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An Event-Triggered H_∞ Filter for Delta Operator T–S Fuzzy Nonlinear Networked Systems with Delays Under High-Frequency Sampling

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This paper designs an event-triggered filter for Delta operator T-S fuzzy nonlinear networked systems with delay under high-frequency sampling. First, Delta operator is used to discretize the T-S fuzzy model to describe the nonlinear system and filter to improve the modeling accuracy. Then, an event-triggered scheme is proposed to reduce the unnecessary waste of the network resources. Second, by constructing a Lyapunov-Krasovskii function with Delta operator, a stability condition with less conservative is obtained. Third, employing the linear matrix inequality, the existence condition of a H_∞ filter is obtained. Finally, a numerical example demonstrates the effectiveness of the proposed method.

1. Introduction

The expansion of the scale of networked systems and the increase in the number of control nodes have led to increasing attention to the handling of high-frequency sampling, network time delay, external interference, and communication congestion [1-3]. Therefore, the research on filtering of networked systems under event-triggered is necessary.

Tackles the event-triggered filtering problem for a class of nonlinear networked control systems subject to the prescribed disturbance attenuation level in [4]. Is devoted to the investigation of reduced-order dissipative filtering for T-S fuzzy Markov jump systems with the event-triggered mechanism in [5]. Focuses on the finite-time H_∞ filtering design problem for a class of switched nonlinear systems via the T-S fuzzy model method. A novel event-triggered scheme is proposed with mixed switching signals. The information of system output and switching signals is available to the filter at each event-triggered sampling instant in [6].

The accuracy of modeling is crucial to the control accuracy of the networked system, and the discrete system is described by Delta operator under high-frequency sampling [7], so that the parameters of the discrete system have smaller errors with the original system parameters. Zhang's team has done a lot of research [8-14]. It is also combined with T-S fuzzy theory to describe the nonlinear system model [15-18]. Therefore combining Delta operator discretization method with T-S fuzzy theory in order to improve the modeling accuracy of nonlinear systems under high frequency sampling is a motivation for this paper.

Periodic transmission of data will transmit the sampled data at each moment to the next link, but usually some data are useless and the network bandwidth is limited, in order to improve the utilization of network resources, the event trigger mechanism is designed to make a judgment on the signal that is about to be transmitted, and if it meets the set threshold, it will be transmitted to the next control node through the network, and if not, it will not be transmitted, which can effectively save network resources [19]. Establishing a reasonable trigger mechanism is another motivation for this paper's research.

In this paper, an event-triggered H_∞ filtering method is proposed for T-S fuzzy network system considering high frequency sampling and time delay. In Sect. 1, the Delta operator discrete networked system is used and a reasonable event-triggered mechanism is designed to obtain a filtered error system with time delay included. By constructing a Delta operator

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Lyapunov-Krasovskii function, stability analysis is derived in Sect. 2. In Sect. 3, the design method of H_∞ filter is given based on linear matrix inequality. In Sect. 4, numerical simulation result is provided to show the effectiveness of the proposed algorithm. Finally, conclusions are given in Sect. 5.

2. System Description

Consider the following nonlinear networked systems described by Delta operator T–S fuzzy rules:

Plant rule i *: IF* $\theta_1(t)$ *is* $\tilde{M}_{i1}$, $\theta_2(x(t))$ *is* $\tilde{M}_{i2}$, ……, *and* $\theta_r(t)$ *is* $\tilde{M}_{i\varphi}$, *THEN*

$$\begin{cases}\delta x(t) = A_i x(t) + B_i \omega(t) \\ y(t) = C_i x(t) + D_i \omega(t) \\ z(t) = L_i x(t)\end{cases} \tag{1}$$

where $x(t) \in R^{n_x}$ is the system state vector; $y(t) \in R^{ny}$ is the measured output; $z(t) \in R^{n_z}$ is the signal to be estimated, and $\omega(t) \in R^{n_\omega}$ is the disturbance input which belongs to $\omega(t) \in L_2[0,\infty)$; A_i, B_i, C_i, D_i, and E_i are system matrices with suitable dimensions, $\tilde{M}_{i\varphi}$ presents the T-S fuzzy set, $\varphi = 1,2,\cdots p$ and $i = 1,2,3\cdots r$ are IF-THEN fuzzy rule numbers, $\theta_1(t), \theta_2(t)....\theta_r(t)$ are premise variable.

