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Finite-Time Synchronization of an Uncertain Complex Dynamical Network Based on Sliding Mode Control Technology

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ABSTRACT

This paper investigates the problem of finite-time synchronization for the uncertain complex system. For generic networks with unknown dynamics of node and unknown coupling functions, a new controller is designed. Then the Lyapunov stability theory is used to prove that the designed controller can ensure that the synchronization state is asymptotically stable. The main characteristic of this designed controller is rather simple in form and the topological structure of the network can be selected as needed. Finally, Lorenz system is taken as an example for numerical simulation demonstration to verify the effectiveness and feasibility of the proposed approach.

1. Introduction

In recent years, complex dynamic networks have attracted the attention of many people in related fields. There are various complex networks in daily life, and their analysis and control is a hot issue in recent years. Complex network is a system structure composed of a large number of interrelated dynamic nodes. Different nodes represent different individuals in different environments. For example, the nervous system can be regarded as a large number of nerve cells connected with each other through nerve fibers[1][2]. Examples of complex networks include the Internet, the World Wide Web, food webs, electric power grids, cellular and metabolic networks, etc. There are a large number of complex networks in our life. It is of great practical significance to study the operation mechanism, dynamic behavior, synchronization ability and anti-interference ability of these complex networks in order to better manage and make use of these real complex networks.

Among many dynamical behaviors of complex networks, synchronization is one of the most valuable research topic. In fact, synchronization is a typical collective behavior and basic movement in nature. For example, the synchronization of coupled oscillators can explain many natural phenomena well. In addition, some synchronization phenomena are very useful in our daily lives, such as the synchronous transmission of digital or analog signals in communication networks[3]-[9]. However, the current research on synchronization of complex networks mostly assumes that the internal coupling of the complex network is known, that is, it is assumed that the internal coupling function of any two nodes in the complex network is known, and the structure of the complex network is also known. However, in reality, it is difficult to determine the coupling relationship and connection structure between nodes in a complex network. Therefore, it is more meaningful to study the coupling relationship between nodes and the unknown connection structure in complex networks.

Although there have been studies on complex networks with uncertain coupling functions, the designed controller is very complex. In addition, it should be pointed out that although many network synchronization strategies have been proposed, the synchronization time cannot be guaranteed.[3] In some cases, it may take a very long time to achieve synchronization. From the perspective of practical applications, it is more reasonable to achieve synchronization in a limited time. Moreover, the limited time synchronization of the complex network means faster synchronization speed and lower synchronization cost[10]. Meanwhile, the finite-time control techniques have demonstrated better robustness and disturbance rejection properties[22]. Finite-time stability is of finite-time control. This means the optimality in settling time[23]. Therefore, the research on the limited time synchronization of complex network is very valuable and meaningful.

This paper is organized as follows. In Section 2, the uncertain complex dynamic network model and several related lemmas are given. In Section 3, the finite time synchronization criterion of network is discussed. In Section 4, a specific example is given to

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prove the effectiveness of the main results obtained in Section 3. Finally, a conclusion is drawn in Section 5.

2. The problem statement and preliminaries

In this section, an uncertain complex dynamic network model is introduced, and some preliminary definitions and lemmas are given.

2.1 An Uncertain Complex Dynamical Network Model

Consider a continuous time uncertain complex dynamic network consisting of N identical nodes. The system is described as follows:

$$\dot{y}_i = f(y_i,t) + h_i(y_1, y_2, ..., y_n) + u_i \tag{1}$$

where $i = 1,2,...,n$; $y_i = (y_{i1,}y_{i2,...,}y_{in})^T \in R^n$ is the state vector of the i th node; $f : \Omega \times R^+ \to R^n$ is a smooth nonlinear vector field; $h_i : \Omega \times \Omega \times ... \times \Omega \times R^+ \to R^n$ are nonlinear smooth diffusive coupling functions; $u_i \in R^n$ are the control inputs.

