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Unmanned Helicopter Prescribed Performance Attitude Control and Parameters Tuning Based on Grey Wolf Optimization

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ABSTRACT

Prescribed performance control (PPC) has received increasing attention because it can obtain high quality control performance of the system in recent years, especially in the fields of the spacecraft and unmanned aerial vehicle. However, the designed controllers are always complex and there are a lot of control parameters to be tuned to guarantee the high-quality performance of the system. In this paper, a PID-liked prescribed performance controller is used to control a helicopter plant. A grey wolf optimization (GWO) algorithm is used to optimize the 9 PPC parameters. And the artificial bee colony (ABC) algorithm and sparrow search algorithm (SSA) are used to compare to GWO. The result of the numerical simulations shows that the application of the GWO guarantees the high-quality performance of the PPC.

1. Introduction

Unmanned helicopters have been widely used in many military and civilian fields because of their good maneuverability, vertical hovering, low risk of casualties, and ease of use (Xu, G. et al., 2020). Additionally, flight stability, high quality transient performance (convergence rate, maximum overshoot, et al.) is also the key to the study of attitude control of unmanned helicopters. However, the unmanned helicopters attitude control system is an under-actuated and multi-input, multi-output nonlinear system with strong coupling, and the model is uncertain and subject to external interference (Li, Y. and Song, S., 2012; He, H., et al., 2018). These characteristics limit the control performance of the system. However, the widely used PID (proportional, integral, derivative) control and LQR algorithm require approximate linearization (Li, Y. and Song, S., 2012) and cannot meet the requirements of high precision control.

Prescribed performance control (PPC), as a potential way to quantitatively describe the transient performance and steady-state performance of a system, has received much attention in recent years. PPC method was first proposed by Bechlioulis and Rovithakis (Bechlioulis C. and Rovithakis G. A., 2008; Bechlioulis, C. P. and Rovithakis, G. A., 2008; Bechlioulis, C. et al., 2009; Bechlioulis, C. P. and Rovithakis, G. A., 2011). It predefines the tracking error of the control system by introducing the prescribed performance function (PPF) and an error transformation technique. By selecting a suitable PPF, the tracking error of the system converges to an arbitrarily small set of residuals, the convergence rate is greater than a preset constant, and the maximum overshoot is less than a preset value.

In this paper, a PID-liked prescribed performance controller is designed to achieve the high-quality attitude control performance of the unmanned helicopters. This controller has a lot of parameters, thus parameters tuning becomes a big challenge. And manually tuning is time-consuming and unrealistic and hardly to achieve required performance (Soni, V. et al., 2016). Reference (Tang, J. et al., 2010) adopts an iterative learning algorithm to tune PID controller parameters. Reference (Poksawat, P. et al., 2016) designs an automatic PID controller parameter optimizing algorithm to tune the controller parameters. Other tuning methods such as Ziegler-Nichols, Haalman and λ-Tuning, Internal Model Principle (IMC) and so on are used in PID parameters tuning (Wu, H. et al., 2014). However, above mentioned tuning methods rely on the accurate system models or only suitable for linear system and always difficult to balance the transient performance and steady-state performance.

Recently, many soft computing algorithms have been implemented in many fields. Such as reference (Wang, H. and L. Liu, 2019), which uses partheno-genetic algorithm and simulated annealing algorithm in degree constrained minimum spanning tree problem; reference (Liu, H., X. Lv, and J. Xiao, 2018), which uses particle swarm optimization (PSO) algorithm for water environment