Hence, the Delta operator T–S fuzzy nonlinear networked systems are inferred as follows:

$$\begin{cases}\delta x(t) = \sum_{i=1}^{r} \tilde{m}_i(\theta(t))[A_i x(t) + B_i \omega(t)] \\ y(t) = \sum_{i=1}^{r} \tilde{m}_i(\theta(t))[C_i x(t) + D_i \omega(t)] \\ z(t) = \sum_{i=1}^{r} \tilde{m}_i(\theta(t)) L_i x(t)\end{cases} \tag{2}$$

where $\theta = [\theta_1, \cdots, \theta_r]^T$, $\tilde{m}_i(\theta(t)) = \dfrac{m_i(\theta(t))}{\sum_{i}^{r} m_i(\theta(t))}$, $\tilde{m}_i(\theta(t)) = \prod_{\varphi=1}^{p} M_{i\varphi}(\tilde{m}_\varphi(t))$, $M_{i\varphi}(\bullet)$ represents the grade of membership for $\theta_j(t)$ in $M_{i\varphi}$, $0 \le \tilde{m}_i(\theta(t)) \le 1 (i = 1,2,\cdots,r)$, $\sum_{i=1}^{r} \tilde{m}_i(\theta(t)) > 0$.

It can be seen that $\tilde{m}_i(\theta(t)) \ge 0$, $\sum_{i=1}^{r} \tilde{m}_i(\theta(t)) = 1$, $(\mathrm{i} = 1,2,\cdots,r)$.

We introduce the following Delta operator fuzzy filter to estimate the unknown system states in (3):

Plant rule j *:IF* $\theta_1(t)$ *is* $\tilde{M}_{j1}$, $\theta_2(x(t))$ *is* $\tilde{M}_{j2}$, ……, *and* $\theta_r(t)$ *is* $\tilde{M}_{j\varphi}$, *THEN*

$$\begin{cases}\delta x_f(t) = A_{fj} x_f(t) + B_{fj} \hat{y}(t) \\ z_f(t) = C_{fj} x_f(t)\end{cases} \tag{3}$$

where $x_f(t) \in R^{n_x}$ is the filter state vector, the estimated signal vector $z(t)$ is $z_f(t) \in R^{n_z}$, A_{fj}, B_{fj}, C_{fj} are the filter parameters matrices to be determined.

Then, the inferred Delta operator fuzzy filter is defined as follows:

$$\begin{cases}\delta x_f(t) = \sum_{j=1}^{r} \tilde{m}_j(\theta(t))[A_{fj} x_f(t) + B_{fj} \hat{y}(t)] \\ z_f(t) = \sum_{j=1}^{r} \tilde{m}_j(\theta(t)) C_{fj} x_f(t)\end{cases} \tag{4}$$

Event-triggered mechanism:

The event-triggered mechanism is introduced. An event monitoring end is set after the sampler, and the current sampling signal needs to meet the trigger threshold of the event monitoring end before it can be transmitted to the next node, and the trigger conditions are set as follows.

$$e_k^T(t)\Lambda_1 e_k(t) \ge \varepsilon y^T(i_k h)\Lambda_2 y(i_k h) \tag{5}$$

where Λ_i $(i = 1,2)$ are positive triggering parameters to be designed, ε is a given constant which belongs to $[0,1)$, $y(t_k h)$ represents the latest transmitted data, $y(i_k h)$ represents the current sampling data, and the threshold error $e_k(t) = y(i_k h) - y_e(t_k h)$.

Then, $\hat{y}(t)$ can be represented as

$$\hat{y}(t) = y(t - \tau(t)) - e_k(t) \tag{6}$$

By combining (1) with (6), the filtering error system can be represented by:

$$\begin{cases}\delta\xi(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \tilde{m}_i \tilde{m}_j [A_{ij}\xi(t) + B_{ij}\xi(t - \tau(t)) \\ \qquad + B_{ij\omega}\bar{\omega}(t) - B_{ije} e_k(t)] \\ e(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \tilde{m}_i \tilde{m}_j C_{ij}\xi(t)\end{cases} \tag{7}$$

where $\xi(t) = \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}$, $\bar{\omega}(t) = \begin{bmatrix} \omega(t) \\ \omega(t - \tau(t)) \end{bmatrix}$, $A_{ij} = \begin{bmatrix} A_i & 0 \\ 0 & A_{fj} \end{bmatrix}$, $B_{ij} = \begin{bmatrix} 0 \\ B_{fj} C_i \end{bmatrix}$, $B_{ij\omega} = \begin{bmatrix} B_i & 0 \\ 0 & B_{fj} D_i \end{bmatrix}$, $B_{ije} = \begin{bmatrix} 0 \\ B_{fj} \end{bmatrix}$, $C_{ij} = \begin{bmatrix} L_i & -C_{fj} \end{bmatrix}$.

Lemma 1[20]: Consider the interconnected feedback system with two subsystems S_1 and S_2:

$$\begin{cases} S_1 : y_\delta(t) = G\delta(t) \\ S_2 : \delta(t) = \Delta y_\delta(t) \end{cases} \tag{8}$$

The interconnected system formed by S_1 and S_2 is robustly asymptotically stable if there exists a matrix $X \in \chi$ with $\chi = \{X : X \text{ is nonsingular and } \|X \circ \Delta \circ X^{-1}\| \le \beta_1\}$ such that, for S_1, the condition holds $\|X \circ G \circ X^{-1}\|_\infty < \beta_2$, $\beta_1\beta_2 < 1$.

