \qquad (5)$$

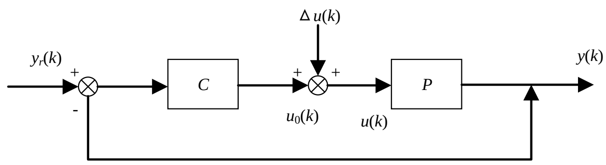

Fig. 3. Equivalent System I

3.2 Parameter estimates converge to non-real values

If parameter estimates converge to non-real values, denoted by P_0 (vector form θ_0), and the switching mechanism switch to adaptive controller finally, the time-varying controller $C(k)$ (vector form $\theta_c(k)$) in Figure 2 will converge to a certain time-invariant controller $C_0=f(P_0)$ (vector form θ_{c0}), if only the mapping $f(.)$ is continuous. Then we can construct a virtual equivalent system of switching MMAC in the input-output sense, see Figure 4, where

$$
\begin{aligned}
e(k)&=y(k)-\phi^T(k-d)\theta_0 \\
&=y(k)-\phi^T(k-d)\hat{\theta}(k)+\phi^T(k-d)\hat{\theta}(k)-\phi^T(k-d)\theta_0
\end{aligned}
\qquad (6)
$$

$$\Delta u(k)=u(k)-u_0(k)=\phi_c^T(k)\theta_c(k)\text{-}\phi_c^T(k)\theta_{c0} \qquad (7)$$

$\phi^T(k-d)$ is the regression vector of parameter estimation, $\hat{\theta}(k)$ is the estimated parameter vector, and the corresponding transfer function of $\hat{\theta}(k)$ is$P_m(k)$.

As parameter estimates converge, i.e. $\hat{\theta}(k)\rightarrow\theta_0$, we have $\theta_c(k)\rightarrow\theta_{c0}$, that means

$$\Delta u(k)=o(\|\phi_c(k)\|) \qquad (8)$$

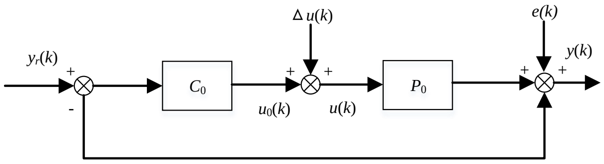

Fig. 4. Equivalent System II

3.3 Parameter estimates converge to non-real values

In this situation, we have to limit the adaptive controller in switching MMAC as designed by one-step-ahead strategy[23]. Otherwise we cannot have the property of $\Delta u(k)$ as in equations (5) and (8), which is critical to virtual equivalent system method. We may still use Figure 3. as the virtual equivalent system of this situation.

4. Main results

With the help of the virtual equivalent systems of switching MMAC, we have the following theorems for different situations of parameter estimation.

4.1 Parameter estimates converge to its real values

Theorem 1

If a switching MMAC system has the following properties:

1) The parameter estimates converge to its real values;

2) After limited number of switches, the switching mechanism switches to adaptive controller finally;

3) The mapping from estimated parameters into controller parameters is continuous;

4) The controller is well defined such that <C,P> constitutes a stable closed-loop system.

Then the switching MMAC system is stable and convergent.

Proof.

Conditions 1), 2), 3) guarantee that the virtual equivalent system exists.

Decompose Figure 1 into two subsystems; see Figure 5 and Figure 6.

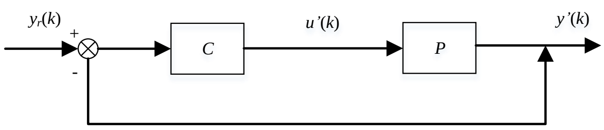

Fig. 5. Subsystem 1

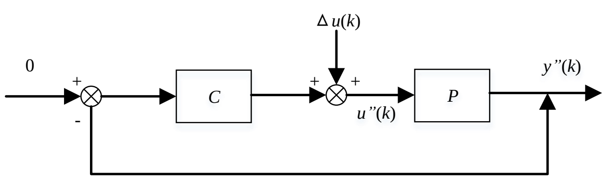

Fig. 6. Subsystem 2

By superposition principle of linear system, we have

$$y(k)=y'(k)+y''(k) \tag{9}$$

$$u(k)=u'(k)+u''(k) \tag{10}$$

By condition 4), subsystem 1 is a stable system. Then we get

$$|y'(k)| < \infty \tag{11}$$

$$|u'(k)| < \infty \tag{12}$$

As for subsystem 2, it is also a stable closed-loop system. And by conditions 1) and 3) we know that equation (5) holds.