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quality assessment; reference (Wang, Y., et al., 2018), which applies an improved ant colony algorithm in robot path planning. In terms of control parameter optimizing, reference (Mpanza, L. J. and Pedro, J. O., 2019) adopts an ant colony optimization (ACO) algorithm to optimize a sliding mode controller with 8 parameters. Reference (Gomez, V. et al., 2020) uses a particle swarm optimization algorithm for multi-objective problems to tune a complex PID controller for unmanned aerial vehicles. In (Shen, S. and Xu, J., 2020), an adaptive genetic algorithm-particle swarm optimization (AGA-PSO) is adopted to tune the complex controller parameters. Many references show that better system performance can be obtained by using metaheuristic optimization algorithms for parameter tuning compared to traditional tuning techniques. Grey Wolf Optimizer (GWO) is a new metaheuristic swarm intelligence algorithm first presented in reference (Mirjalili, S. et al., 2014) by Mirjalili et al. in 2014. Many applications of GWO have been proposed in references. Reference (Soni, V. et al., 2016) suggests a "hybrid GWO and Pattern Search Algorithm (hGWO-PS)" to optimize a 2DOF-PID controller. Reference (Precup, R.-E. et al., 2017) adopts GWO to optimize a PI fuzzy controller. The GWO algorithm has following advantages: simple, flexible and robust; local minima avoidance; few parameters; no derivation information is needed in the initial search; model free optimization algorithm (Panda, M. and Das, B., 2019; Faris, H. et al., 2018). Thus, the GWO algorithm is proposed to optimize the prescribed performance controller in this paper.

The rest sections of the paper are organized as: Section 2 introduces the mathematical model of the 3-degree-of-freedom (3-DOF) unmanned helicopter attitude control system. In section 3, a PID-liked prescribed performance controller is given and an objective function is introduced. In section 4, the mathematical model of GWO algorithm is introduced. Section 5 presents the simulation results. The conclusions of this paper are given in section 6.

2. Dynamic Model of 3-DOF Unmanned Helicopter Plant

In this section, the unmanned helicopter plant and coordinate systems are introduced, then the dynamics model of the plant is presented. In this paper, the dynamic model of unmanned helicopter plant in reference (Zhu, B. et al., 2019) is adopted. A 3-DOF unmanned helicopter plant and its components is shown in Fig. 1.

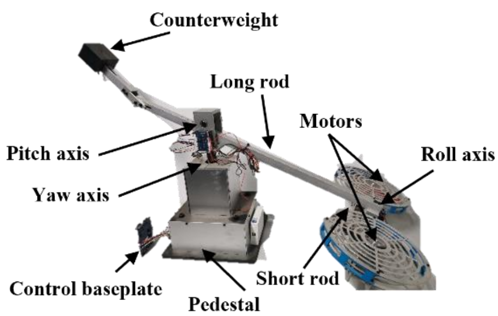


Fig. 1. 3-DOF unmanned helicopter plant

A global coordinate system (Cartesian coordinate system) and a body coordinate system (Cartesian coordinate system) is provided to describe the attitude of the helicopter plant (see Fig. 2).

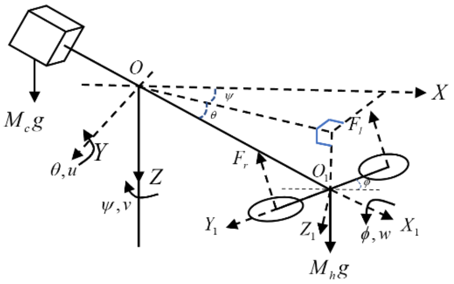


Fig. 2. Coordinate system of the plant

Following assumptions are given to simplify the model before the dynamic modelling:

(1) The helicopter plant system is a rigid system.
(2) The helicopter plant system is symmetrical.
(3) Linear DC motors.

The dynamics of the three channels (pitch, yaw, roll) can be visualized in Fig. 3. The attitude angle vector is $\vec{X} = [\theta, \psi, \phi]$ (pitch, yaw, roll, respectively), attitude angular velocity vector is $\vec{\omega} = [u, v, w]$ (pitch, yaw, roll, respectively). $\vec{F} = [F_l, F_r]$ is the lifting force vector of the DC motors. The symmetric inertia matrix is as follows:

$$J = \begin{bmatrix} J_x & J_{xy} & J_{xz} \\ J_{xy} & J_y & J_{yz} \\ J_{xz} & J_{yz} & J_z \end{bmatrix}$$

where $J_{xy} = 0, J_{xz} = 0, J_{yz} = 0$.