Lemma 2[21] (Schur complement): For a given symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} < 0$, where $S \in R^{r*r}$, $S_{21} = S_{12}^T$, the following conditions are equivalent:

(1) $S < 0$;

(2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0$, $S_{11} - S_{12}^T S_{22}^{-1} S_{12} < 0$。

Lemma 3[22] : The property of Delta operator: for any time function $x(t)$ and $y(t)$, where T is a sampling period:

$$\delta\left(x(t)y(t)\right)=\delta\left(x(t)\right)y(t)+x(t)\delta\left(y(t)\right)+T\delta\left(x(t)\right)\delta\left(y(t)\right)$$

Lemma 4[23]: For any positive definite matrix $W\in R^{n*n}$ and $W^T=W>0$ with appropriate dimensions, two positive integers r and r_0 satisfying, the following inequality holds:

$$\left(\sum_{i=r_0}^{r}x(i)\right)^T W\left(\sum_{i=r_0}^{r}x(i)\right)\le(r-r_0+1)\sum_{i=r_0}^{r}x^T(i)Wx(i)$$

According to Lemma 1, defining $\tau_{12}=\tau_2-\tau_1$ and $y_\delta(\mathrm{t})=\dot{\xi}(\mathrm{t})$, we have

$$\begin{aligned}\delta(\mathrm{t})&=\frac{2}{\tau_{12}}\xi(\mathrm{t}-\tau(\mathrm{t}))-\frac{1}{\tau_{12}}\left[\xi(\mathrm{t}-\tau_1)+\xi(\mathrm{t}-\tau_2)\right]\\&=\frac{1}{\tau_{12}}\int_{-\tau_2}^{-\tau(\mathrm{t})}\dot{\xi}(\mathrm{t}+\theta)d\theta-\frac{1}{\tau_{12}}\int_{-\tau(\mathrm{t})}^{-\tau_1}\dot{\xi}(\mathrm{t}+\theta)d\theta\\&=\frac{1}{\tau_{12}}\int_{-\tau_2}^{-\tau_1}k(\theta)\dot{\xi}(\mathrm{t}+\theta)d\theta\end{aligned}$$

$$\delta(\mathrm{t}):=\Delta(\mathrm{t})y_\delta(\mathrm{t})$$

$$\xi(\mathrm{t}-\tau(\mathrm{t}))=\frac{\tau_{12}}{2}\delta(\mathrm{t})+\frac{1}{2}\left[\xi(\mathrm{t}-\tau_1)+\xi(\mathrm{t}-\tau_2)\right]$$

Then the filtering error system can be transformed into the following equivalent system:

$$\begin{cases}\delta\xi(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{m}_i\tilde{m}_j[A_{ij}\xi(t)+\frac{1}{2}B_{ij}\xi(t-\tau_m)\\\qquad+\frac{1}{2}B_{ij}\xi(t-\tau_M)+\frac{1}{2}(\tau_M-\tau_m+T)B_{ij}\sigma(t)\\\qquad+B_{ij\omega}\bar{\omega}(t)+B_{ije}e_k(t)]\\e(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{m}_i\tilde{m}_jC_{ij}\xi(t)\end{cases}\tag{9}$$

Our attention is focused on designing the filter of the Delta operator T-S fuzzy by dynamic event-triggered. When the filtering error system (9) satisfies conditions (1) and (2), the system has H_∞ perturbation performance index γ.

(1) For the case $\bar{\omega}(t)=0$, the filtering error system (9) is said to be asymptotically stable;

(2) Under zero initial condition, $e(t)$ satisfies $\|e(t)\|_2<\gamma\|\bar{\omega}(t)\|_2$,

where $\gamma>0$ is H_∞ performance level.

3. Stability Analysis

Theorem 1 For given scalars $0<\varepsilon<1$, $0<\tau_m\le\tau_M$, $\gamma>0$, the Delta operator filtering error system (9) is asymptotically stable with the prescribed H_∞ performance index γ, if there exist positive matrices

$P>0$, $Q>0$, $S_i>0\ (i=1,2)$, $U_i>0\ (i=1,2)$, $\Lambda_i>0\ (i=1,2)$, A_{fj}, B_{fj}, C_{fj},

such that:

$$\Xi_{ii}<0\tag{10}$$

$$\Xi_{ii}+(\Xi_{ij}+\Xi_{ji})<0,\ j\ne i\tag{11}$$

for

$$\Xi_{ij}=\begin{bmatrix}\varphi_{11}&PA_{ij}&\frac{1}{2}PB_{ij}&\frac{1}{2}PB_{ij}&\frac{1}{2}\bar{\tau}_MPB_{ij}&PB_{ije}&PB_{ij\omega}\\ *&\varphi_{22}&\varphi_{23}&\varphi_{24}&\frac{1}{2}\bar{\tau}_MPB_{ij}&PB_{ije}&PB_{ij\omega}\\ *&*&\varphi_{33}&\varphi_{34}&\varphi_{35}&0&\varphi_{37}\\ *&*&*&\varphi_{44}&\varphi_{45}&0&\varphi_{47}\\ *&*&*&*&\varphi_{55}&0&\varphi_{57}\\ *&*&*&*&*&-\Lambda&0\\ *&*&*&*&*&*&\varphi_{77}\end{bmatrix}$$