Then we have

$$|y''(k)| = o(\|\phi_c(k)\|) \tag{13}$$

$$|u''(k)| = o(\|\phi_c(k)\|) \tag{14}$$

By (9) and (10), it is obvious that

$$y(k)=y'(k)+y''(k)=y'(k)+o(\|\phi_c(k)\|) \tag{15}$$

$$u(k)=u'(k)+u''(k)=u'(k)+o(\|\phi_c(k)\|) \tag{16}$$

From now on, we use reduction to absurdity to get our result.

Suppose the virtual equivalent system is not stable, i.e.$y(k)$and $u(k)$ are both unbounded. Then $\|\phi_c(k)\|$ is also unbounded. There must exist a subsequence $\|\phi_c(l)\|$ that goes to infinity.

For elements of $\|\phi_c(l)\|$, we have

$$\begin{aligned} y(l)&=y'(l)+y''(l)=y'(l)+o(\|\phi_c(l)\|) \\ y(l-1)&=y'(l-1)+y''(l-1)=y'(l-1)+o(\|\phi_c(l-1)\|) \\ y(l-2)&=y'(l-2)+y''(l-2)=y'(l-2)+o(\|\phi_c(l-2)\|) \\ &\vdots \end{aligned} \tag{17}$$

$$\begin{aligned} u(l)&=u'(l)+u''(l)=u'(l)+o(\|\phi_c(l)\|) \\ u(l-1)&=u'(l-1)+u''(l-1)=u'(l-1)+o(\|\phi_c(l-1)\|) \\ u(l-2)&=u'(l-2)+u''(l-2)=u'(l-2)+o(\|\phi_c(l-2)\|) \\ &\vdots \end{aligned} \tag{18}$$

From Lemma 1 (See Appendix), we know that

$$\|\phi_c(l-i)\| = O(\|\phi_c(l)\|+M), i=1,2,\dots,I \qquad 0<M<\infty \tag{19}$$

where I is a limited integer.

Then we obtain the following inequalities

$$|y(l-1)|^2 \le |y'(l-1)|^2 + o(\|\phi_c(l)\|^2) \tag{20}$$

$$\vdots$$

$$|u(l-1)|^2 \le |u'(l-1)|^2 + o(\|\phi_c(l)\|^2) \tag{21}$$

$$\vdots$$

$$|y_r(l)|^2 = |y_r(l)|^2 \qquad (22)$$

$$|y_r(l-1)|^2 = |y_r(l-1)|^2 \qquad (23)$$

$$\vdots$$

Making sums from Equation (20) to Equation (23) and taking Equations (11)-(12) into account, we get

$$\frac{\|\phi_c(l)\|^2}{\|\phi_c(l)\|^2} \le 0 \qquad (24)$$

That is obviously absurd. Then the assumption that the virtual equivalent system is unstable can't hold.

It means the virtual equivalent system is stable, i.e.

$$|y(k)| < \infty \qquad (25)$$

$$|u(k)| < \infty \qquad (26)$$

Further we have

$$\|\phi_c(l)\| \le \infty \qquad (27)$$

Then Equations (15) - (16) yield

$$y(k) \to y'(k) \qquad (28)$$

$$u(k) \to u'(k) \qquad (29)$$

That means the virtual equivalent system is convergent. So, the switching MMAC system is stable and convergent. That completes the proof.