Thus, the dynamic model of the 3-DOF unmanned helicopter plant can be written in the following form:

$$\begin{aligned} &\dot{\theta} = u \\ &\dot{u} = \left[\left(M_c g l_c - M_h g l_m \right) \cos\theta + \left(F_l + F_r \right) l_m \cos\phi \right] / J_y \\ &\dot{\psi} = v \\ &\dot{v} = \left(F_l + F_r \right) l_m \sin\phi \cos\theta / J_z \\ &\dot{\phi} = w \\ &\dot{w} = \left[\left(F_l - F_r \right) l_r \right] / J_x \end{aligned} \tag{1}$$

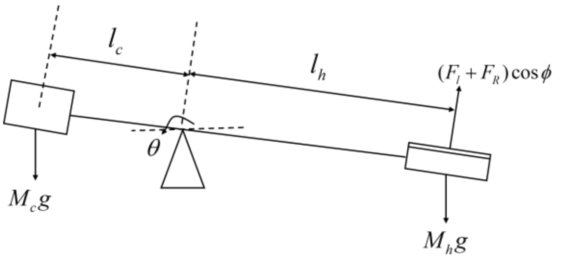
where M_c is the mass of counterweight , M_h is the total mass of the short rod and two motors, l_c is the distance from the counterweight to yaw axis, l_h is the distance from roll axis to yaw axis, l_r is the distance from the center of gravity of the motor to roll axis.

To facilitate the design of the controller and according to Fig. 3, the lifting forces of the motor is converted into the control force of three channels, as follows

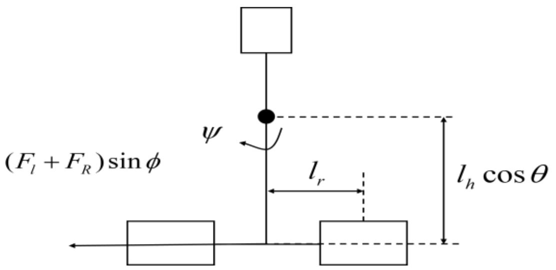
$$\begin{aligned} &F_\theta = \left(F_l + F_r \right) \cos\phi \\ &F_\psi = \left(F_l + F_r \right) \sin\phi \\ &F_\phi = F_l - F_r \end{aligned} \tag{2}$$

The dynamic model equation (1) can be rewritten as:

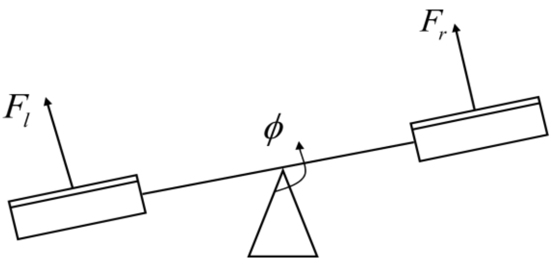
$$\begin{aligned} &\dot{\theta} = u \\ &\dot{u} = [(M_c g l_c - M_h g l_h) \cos\theta + F_\theta l_h] / J_y \\ &\dot{\psi} = v \\ &\dot{v} = F_\psi l_h \cos\theta / J_z \\ &\dot{\phi} = w \\ &\dot{w} = F_\phi l_x / J_x \end{aligned} \tag{3}$$



(a) Pitch channel



(b) Yaw channel



(c) Roll channel

Fig. 3. Three attitude channels of the plant. (a) Pitch channel, (b) Yaw channel, (c) Roll channel

3. Prescribed Performance Controller Design

In this section, the prescribed performance function and error transformation method are introduced. A PID-liked prescribed performance controller is presented. Then a discrete objective function is introduced to quantitatively describe the performance of the helicopter control system.