in which

$\varphi_{11}=(T-2)P+\tau_mS_1+(\tau_M+T)S_2$,

$\varphi_{22}=PA+A^TP-\frac{1}{\tau_m}S_1-\frac{1}{\tau_M+T}S_2+(\frac{\tau_M-\tau_m}{T}+2)Q+R_1+R_2+C_{ij}^TC_{ij}$,

$\varphi_{23}=\frac{1}{2}PB_{ij}+\frac{1}{\tau_m}S_1$, $\varphi_{24}=\frac{1}{2}PB_{ij}+\frac{1}{\tau_M+T}S_2$

$\varphi_{33}=-\frac{1}{4}Q-R_1+\frac{1}{\tau_m}TS_1+\frac{1}{4}\varepsilon C_i^T\Lambda C_i$,

$\varphi_{34}=-\frac{1}{4}Q+\frac{1}{4}\varepsilon C_i^T\Lambda C_i$,

$\varphi_{35}=\varphi_{45}=-\frac{1}{4}\bar{\tau}_MQ+\frac{1}{4}\bar{\tau}_M\varepsilon C_i^T\Lambda C_i$,

$\varphi_{37}=\varphi_{47}=\frac{1}{2}\varepsilon C_i^T\Lambda\left[0\quad D_i\right]$,

$\varphi_{57}=\frac{1}{2}\bar{\tau}_M\varepsilon C_i^T\Lambda\left[0\quad D_i\right]$,

$\varphi_{44}=-\frac{1}{4}Q-R_2+\frac{1}{\tau_M}S_2+\frac{1}{4}\varepsilon C_i^T\Lambda C_i$,

$\varphi_{55}=-\frac{1}{4}\bar{\tau}_M{}^2Q+\frac{1}{4}\bar{\tau}_M{}^2\varepsilon C_i^T\Lambda C_i$,

$\varphi_{77}=-\gamma^2I+\varepsilon\left[0\quad D_i\right]^T\Lambda\left[0\quad D_i\right]$,

$\bar{\tau}_M=\tau_M-\tau_m+T$.

Proof We construct a Lyapunov-Krasovskii functional candidate as

$$V=V_1+V_2+V_3+V_4$$

$$V_1=\xi^T(t)P\xi(t)$$

$$V_2=T\sum_{i=\tau_m/T}^{\tau_M/T+1}\sum_{j=1}^{i}\xi^T(t-jT)Q\xi(t-jT)$$

$$V_3=T\sum_{i=1}^{\tau_m/T}\xi^T(t-iT)R_1\xi(t-iT)+T\sum_{i=1}^{\tau_M/T+1}\xi^T(t-iT)R_2\xi(t-iT)$$

$$V_4=T^2\sum_{i=1}^{\tau_m/T}\sum_{j=1}^{i}\delta\xi^T(t-jT)S_1\delta\xi(t-jT)+T^2\sum_{i=1}^{\tau_M/T+1}\sum_{j=1}^{i}\delta\xi^T(t-jT)S_2\delta\xi(t-jT)$$

Taking the delta operator manipulations of $V(t)$, we can obtain:

$$\delta V_1 = \xi^T(t)P\delta\xi(t) + \delta\xi^T(t)P\xi(t) + T\delta\xi^T(t)P\delta\xi(t)$$

$$\begin{aligned}\delta V_2 &= \sum_{i=\tau_m/T}^{\tau_M/T+1}\sum_{j=1}^{i}\xi^T(t-jT+T)Q\xi(t-jT+T) \\ &-\sum_{i=\tau_m/T}^{\tau_M/T+1}\sum_{j=1}^{i}\xi^T(t-jT)Q\xi(t-jT) \\ &= \sum_{i=\tau_m/T}^{\tau_M/T+1}\xi^T(t)Q\xi(t) - \sum_{i=\tau_m/T}^{\tau_M/T+1}\xi^T(t-iT)Q\xi(t-iT) \\ &\le (\frac{\tau_M-\tau_m}{T}+2)\xi^T(t)Q\xi(t) \\ &-[\frac{1}{2}\xi(t-\tau_m)+\frac{1}{2}\xi(t-\tau_M-T) \\ &+\frac{1}{2}(\tau_M-\tau_m+T)\sigma(t)]^T \\ &Q[\frac{1}{2}\xi(t-\tau_m)+\frac{1}{2}\xi(t-\tau_M-T) \\ &+\frac{1}{2}(\tau_M-\tau_m+T)\sigma(t)]\end{aligned}$$