5. Parameter estimates converge to non-true values

Theorem 2

If a switching MMAC system has the following properties:

1) The parameter estimates converge, $P_m(k)$ is uniformly controllable and the estimation error satisfies.

$$y(k)\text{-}\phi^T(k-d)\hat{\theta}(k)\text{=}o(\alpha+\|\phi(k-d)\|) \quad ,\alpha>0 \qquad (30)$$

2) After limited number of switches, the switching mechanism switches to adaptive controllerfinally;

3) The mapping from estimated parameters into controller parameters is continuous;

4) The controller is well defined such that $<C_0,P_0>$ constitutes a stable closed-loop system

Then the switching MMAC system is stable and convergent.

Proof.

Conditions 1), 2), 3) guarantee that the virtual equivalent system exists.

Decompose Figure 4 into three subsystems; see Figure 7, Figure8 and Figure 9.

In Figure 8, condition 1) and condition 3) guarantee

$$\Delta u(k)\text{=}o(\|\phi_c(k)\|) \qquad (31)$$

And from the definition of $\phi_c^T(k)$ in equation (4), we have

$$\|\phi_c(k)\| = O(\|\phi(k-d)\|)\text{+}M, 0<M<\infty \qquad (32)$$

Then Equation (31) and Equation (32) indicate

$$\Delta u(k)\text{=}o(\alpha+\|\phi(k-d)\|) \quad ,\alpha>0 \qquad (33)$$

And we also have (see Lemma 2 in Appendix)

$$\|\phi(k-d-i)\| = O(\|\phi(k-d-i)\|) \quad ,i=1,2,...,I \qquad (34)$$

where $I$is a limited integer.

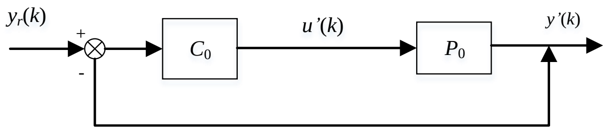

Figure. 7 Subsystem 1

From Equation (30) of condition 1 and Equation (6), we know that in Figure 9.

$$e(k)\text{=}o(\alpha+\|\phi(k-d)\|), \quad \alpha>0 \qquad (35)$$

Then we can develop the result of Theorem 2 following the similar procedures of the proof of Theorem 1. Details omitted.

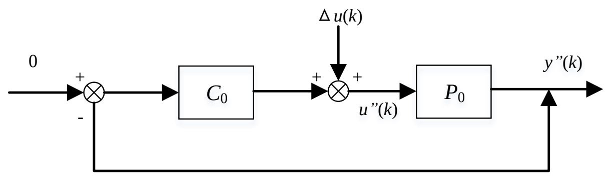

Figure. 8 Subsystem 2

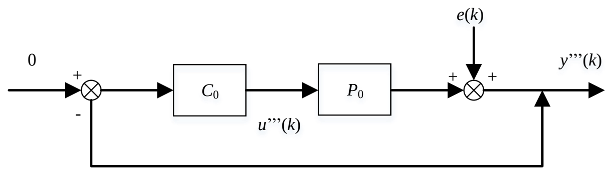

Figure. 9 Subsystem 3

4.3 Parameter estimates may not converge

Theorem 3

If a switching MMAC system with one-step-ahead adaptive controller, has the following property:

The parameter estimation error satisfies

$$y(k)\text{-}\phi^T(k-d)\hat{\theta}(k)\text{=}o(\alpha+\|\phi(k-d)\|) \quad ,\alpha>0 \qquad (36)$$

2) $B(q^{-1})$is Hurwitz stable and$b_0 \neq 0$;

3) The control signal $u(k)$ exists;

4) After limited number of switches, the switching mechanism switches to adaptive controller finally.

Then the switching MMAC system is stable and convergent.

Proof.

The virtual equivalent system and its decomposition subsystems are shown in Figure 3, Figure 5 and Figure 6.

First we introduce one-step-ahead adaptive control strategy.