In addition, $h_i(y_1, y_2, ..., y_n)$ is a general function, which can represent not only the linear connection between node states, but also the nonlinear connection between node states. For example

When

$$h_i = c\sum_{j=1}^{n} a_{ij}\Gamma x_j \tag{2}$$

Where c is the coupling strength between the network nodes, Γ is the internal coupling matrix, a_{ij} is matrix element of the coupling matrix representing the topological, and $\sum_{j=1}^{n} a_{ij} = 0$ Here $h_i(y_1, y_2, ..., y_n)$ can indicate that the connection between nodes is linear.

When

$$h_i = c\sum_{j=1}^{n} a_{ij} H(x_j) \tag{3}$$

Where $H(\bullet)$ is the internal coupling function between two adjacent nodes. Here $h_i(y_1, y_2, ..., y_n)$ can indicate that the connection between nodes is nonlinear.

Whether it is linear or nonlinear connection, the internal coupling function in the network model is always the same. However, in fact, h can represent not only the same internal coupling function, but also different internal coupling functions. For example, there are two nodes $i_0, j_0 (i_0 \neq j_0)$,and

$$h_{i_0} = c\sum_{j=1}^{n} a_{i_0 j}\Gamma x_j, h_{j_0} = c\sum_{j=1}^{n} a_{j_0 j} H(x_j) \tag{4}$$

This situation can not be expressed by linear connection structure or nonlinear connection structure which this paper studies is this situation. This paper studies the synchronization problem of complex networks with uncertain internal connection structure.

2.2 preliminary definitions and lemmas

Consider that the j th isolated node dynamical sub-network of network (1) is taken as the synchronization target, and its dynamic equation is expressed as

$$\dot{x}_j = f(x_j, \ t) \tag{5}$$

where $1 \leq j \leq n$, $j \neq i$

Network synchronizationisa typical collective behavior [3]. Finite-time synchronization means that the drive and response vectors synchronize within finite time[10]. In the following, a rigorous mathematical definition is introduced for the concept of network synchronization in finite-time.

Define system (5) as the master system and system (1) as the slave system. If there exists a constant $t_1 > 0$,such that

$$\lim_{t \to t_1} \| y_i(t) - x_j(t) \| \equiv 0 \tag{6}$$

where $1 \leq i \leq n, 1 \leq j \leq n, i \neq j$ Then, it can be said that the network (1) realizes limited time synchronization

Define the error vector as

$$e_i(t) = y_i(t) - x_j(t) \tag{7}$$

Then the objective of controller u_i is to guide the dynamical network (1) to synchronize in finite-time. The precise definition is as follows.

Definition 1. The complex network (1) is said to be stochastically synchronized in finite time if, for a suitable designed feedback controller, there exists a constant $t_1 > 0$,such that

$$\lim_{t \to t_1} \| e_i(t) \| = 0 \tag{8}$$

and

$$\| y_i(t) - x_j(t) \| \equiv 0 \tag{9}$$

where $1 \leq i \leq n, 1 \leq j \leq n, i \neq j$

From Eq. (4), the error dynamics is obtained as follows

$$\dot{e}_i(t) = f(y_i, t) - f(x_j, t) + h_i(y_1, y_2, ..., y_n) + u_i \tag{10}$$

Lemma 1 [3]. Assume that a continuous, positive-definite function V(t) satisfies the following differential inequality:

$$\dot{V}(t) \leq -\alpha V^{\eta}(t), \forall t \geq t_0, V(t_0) \geq 0 \tag{11}$$

where $\alpha > 0, 0 < \eta < 1$ are two constants. Then, for any given t_0 ,*V(t)* satisfies the following inequality:

$$V^{1-\eta}(T) \leq V^{1-\eta}(t_0) - \alpha(1-\eta)(t - t_0), t_0 \leq t \leq t_1 \tag{12}$$

and satisfies the following equality:

$$V(t) \equiv 0, \forall t > t_1 \tag{13}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{\alpha(1-\eta)} \tag{14}$$