$$\begin{aligned}\delta V_3 &= \sum_{i=1}^{\tau_m/T}\xi^T(t-iT+T)R_1\xi(t-iT+T) \\ &+\sum_{i=1}^{\tau_M/T+1}\xi^T(t-iT+T)R_2\xi(t-iT+T) \\ &-\sum_{i=1}^{\tau_m/T}\xi^T(t-iT)R_1\xi(t-iT) \\ &+\sum_{i=1}^{\tau_M/T+1}\xi^T(t-iT)R_2\xi(t-iT) \\ &= \xi^T(t)(R_1+R_2)\xi(t) - \xi^T(t-\tau_m)R_1\xi(t-\tau_m) \\ &-\xi^T(t-\tau_M-T)R_2\xi(t-\tau_M-T)\end{aligned}$$

$$\begin{aligned}\delta V_4 &= T[\sum_{i=1}^{\tau_m/T}\sum_{j=1}^{i}\delta\xi^T(t-jT+T)S_1\delta\xi(t-jT+T) \\ &+\sum_{i=1}^{\tau_M/T+1}\sum_{j=1}^{i}\delta\xi^T(t-jT+T)S_2\delta\xi(t-jT+T) \\ &-\sum_{i=1}^{\tau_m/T}\sum_{j=1}^{i}\delta\xi^T(t-jT)S_1\delta\xi(t-jT) \\ &-\sum_{i=1}^{\tau_M/T+1}\sum_{j=1}^{i}\delta\xi^T(t-jT)S_2\delta\xi(t-jT)] \\ &\le \delta\xi^T(t)(\tau_m S_1+\tau_M S_2)\delta\xi(t) \\ &-\frac{1}{\tau_m}[\xi(t)-\xi(t-\tau_m)]^T S_1[\xi(t)-\xi(t-\tau_m)] \\ &-\frac{1}{\tau_M+T}[\xi(t)-\xi(t-\tau_M-T)]^T \\ &S_2[\xi(t)-\xi(t-\tau_M-T)]\end{aligned}$$

$$\begin{aligned}0 &= -\delta\xi^T(t)P[\delta\xi(t)-A_{ij}\xi(t)-\frac{1}{2}B_{ij}\xi(t-\tau_m) \\ &-\frac{1}{2}B_{ij}\xi(t-\tau_M)-\frac{1}{2}\bar{\tau}_M B_{ij}\sigma(t)-B_{ij\omega}\bar{\omega}(t) \\ &-B_{ije}e_k(t)]-[\delta\xi(t)-A_{ij}\xi(t)-\frac{1}{2}B_{ij}\xi(t-\tau_m) \\ &-\frac{1}{2}B_{ij}\xi(t-\tau_M)-\frac{1}{2}\bar{\tau}_M B_{ij}\sigma(t) \\ &-B_{ij\omega}\bar{\omega}(t)-B_{ije}e_k(t)]^T P\delta\xi(t)\end{aligned}$$

From Theorem 1, we know that:

$$\begin{aligned}&\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{m}_i\tilde{m}_j(x(t))\Xi_{ij} \\ &=\sum_{i=1}^{r}\sum_{i=1}^{r}\tilde{m}_i^2(x(t))(\Xi_{ii}+(\Xi_{ij}+\Xi_{ji}))\end{aligned}$$

Defined variable $\eta(t)$:

$$\eta(t)=\begin{bmatrix}\eta_1(t) & \eta_2(t)\end{bmatrix}$$

$$\eta_1(t)=\begin{bmatrix}\delta\xi(t) & \xi(t) & \xi(t-\tau_m) & \xi(t-\tau_M-T)\end{bmatrix}$$

$$\eta_2(t)=\begin{bmatrix}\sigma(t) & e_k(t) & \bar{w}(t)\end{bmatrix}$$

According to the Lemma 2 complementary:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{m}_i\tilde{m}_j\eta^T(t)\Xi_{ij}\eta(t)\le 0$$

$$\Rightarrow \delta V(t)+e^T(t)e(t)-\gamma^2\tilde{\omega}^T(t)\tilde{\omega}(t)\le 0$$

(1) For the case $\bar{\omega}(t)=0$, $\delta V(t)+e^T(t)e(t)\le 0 \Rightarrow \delta V(t)\le 0$, the filtering error system (9) is asymptotically stable;

(2) For the case $\bar{\omega}(t)\ne 0$,

$\delta V(t)+e^T(t)e(t)-\gamma^2\bar{\omega}^T(t)\bar{\omega}(t)\le 0$,

$\Rightarrow e^T(t)e(t)-\gamma^2\bar{\omega}^T(t)\bar{\omega}(t)\le 0$, satisfying that for any non-zero $\bar{\omega}(t)\in L_2[0,\infty)$ there is $\|e(t)\|_2<\gamma\|\bar{\omega}(t)\|_2$.