Rewrite equation (1) in prediction form[23]

$$y(k+d)=\phi^T(k)\hat{\theta}(k) \tag{37}$$

Then one-step-ahead adaptive control signal $u(k)$ is decided by

$$\varphi^T(k)\hat{\theta}(k)=y^*(k+d) \tag{38}$$

$y^*(k+d)$is identical to $y_r(k+d)$. Equation (39) means

$$\begin{aligned}u(k)=&\frac{1}{\hat{\theta}_{n+1}}[-\hat{\theta}_1 y(k)--\hat{\theta}_2 y(k-1)\text{-...-}\hat{\theta}_n y(k-n+1)\text{-}\hat{\theta}_{n+2}u(k-1)\\ &\text{-...-}\hat{\theta}_{n+m+d}u(k-m-d+1)+y^*(k+d)]\end{aligned} \tag{39}$$

Accordingly

$$\begin{aligned}u_0(k)=&\frac{1}{\theta_{n+1}}[-\hat{\theta}_1 y(k)--\hat{\theta}_2 y(k-1)\text{-...-}\hat{\theta}_n y(k-n+1)\text{-}\hat{\theta}_{n+2}u(k-1)\\ &\text{-...-}\hat{\theta}_{n+m+d}u(k-m-d+1)+y^*(k+d)]\end{aligned} \tag{40}$$

From Equations (39) and (40), we have

$$\theta_{n+1}u_0(k)\text{-}\hat{\theta}_{n+1}u(k)=\theta_{n+1}u^{'}(k)-\theta_{n+1}u(k)+(\theta_{n+1}-\hat{\theta}_{n+1})u(k) \tag{41}$$

Further, it is obvious that

$$\begin{aligned}\Delta u(k)&=\frac{1}{\vartheta_{n+1}}[\theta_{n+1}u_0(k)\text{-}\hat{\theta}_{n+1}u(k)+(\hat{\theta}_{n+1}\text{-}\theta_{n+1})u(k)]\\ &=\frac{1}{\vartheta_{n+1}}[\varphi^T(k-d)\theta\text{-}\varphi^T(k-d)\hat{\theta}(k)]\\ &=\frac{1}{\vartheta_{n+1}}[y(k)\text{-}\varphi^T(k-d)\hat{\theta}(k)]\end{aligned} \tag{42}$$

Here, $\theta_{n+1}$is b_0 in Equation (1).

Based on condition (1), we obtain

$$\Delta u(k)=o(\alpha+\|\phi(k-d)\|) \quad ,\alpha>0 \tag{43}$$

The remained procedures are similar to the proof of Theorem 1. Details are omitted.

Note: in reference[23], it givesthe following property under some certain conditions.

$$y(k)\text{-}\phi^T(k-d)\hat{\theta}(k-1)=o(\alpha+\|\phi(k-d)\|) \quad ,\alpha>0 \tag{44}$$

and

$$\lim_{t\to\infty}\left\|\hat{\theta}(t)\text{-}\hat{\theta}(t-k)\right\|\to 0 \tag{45}$$

Then we know that Equation (36) holds.

5. Concluding remarks

Based on virtual equivalent system concept methodology, we developed some general criteria for judging the stability and convergence of switching MMAC systems in which the adaptive control strategy and parameter estimation algorithm are arbitrary to some extent. Thus we argue that virtual equivalent system could provide a unified theoretical framework or a general theory for switching MMAC system. In the future research work, we will focus on extending multiple model adaptive control (combining switching strategy and weighting strategy) to solve the fault-tolerant control of complex control problems, linear time-varying control problem, and nonlinear control problem.

The advantage of switching MMAC is the fastness in response to the system parameter change, but its drawback is the robustness to the disturbances and noises; and the advantage of weighted MMAC is the smooth of adaptive process under noise or disturbance, but its drawback is the slow response to the system parameter change. In the future, we have a plan to make use of advantages of these two schemes. For this goal, an MMAC scheme based on switch/weighting intelligent fusion algorithm will be considered. The corresponding research work are as follows: 1) switching/weighting intelligent fusion algorithm against disturbances and noises of the system; 2) stability and convergence analysis of the corresponding closed-loop control system. To be specific, the switching/weighting intelligent fusion MMAC will adopt weighting algorithm to unify the switching function and weighting function to construct a new type of multiple model adaptive control system.