3.1 Prescribed Performance Function and Error Transformation

The control purpose of PPC method is:

(1) The tracking error of the system converges to an arbitrarily small set of residuals.
(2) The convergence rate is not less than a preset constant.
(3) The maximum overshoot is less than a sufficiently small preset value.

Thus, the smooth function $\rho(t):\mathbb{R}^{+}\to\mathbb{R}^{+}$ is introduced as PPF to define the error boundary, as follows

$$\rho(t) = (\rho_0 - \rho_\infty)e^{-st} + \rho_\infty \tag{4}$$

where the parameters in equation (4) satisfy $\rho_0 > 0, \rho_\infty > 0, s > 0$, which define the initial error bound, ultimate error bound, and convergence rate of boundary, respectively. Fig. 4 illustrates the exponential PPF. PPF also has the following properties:

(1) $\rho(t) > 0, \forall t > 0$ and is strictly decreasing.

(2) $\lim_{t\to\infty}\rho(t) = \rho_\infty > 0$.

(3) $\rho(0) = \rho_0 > e(0)$.

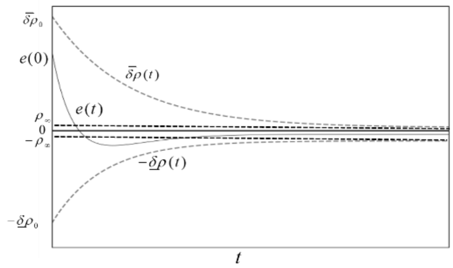


Fig. 4. Exponential prescribed performance function

The following inequality is given to achieve the preset transient performance and steady-state performance.

$$-\underline{\kappa}(t) < e(t) < \bar{\kappa}(t), \forall t > 0 \tag{5}$$

where $\underline{\kappa}$ and $\bar{\kappa}$ are positive constants, $e(t) = x(t) - x_d(t)$ is the original error of the attitude angle, $x_d(t)$ is the desired attitude angle.

An error transformation $f_{tr}(\varepsilon)$ is introduced to transform the inequality constraint (5) to equivalent "unconstrained" one:

$$e(t) = f_{tr}(\varepsilon)\rho(t) \tag{6}$$

The error transformation $f_{tr}(\varepsilon)$ is defined as:

$$f_{tr}(\varepsilon) = \frac{\bar{\kappa}e^{\varepsilon} - \underline{\kappa}e^{-\varepsilon}}{e^{\varepsilon} + e^{-\varepsilon}} \tag{7}$$

where ε is the transformed error. And $f_{tr}(\varepsilon)$ satisfies the following properties

(1) $-\underline{\kappa} < f_{tr}(\varepsilon) < \bar{\kappa}$

(2) $\lim_{\varepsilon\to+\infty} f_{tr}(\varepsilon) = \bar{\kappa}$ and $\lim_{\varepsilon\to-\infty} f_{tr}(\varepsilon) = -\underline{\kappa}$.

According to equation (6) and (7), the expression of ε is the inverse function of (7) which can be written as:

$$\varepsilon = \frac{1}{2}\ln\left(\frac{\underline{\kappa} + \lambda}{\bar{\kappa} - \lambda}\right) \tag{8}$$

where $\lambda = e/\rho$ is the normalization of original error.

The derivative of λ is

$$\dot{\lambda} = \frac{\dot{e}\rho - e\dot{\rho}}{\rho^2} \tag{9}$$

The time derivative of ε is

$$\dot{\varepsilon} = \frac{\partial\varepsilon}{\partial\lambda}\frac{d\lambda}{dt} = \frac{1}{2}\frac{\bar{\kappa} + \underline{\kappa}}{(\underline{\kappa} + \lambda)(\bar{\kappa} - \lambda)}\frac{\dot{e}\rho - e\dot{\rho}}{\rho^2} \tag{10}$$

In this paper, $\bar{\kappa} = 1, \underline{\kappa} = 1$. It can be proved that by using the transformed error ε to design the controller, the system can satisfy the inequality (5) and achieve the prescribed performance (Sun, R. et al., 2018).