Proof. Consider the following differential equation

$$\dot{X}(t) = -cX^{\eta}(t), X(t_0) = V(t_0) \tag{15}$$

Although this differential equation does not satisfy the global Lipschitz condition, the unique solution to this equation can be found as

$$X^{1-\eta}(t) = X^{1-\eta}(t_0) - c(1-\eta)(t-t_0) \tag{16}$$

and

$$x(t) \equiv 0, \forall t \geq t_1 \tag{17}$$

It is direct to prove that $x(t)$ is differential for $t > t_0$,one can obtains

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1-\eta)(t-t_0), t_0 \leq t \leq t_1 \tag{18}$$

and

$$V(t) = 0, \forall t \geq t_1 \tag{19}$$

With t_1 given in (14). The proof is completed.

Lemma 2 [3] . Suppose $x_1, x_2, ..., x_n$ are positive number and $0 \leq \alpha \leq 2$, then the following inequality holds

$$\sum_{i=1}^{n} |x_i|^{\alpha} \geq \left(\sum_{i=1}^{n} x_i^2\right)^{\frac{\alpha}{2}} \tag{20}$$

3. Finite-time synchronization of complex networks

In this section, a finite time synchronization controller is designed, and the Lyapunov stability theory is used to prove that the modified controller can ensure the gradual stability of the synchronization state in a finite time.

The sliding mode function is defined as

$$s_i(t) = e_i(t) + \alpha \int_0^t \operatorname{sgn}(e_i(\tau)) |e_i(\tau)|^{\beta} d\tau \tag{21}$$

Where $\alpha > 0$ and $0 < \beta < 1$ are constants.

Then taking time derivative of the sliding mode function, one can gain

$$\dot{s}_i(t) = \dot{e}_i(t) + \alpha \operatorname{sgn}(e_i(\tau)) |e_i(\tau)|^{\beta} \tag{22}$$

When the sliding surface is stable, i.e. $\dot{s}_i(t) = 0$,there must be

$$\dot{e}_i(t) = -\alpha \operatorname{sgn}(e_i(\tau)) |e_i(\tau)|^{\beta} \tag{23}$$

Theorem 1. Consider the sliding mode dynamics Eq. (23). The sliding surface will be asymptotically stable, its trajectories converge to the equilibrium $e_i(t) = 0$ in a finite time t_1 .

Proof. Define a Lyapunov function as follows:

$$V_1 = \frac{1}{2} \sum_{i=1}^{n} e_i^2 \tag{24}$$

Then taking time derivative of the Lyapunov function V_1, one can gain

$$\dot{V}_1 = \sum_{i=1}^{n} e_i \dot{e}_i \tag{25}$$

Replacing $\dot{e}_i$ from Eq. (23) into the equation above, it yields

$$\dot{V}_1 = -k_i \sum_{i=1}^{n} |e_i|^{1+\beta} \tag{26}$$

From Lemma 2, Eq. (20), one can obtain

$$\begin{aligned} \dot{V}_1 &\leq -2^{\frac{1+\beta}{2}} k_i \left(\frac{1}{2} \sum_{i=1}^{n} e_i^2\right)^{\frac{1+\beta}{2}} \\ &= -2^{\frac{1+\beta}{2}} k_i V^{\frac{1+\beta}{2}} \end{aligned} \tag{27}$$

Therefore based on Lemma 1, The sliding surface will be asymptotically stable in a finite time t_1 . Hence the proof is completed.