In summary, the filtering error system (9) satisfies the H_∞ performance index γ.

The proof is completed.

4. Filter Design

Theorem 2 For given scalars $0<\varepsilon<1$, $0<\tau_m\le\tau_M$, $\gamma>0$, the Delta operator filtering error system (9) is asymptotically stable with the prescribed H_∞ performance index γ, if there exist positive matrices $\bar{P}>0$, $\bar{Q}>0$, $\bar{S}_i>0$ $(i=1,2)$, $\bar{U}_i>0$ $(i=1,2)$, $\Lambda_i>0$ $(i=1,2)$, A_{fi}, B_{fi}, C_{fi}, such that:

$$\bar{\Xi}_{ii}<0 \tag{12}$$

$$\bar{\Xi}_{ii}+(\bar{\Xi}_{ij}+\bar{\Xi}_{ji})<0,\ j\ne i \tag{13}$$

for

$$\bar{\Xi}_{ij}=\begin{bmatrix}\phi_{11} & \phi_{12} & \frac{1}{2}\phi_{13} & \frac{1}{2}\phi_{14} & \frac{1}{2}\bar{\tau}_M\phi_{15} & \phi_{16} & \phi_{17} & 0 \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \frac{1}{2}\bar{\tau}_M\phi_{25} & \phi_{26} & \phi_{27} & \phi_{28} \\ * & * & \phi_{33} & \phi_{34} & \phi_{35} & 0 & \phi_{37} & 0 \\ * & * & * & \phi_{44} & \phi_{45} & 0 & \phi_{47} & 0 \\ * & * & * & * & \phi_{55} & 0 & \phi_{57} & 0 \\ * & * & * & * & * & -\Lambda & 0 & 0 \\ * & * & * & * & * & * & \phi_{77} & 0 \\ * & * & * & * & * & * & * & -I\end{bmatrix}$$

in which

$\phi_{11}=(T-2)P+\tau_m\bar{S}_1+(\tau_M+T)\bar{S}_2$,

$$\phi_{22}=\hat{\phi}_{22}+\hat{\phi}_{22}^T-\frac{1}{\tau_m}\bar{S}_1-\frac{1}{\tau_M+T}\bar{S}_2 +(\frac{\tau_M-\tau_m}{T}+2)\bar{Q}+\bar{R}_1+\bar{R}_2,$$

$$\phi_{12}=\hat{\phi}_{22}=\begin{bmatrix}\bar{P}A_i & \bar{A}_{fj}\\ FA_i & \bar{A}_{fj}\end{bmatrix},\phi_{16}=\phi_{26}=\begin{bmatrix}\bar{B}_{fj}\\ \bar{B}_{fj}\end{bmatrix},$$

$$\phi_{13}=\phi_{14}=\phi_{15}=\phi_{25}=\begin{bmatrix}\bar{B}_{fj}C_i\\ \bar{B}_{fj}C_i\end{bmatrix},\phi_{28}=\begin{bmatrix}L_i^T\\ -C_{fj}^T\end{bmatrix},$$

$$\phi_{17}=\phi_{27}=\begin{bmatrix}\bar{P}B_i & \bar{B}_{fj}D_i\\ FB_i & \bar{B}_{fj}D_i\end{bmatrix},$$

$$\phi_{33}=\frac{1}{4}\varepsilon C_i^T\Lambda C_i+\frac{1}{\tau_m}\bar{S}_1-\frac{1}{4}\bar{Q}-\bar{R}_1,$$

$$\phi_{34}=\frac{1}{4}\varepsilon C_i^T\Lambda C_i-\frac{1}{4}\bar{Q},$$

$$\phi_{35}=\phi_{45}=\frac{1}{4}\tau_M\varepsilon C_i^T\Lambda C_i-\frac{1}{4}\tau_M\bar{Q},$$

$$\phi_{37}=\phi_{47}=\frac{1}{2}\varepsilon C_i^T\Lambda\begin{bmatrix}0 & D_i\end{bmatrix},$$

$$\phi_{44}=\frac{1}{4}\varepsilon C_i^T\Lambda C_i+\frac{1}{\tau_M+T}\bar{S}_2-\frac{1}{4}\bar{Q}-\bar{R}_2,$$

$$\phi_{55}=\frac{1}{4}\tau_M{}^2\varepsilon C_i^T\Lambda C_i-\frac{1}{4}\tau_M{}^2\bar{Q},$$

$$\phi_{57}=\frac{1}{2}\tau_M\varepsilon C_i^T\Lambda\begin{bmatrix}0 & D_i\end{bmatrix},$$

$$\phi_{77}=-\gamma^2I+\varepsilon\begin{bmatrix}0 & D_i\end{bmatrix}^T\Lambda\begin{bmatrix}0 & D_i\end{bmatrix},$$

In addition, if the above conditions are feasible, the parameter matrices of the fuzzy filter are given by

$$A_{fj}=F^{-1}\bar{A}_{fj},B_{fj}=F^{-1}\bar{B}_{fj},C_{fj}=\bar{C}_{fj}.$$

Proof Define:

$$P=\begin{bmatrix}\bar{P} & S\\ S^T & W\end{bmatrix},\quad H=\begin{bmatrix}I & 0\\ 0 & SW^{-1}\end{bmatrix},$$

$$J=diag\{H,H,H,H,H,H,I,I\},$$

where $\bar{P}>0$, $W>0$ and S is invertible.