References

[1] B. Martensson, Adaptive stabilization,Ph.D. dissertation, Lund Instituteof Technology, Lund, Sweden, 1986.

[2] M. Fu and B. R. Barmish, Adaptive stabilization of linear systemsvia switching control, IEEE Trans. Automat. Contr., vol. 31, pp.1097–1103, Dec. 1986.

[3] K. Poolla and S. J. Cusumano, A new approach to adaptive robustcontrol—Parts I and II, Coordinated Science Laboratory, Univ. Illinois,Urbana, Tech. Rep., Aug. 1988.

[4] D. E. Miller and E. J. Davison, An adaptive controller which providesLyapunov stability,IEEE Trans. Automat. Contr., vol. 34, pp. 599–609,June 1989.

[5] D. E. Miller, Adaptive stabilization using a nonlinear time-varyingcontroller, IEEE Trans. Automat. Contr., vol. 39, pp. 1347–1359, July1994.

[6] R. H. Middleton, G. C. Goodwin, D. J. Hill, and D. Q. Mayne, Designissues in adaptive control, IEEE Trans. Automat. Contr., vol. AC-33,pp. 50–58, Jan. 1988.

[7] A. S. Morse, D. Q. Mayne, and G. C. Goodwin, Applications of hysteresisswitching in parameter adaptive control, IEEE Trans. Automat.Contr., vol. 37, pp. 1343–1354, Sept. 1992.

[8] S. R. Weller and G. C. Goodwin, Hysteresis switching adaptive controlof linear multivariable systems, IEEE Trans. Automat. Contr., vol. 39,pp. 1360–1375, July 1994.

[9] A.S. Morse, Supervisory control of families of linear set pointcontrollers, in Proc. IEEE 32nd Conf. Decision Contr., San Antonio,TX, Dec. 1993.

[0] K.S. Narendra and S. Mukhopadhyay, Intelligent control using neuralnetworks, in Intelligent Control, M. M. Gupta and N. K. Sinha, Eds.New York: IEEE, 1994.

[11] K.S. Narendra and J. Balakrishnan, Improving transient response of adaptive control systems using multiple models and switching, IEEE. Trans. Automat.Contr, Vol 40,pp.1861-1866, 1994.

[12] K.S.Narendraand J.Balakrishnan, Adaptive control using multiple models, IEEE.Trans.Automat.Contr, Vol.42, pp.171-187, 1997.

[13] T. Autenrieth and E. Rogers., Performance enhancements for a class of multiple model adaptive control schemes. Int.J.Adapt. and Signal.Process, Vol.13, pp105-127, 1999.

[14] K.S. Narendraand C. Xiang, Adaptive control of discrete-time systems using multiple models, IEEE. Trans. Automat.Contr., Vol.45, No.9,pp1669-1685. 2000.

[15] Xiaoli Li, Wei Wang, Shuning Wang, Multiple model adaptive control for discrete time systems. American Control Conference'2001, Arlington, Virginia, USA , pp4820-4825. 2001.

[16] Lingji Chen, K.S. Narendra, Nonlinear adaptive control using neural networks and multiple models. Automatica, Vol.37, No. 8,pp.12451255. 2001.

[17] K.S. Narendra, and Osvaldo Driollet, Stochastic Adaptive Control Using Multiple Estimation Models. Int. J. Adapt. Control and Signal Process, Vol.15, pp.287-317, 2001.

[18] Sajjad Fekri, Michael Athans and Antonio Pascoal, Issues, progress and new results in robust adaptive control, Int. J. Adapt. Control and Signal Process, Vol.20(10),pp.519-579. 2006

[19]K. J. Astrom, B. Wittenmark, Adaptive Control (2nd edn). Addison-Wesley: MA, U.S.A., 1995.

[20]G. C. Goodwin, Adaptive control: where to now, Adaptive Systems for Signal Processing, Communications, and Control Symposium 2000.