Theorem 2. The error system (7) can be finite-timely stabilized by the controller

$$\begin{aligned} u_i = {} & f(x_j, t) - f(y_i, t) - h_i(y_1, y_2, ..., y_n) - \delta \operatorname{sgn} s_i(t) \\ & - k_i [\operatorname{sgn}(e_i(t)) e_i(t)]^{\beta} \end{aligned} \tag{28}$$

where $1 \leq i \leq n, 1 \leq j \leq n, i \neq j 0 < \beta < 1, k_i > 0$

Proof. Define a Lyapunov function as follows:

$$V_2 = \frac{1}{2} \sum_{i=1}^{n} s_i^2 \tag{29}$$

Then taking time derivative of the Lyapunov function V_2, one can gain

$$\dot{V}_2 = \sum_{i=1}^{n} s_i \dot{s}_i \tag{30}$$

Replacing $\dot{s}_i$ from Eq. (22) into the equation above, it yields

$$\dot{V}_2 == \sum_{i=1}^{n} s_i [\dot{e}_i(t) + \alpha \operatorname{sgn}(e_i(\tau)) |e_i(\tau)|^{\beta}] \tag{31}$$

Consider the Eq. (28)and(10), one can gain

$$\dot{V}_2 = -k_i \sum_{i=1}^{n} |s_i| \tag{32}$$

From Lemma 2, Eq. (20), one can obtain

$$\begin{aligned} \dot{V}_2 &\leq -2^{\frac{1+\beta}{2}} k_i \left(\frac{1}{2} \sum_{i=1}^{n} s_i^2\right)^{\frac{1+\beta}{2}} \\ &= -2^{\frac{1+\beta}{2}} k_i V_2^{\frac{1+\beta}{2}} \end{aligned} \tag{33}$$

From Lemma 1, the error system (7) can be finite-timely stabilized by the controller u_i. Then the slave system (1) will synchronize the master system (5) in a finite time.

The above formula proves that the system approaches the sliding mode surface in a finite time and does not leave the sliding mode surface. In addition, since the sliding mode surface is stable, the closed-loop system is asymptotically stable.

4. Simulations

In this section, an example is given to illustrate the effectiveness of the above synchronization controller.

In simulation, consider a dynamical network consisting of 3 identical Lorenz systems. Take a single node (1) in the network as the synchronization target. Here, node dynamics is described by

$$\begin{pmatrix}\dot{x}_{j1}\\ \dot{x}_{j2}\\ \dot{x}_{j3}\end{pmatrix}=A\begin{pmatrix}x_{j1}\\ x_{j2}\\ x_{j3}\end{pmatrix}+\begin{pmatrix}0\\ -x_{j1}x_{j3}\\ x_{j1}x_{j2}\end{pmatrix}$$

and

$$\begin{pmatrix}\dot{y}_{i1}\\ \dot{y}_{i2}\\ \dot{y}_{i3}\end{pmatrix}=A\begin{pmatrix}y_{i1}\\ y_{i2}\\ y_{i3}\end{pmatrix}+\begin{pmatrix}0\\ -y_{i1}y_{i3}\\ y_{i1}y_{i2}\end{pmatrix}$$

where

$$A=\begin{pmatrix}-a & a & 0\\ c & -1 & 0\\ 0 & 0 & -b\end{pmatrix}$$

a=10,b=8/3,c=28,and $1\le i\le 3, 1\le j\le 3, i\ne j$ The networked system (1) is defined as follows:

$$\begin{pmatrix}\dot{y}_{i1}\\ \dot{y}_{i2}\\ \dot{y}_{i3}\end{pmatrix}=A\begin{pmatrix}y_{i1}\\ y_{i2}\\ y_{i3}\end{pmatrix}+\begin{pmatrix}0\\ -y_{i1}y_{i3}\\ y_{i1}y_{i2}\end{pmatrix}+\begin{pmatrix}f_1(y_i)-2f_1(y_{i+1})+f_1(y_{i+2})\\ 0\\ f_2(y_i)-2f_2(y_{i+1})+f_2(y_{i+2})\end{pmatrix}+u_i \quad (34)$$

Where $f_1(y_i)=a(y_{i2}-y_{i1}), f_2(y_i)=y_{i1}y_{i2}-by_{i3}, y_3\equiv y_1, y_4\equiv y_2$

Assume that $k_i=1, \beta=\frac{1}{2}$, the initial value of the system is $x(0)=(5,6,7)$, $y_i(0)=(50+10i, 60+10i, 70+10i)$ and $i=1,2$, $j=3$

As expected, one can find that the trajectories of the closed loop slave system can synchronize the trajectories of the master system within finite-time.