3.2 Prescribed Performance Controller

The helicopter plant attitude control system is nonlinear, channels strongly coupled, under-actuated with multi-input and multi-output. Considering the complexity and feasibility of the controller in practical application of unmanned helicopter, a PID based controller with PPF is adopted.

The roll channel and yaw channel are coupled to each other. Therefore, in each iteration the expected roll angle ϕ_{pre} needs to be updated: $\phi_{pre} = arctan(F_\psi / F_\theta)$ to achieve yaw motion. The structure of the control frame is shown in Fig. 5. And the transformed error ε is used to design the controller. The control force vector $\vec{F}=[F_\theta, F_\psi, F_\phi]$ in equation (2) are calculated as

$$
\begin{aligned}
F_\theta &= k_P^\theta \varepsilon_\theta + k_I^\theta \int \varepsilon_\theta + k_D^\theta \dot{\varepsilon}_\theta \\
F_\psi &= k_P^\psi \varepsilon_\psi + k_I^\psi \int \varepsilon_\psi + k_D^\psi \dot{\varepsilon}_\psi \\
F_\phi &= k_P^\phi \varepsilon_\phi + k_I^\phi \int \varepsilon_\phi + k_D^\phi \dot{\varepsilon}_\phi
\end{aligned} \tag{11}
$$

where k_P^i, k_I^i, k_D^i $(i=\theta,\psi,\phi)$ are control parameters of the prescribed performance controller, $\varepsilon_i, \dot{\varepsilon}_i$ $(i=\theta,\psi,\phi)$ are transformed errors and its derivative of pitch, yaw, roll channels. According to equation (2), the lifting forces $\vec{F}=[F_r, F_l]$ of the DC motors can be calculated as follows:

$$
\begin{aligned}
F_l &= \left(\sqrt{F_\theta^{\,2}+F_\psi^{\,2}}+F_\phi\right)/2 \\
F_r &= \left(\sqrt{F_\theta^{\,2}+F_\psi^{\,2}}-F_\phi\right)/2
\end{aligned} \tag{12}
$$

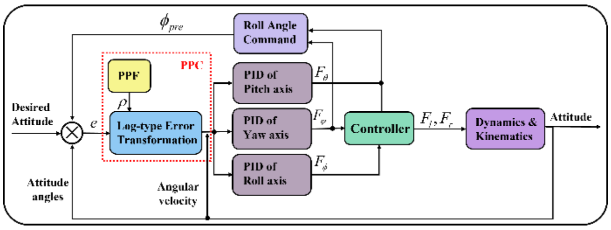


Fig. 5. The structure of controller frame

3.3 Objective Function

In this subsection, an objective function is presented to describe the overall performance of the system. A commonly used integral-type objective function in (Mpanza, L. J. and Pedro, J. O., 2019) is adopted. For the purpose of numerical simulation, the integral-type objective function is changed to a discrete form:

$$
J=\sum_{i=1}^{n} h_i \times \left[\left(\frac{\theta_{pre}-\theta_i}{\theta_{pre}}\right)^2+\left(\frac{\psi_{pre}-\psi_i}{\psi_{pre}}\right)^2+\left(\frac{F_{ri}}{\max\{F_{ri}\}}\right)^2+\left(\frac{F_{li}}{\max\{F_{li}\}}\right)^2\right]/T \\
+\omega\times\frac{\max\{\psi_i\}-\psi_{pre}}{\psi_{pre}} \tag{13}
$$

where h_i is the step of simulation, T is the total simulation time, θ_{pre} and ψ_{pre} are expected attitude angles of pitch channel and yaw channel, respectively. $(\max\{\psi_i\}-\psi_{pre})/\psi_{pre}$ is a correction for overshoot, and ω is the weight.