Then pre-multiply and post-multiply (10) and (11) with J and its transpose J^T. For the sake of convenience, we define some new variables:

$$F=SW^{-1}S^T,\bar{S}_1=HS_1H^T,\bar{S}_2=HS_2H^T,$$
$$\bar{R}_1=HR_1H^T,\bar{R}_2=HR_2H^T,\bar{Q}=HQH^T$$
$$\bar{A}_{fj}=SA_{fj}W^{-1}S^T,\bar{B}_{fj}=SB_{fj},\bar{C}_{fj}=C_{fj}.$$

Furthermore, equivalently under transformation $S^{-T}Wx_f(\mathrm{t})$, the parameters of filter can be yielded as follows：

$$A_{fj}(\mathrm{t})=S^{-T}W(S^{-1}\bar{A}_{fj}(\mathrm{t})S^{-T}W)W^{-1}S^T=F^{-1}\bar{A}_{fj}(\mathrm{t})$$
$$B_{fj}(\mathrm{t})=S^{-T}W(S^{-1}\bar{B}_{fj}(\mathrm{t}))=F^{-1}\bar{B}_{fj}(\mathrm{t})$$
$$C_{fj}(\mathrm{t})=(\bar{C}_{fj}(\mathrm{t})S^{-T}W)W^{-1}S^T=\bar{C}_{fj}(\mathrm{t})$$

Thus, (12) and (13) can be easily obtained from (10) and (11) respectively.

The proof of Theorem 2 is completed.

5. Numerical Simulations

A numerical example is given to illustrate effectiveness of the proposed method.

Example Considering the nonlinear networked system with T-S fuzzy model (1), the system parameters are shown below:

$A_1=\begin{bmatrix}-0.3 & -0.5\\ 0.81 & -0.9\end{bmatrix}$, $B_1=\begin{bmatrix}-1.5\\ 0.1\end{bmatrix}$, $C_1=\begin{bmatrix}-0.5 & 1\end{bmatrix}$, $D_1=0.7$, $L_1=\begin{bmatrix}-0.5 & 0.6\end{bmatrix}$, $A_2=\begin{bmatrix}-0.3 & -0.3\\ -0.81 & -1.08\end{bmatrix}$, $B_2=\begin{bmatrix}-1.5\\ 0.1\end{bmatrix}$, $C_2=\begin{bmatrix}-0.5 & 1\end{bmatrix}$, $D_2=0.2$, $L_2=\begin{bmatrix}-0.3 & 0.6\end{bmatrix}$.

The membership function for the system and the filter are shown as follows:

$$\tilde{m}_1(x)=1-\frac{1}{1+e^{x_0}},\tilde{m}_2(x_1)=1-\tilde{m}_1(x),$$

The sampling period is chosen as $T=0.01$. In addition, let $\tau_m=0.05$, $\tau_M=0.1$, and $\varepsilon=0.01$.

By applying Theorem 2, the filter parameters and event-triggered parameters are obtained as follows:

$A_{f1}=\begin{bmatrix}-1.1480 & -0.0425\\ -0.0122 & -1.2879\end{bmatrix}$, $B_{f1}=\begin{bmatrix}0.0119\\ -0.0119\end{bmatrix}$, $C_{f1}=\begin{bmatrix}-0.2504 & -0.1803\end{bmatrix}$, $A_{f2}=\begin{bmatrix}-1.0225 & -0.0454\\ 0.0399 & -1.3657\end{bmatrix}$, $B_{f2}=\begin{bmatrix}0.0015\\ -0.0158\end{bmatrix}$, $C_{f2}=\begin{bmatrix}-0.3371 & -0.0848\end{bmatrix}$.

$\Lambda=99.4236$, triggered 82 times.