The is state trajectories of Lorenz system shown in Fig. 1-6

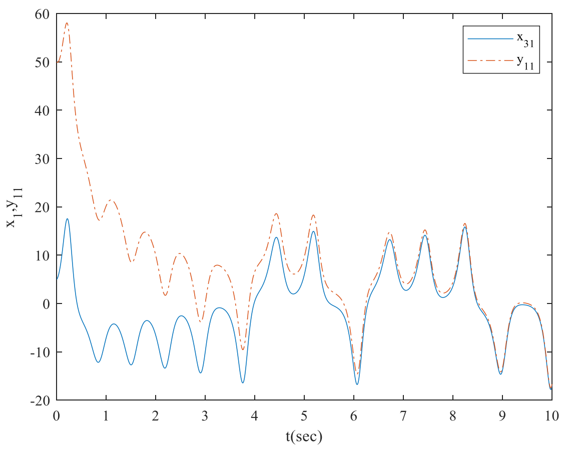

Fig. 1. State trajectories of Lorenz system x_{31} and y_{11}

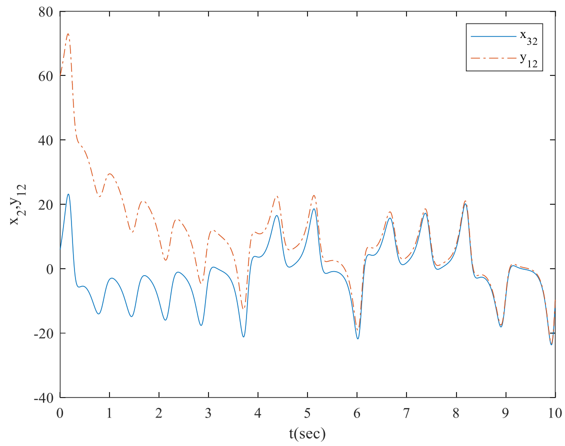

Fig. 2. State trajectories of Lorenz system x_{32} and y_{12}

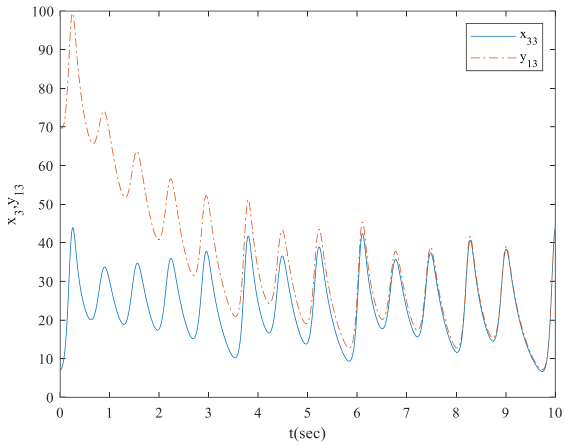

Fig. 3. State trajectories of Lorenz system x_{33} and y_{13}

From Fig.1-Fig.3, we can see the trajectory indicate the synchronization results. The states x_{31} is tracking the state y_{11} in finite time, the states x_{32} is tracking the state y_{12} in finite time,, the states x_{33} is tracking the state y_{13} in finite time, where initial condition with