[21]Ioannou P, Sun J., Robust Adaptive Control, Prentice-Hall: NJ, U.S.A., 1996.
[22]Weicun Zhang, Tianguang Chu, Long Wang, A new theoretical framework for self-tuning control, International Journal of Information Technology, Vol.11 (11), pp.123-139, 2005.
[23]G.C. Goodwin, Sin Kwai Sang, Adaptive Filtering, Prediction and Control, Prentice Hall, 1984.

APPENDIX

Lemma 1.

For the regression vector $\phi_c^T(k)$ as defined in Equation (4), we have the following estimation result:

$$\|\phi_c(l-i)\| = O(\|\phi_c(l)\|)+M,0<M<\infty;i=1,2,...,I$$

where I is a limited integer.

Proof:

Suppose

$$\phi_c^T(k)=[y(k),y(k-1),...y(k-s_1),u(k-s_2),y_r(k),...y_r(k-s_3)]$$

Then we have

$$\phi_c^T(l)=[y(l),y(l-1),...y(l-s_1),u(l-1),...u(l-s_2),y_r(l-s_3)]$$

$$\phi_c^T(l-1)=[y(l-1),y(l-2),...y(l-s_1\text{-}1),u(l-2),...$$
$$u(l-s_2\text{-}1),y_r(l-1),...y_r(l-s_3-1)]$$

By components comparison between $\phi_c^T(l)$ and $\phi_c^T(l)$, we obtain

$$\|\phi_c(l-1)\|^2=\|\phi_c(l)\|^2+y^2(l-s_1\text{-}1)\text{-}y^2(l)+$$
$$u^2(l-s_2\text{-}1)\text{-}u^2(l-1)+y_r^2(l-s_3\text{-}1)\text{-}y_r^2(l)$$

Because $|y_r(k)|<M$,$0<M<\infty$, then we get

$$\|\phi_c(l-1)\|^2\le\|\phi_c(l)\|^2+y^2(l-s_1\text{-}1)+u^2(l-s_2\text{-}1)+M$$

Generally speaking, we have

$s_1\le n$, $s_2\le m$

From Equation (1), we know that

$$y^2(l-s_1\text{-}1)+u^2(l-s_2\text{-}1)\le$$
$$\frac{a_{l-s_1-1}^2+b_{l-s_2-1}^2}{a_{l-s_1-1}^2 b_{l-s_2-1}^2}\{a_{l-s_1-1}^2 y^2(l-s_1\text{-}1)+b_{l-s_2-1}^2 u^2(l-s_2\text{-}1)\}=O(\|\phi_c(l)^2\|)$$

Then we have

$$\|\phi_c(l-1)\|=O(\|\phi_c(l)\|)+M$$

And similarly, for limited integer I, we can get

$$\|\phi_c(l-i)\|=O(\|\phi_c(l)\|)+M,i=1,2,...,I$$

That completes the proof of Lemma 1.

Lemma 2.

For the regression vector $\phi^T(k-d)$ as defined in Equation (6), we have the following estimation:

$$\|\phi(k-d-i)\|=O(\|\phi(k-d)\|),i=1,2,...,I$$

where I is a limited integer.

Proof:

The procedures of the proof are similar to that of Lemma 1. Thus details are omitted here to save space.

Weicun Zhang is associate professor at the School of Automation and Electrical Engineering, University of Science and Technology Beijing. He obtained his MSc degree in Automatic Control (1989) from Beijing Institute of Technology and the Ph.D. degree in Control Theory and Applications (1993) from Tsinghua University, P. R. China. From March 1997 to May 1998, he was a visiting research fellow in Industrial and Operations Engineering Department, University of Michigan at Ann Arbor, From September 2006 to August 2007; he was a visiting professor in Department of Electrical and Computer Engineering, Seoul National University, South Korea. His research interests include: self-tuning adaptive control, multiple model adaptive control and estimation for both linear and nonlinear dynamic systems. As representative research work, he established a Virtual Equivalent System (VES) theory for unified analysis (stability, convergence, and robustness) of self-tuning control systems, which is independent of specific control strategy and parameter estimation algorithm. With the help of VES, He proved the stability of weighted multiple model adaptive control system.