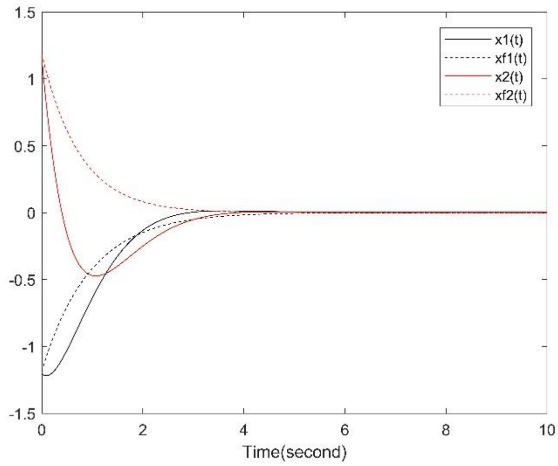

Fig. 1 The response curve of x_1, x_2, x_{f1} and x_{f2}

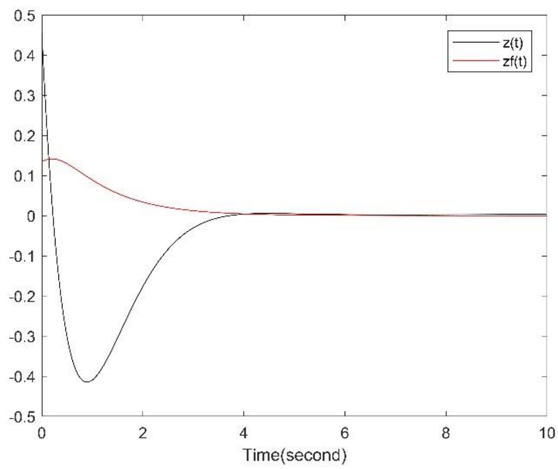

Fig. 2 The response curve of $z(t)$ and $z_f(t)$

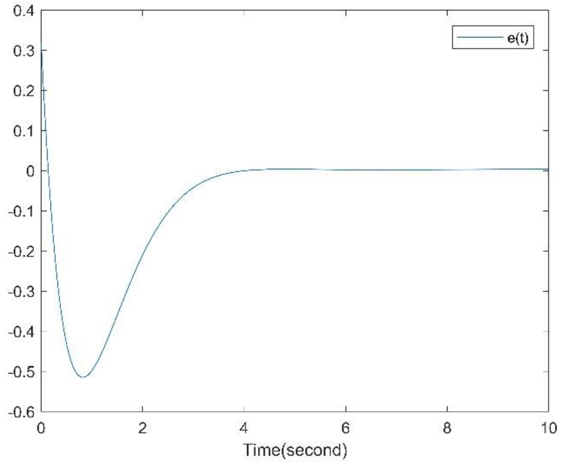

Fig. 3 The response curve of $e(t)$

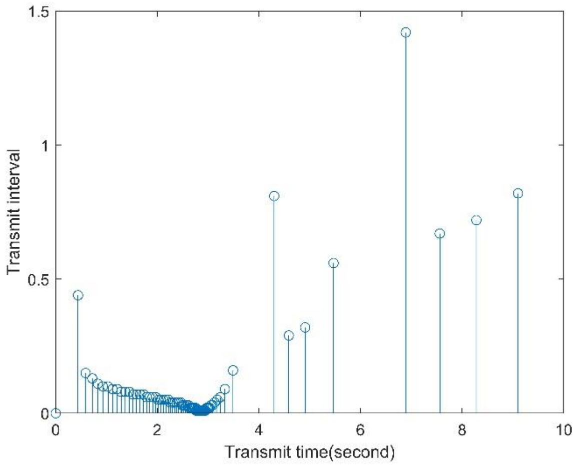

Fig. 4 Triggering instants and intervals

Meanwhile, the minimum performance index of H_∞ obtained is γ=0.6313, better than γ=1.756 in [24].

The external disturbance functions are chosen as $\omega(t)=\begin{bmatrix}0.1\sin(0.001\pi t)\\0.05\sin(0.01\pi t)\end{bmatrix}$, the initial state of the system and filter are $x_0=\begin{bmatrix}-1.2 & 1.2\end{bmatrix}^T$, $x_{f0}=\begin{bmatrix}-1.2 & 1.2\end{bmatrix}^T$.

According to the above parameters, the simulation results shown in Fig. 1-4 can be obtained. According to Fig. 1, the state response curve of the filter is zero after 5s, indicating that the designed filter is asymptotically stable; according to Fig. 2, it can be seen that $z_f(t)$ can track well to describe $z(t)$; Fig. 3 shows that the error gradually decreases with the increase of sampling time, and the response curve of the filtering error roughly converges to zero at 3s, indicating that the designed H_∞ filter is effective; Fig. 4 depicts the trigger time and time interval of the event-triggered scheme, and it can be seen that the designed event-triggered mechanism reduces the amount of signal transmission.

6. Conclusion

In this paper, the problem of event-triggered H_∞ filtering for Delta operator T-S fuzzy nonlinear networked systems with high-frequency sampling has been investigated. The modeling accuracy have been handled effectively by employing a Delta operator T-S fuzzy model. A H_∞ filter has been designed on event-triggered mechanism, we constructed a new filtering error system with delays. A novel sufficient condition has been derived to guarantee the filtering error system to be asymptotically stable. Moreover, the existence conditions of filter have been obtained. Finally, an example has been given to show the effectiveness and advantages of the proposed method in this paper.

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