$$x(0)=(5,6,7)$$
$$y_1(0)=(60,70,80)$$
$$y_2(0)=(70,80,90)$$

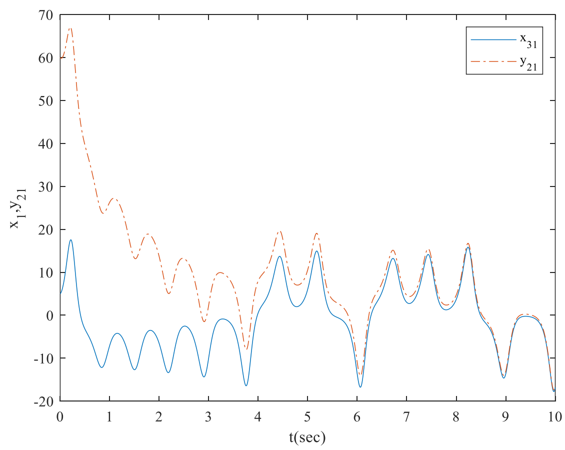

Fig. 4. State trajectories of Lorenz system x_{31} and y_{21}

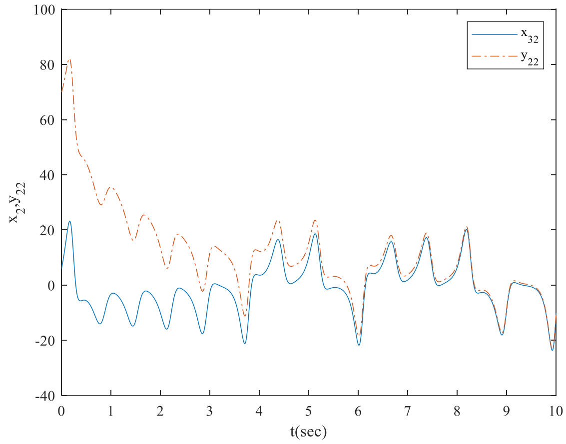

Fig. 5. State trajectories of Lorenz system x_{32} and y_{22}

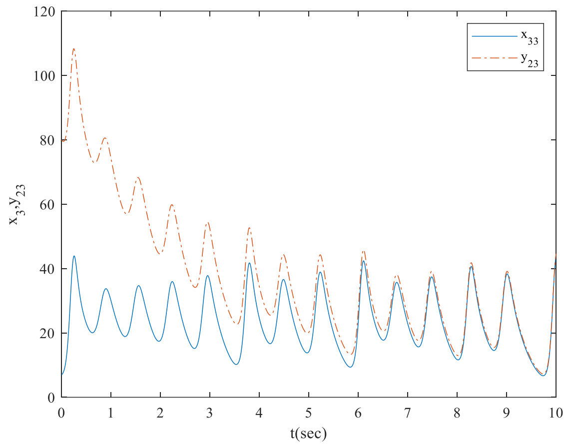

Fig. 6. State trajectories of Lorenz system x_{33} and y_{23}

From Fig.4-Fig.6, we can see the trajectory indicate the synchronization results. The states x_{31} is tracking the state y_{11} in finite time, the states x_{32} is tracking the state y_{12} in finite time,, the states x_{33} is tracking the state y_{13} in finite time, where initial condition with

$$x(0) = (5,6,7)$$

$$y_1(0) = (60,70,80)$$

$$y_2(0) = (70,80,90)$$

It can be seen that under the action of the relevant controll er, the trajectories of all network nodes can be synchronized q uickly in a limited time. These simulation results have illustrat ed the effectiveness of the proposed method.

5. Conclusion

This paper studies the synchronization of uncertain complex dynamic networks in finite time, proposes a finite time synchronization controller, and proves that the designed controller can ensure the asymptotic stability of the synchronization state in finite time by using Lyapunov stability theory. The research results show that this method can realize complex finite time synchronization whether the structure of complex network is known or not and whether the connection structure between node states is linear or nonlinear. The advantage of this method is that it has simple structure, high universality and is more conducive to practical engineering application. In addition, in practical application, we can design a controller with the same structure, and then tune and transform its parameters to obtain a suitable overall controller. Finally, its effectiveness is verified by numerical simulation.

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