4. Grey Wolf Optimization Algorithm

Grey Wolf Optimization (GWO) algorithm is a nature inspired metaheuristic algorithm. GWO algorithm describes the leadership hierarchy and the hunting behaviors of the grey wolves (Panda, M. and Das, B., 2019). The mathematical modeling of the GWO algorithm consists of five steps (for single objective problem) (Mirjalili, S. et al., 2014).

Step1: Social hierarchy

A grey wolf pack consists of 5-12 wolves. And a grey wolf pack has a strict social hierarchy and is often divided into four layers (Fig. 6): alpha wolf (α), beta wolf (β), delta wolf (δ), omega wolf (ω). The social status of each layer is also shown in Fig. 6.

Fig. 6. Hierarchy and social status of grey wolf

Step2: Encircling prey

The encircling behavior of grey wolf group can be described by the following equations,

$$\vec{D}=\left|C\vec{P}_i(t)-\vec{P}(t)\right| \tag{14}$$

$$\vec{P}(t+1)=\vec{P}_i(t)-A\vec{D} \tag{15}$$

where t is the current iteration, $\vec{P}_i(t)$ is the position of prey at t^{th} iteration which represents the current optimal solution, $\vec{P}(t)$ is the position of the current wolf at t^{th} iteration, A and C are two random coefficients that determine the movement of the gray wolf which are calculated as follows,

$$
\begin{aligned}
A &= 2a\vec{r}_1 - a \\
C &= 2\vec{r}_2
\end{aligned} \tag{16}
$$

where r_1, r_2 are random numbers from 0 to 1, a is a coefficient that decreases linearly from 2 to 0 during iteration t which can be calculated as $a=2(t_{max}-t)/t_{max}$, t_{max} is the maximum iterations.

Step3: Hunting

Beside the leader wolf (α wolf), β wolf and δ wolf are also involved in hunting command. And the positions of the α wolf, β wolf and δ wolf always represent the three best current positions of the grey wolf pack in hunting. In the actual optimization process, the position of the prey is always unknown in advance. Therefore, the positions of the α wolf, β wolf and δ wolf are used to update the positions of other wolves. The position update process is as follows,

$$
\begin{aligned}
\vec{D}_\alpha &= \left|C_1\vec{P}_\alpha(t)-\vec{P}(t)\right| \\
\vec{D}_\beta &= \left|C_2\vec{P}_\beta(t)-\vec{P}(t)\right| \\
\vec{D}_\delta &= \left|C_3\vec{P}_\delta(t)-\vec{P}(t)\right|
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
\vec{P}_1(t) &= \vec{P}_\alpha(t)-A_1\vec{D}_\alpha \\
\vec{P}_2(t) &= \vec{P}_\beta(t)-A_2\vec{D}_\beta \\
\vec{P}_3(t) &= \vec{P}_\delta(t)-A_3\vec{D}_\delta
\end{aligned} \tag{18}
$$

$$\vec{P}(t+1)=\frac{\vec{P}_1(t)+\vec{P}_2(t)+\vec{P}_3(t)}{3} \tag{19}$$

Fig. 7 illustrates the position updating of the ω wolf according to equations (17) ~ (19),

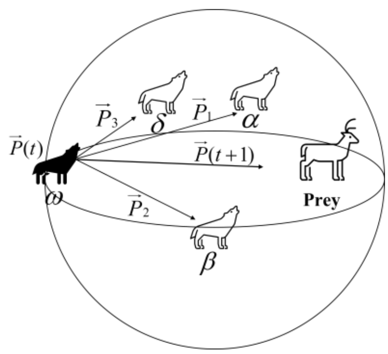


Fig. 7. Position updating of ω wolf according to equations (17) ~ (19)

Step4: Attacking prey

When the grey wolves attack the prey, the hunting behavior ends.

And the attacking behavior is mathematically described by the coefficient a, which linearly decreases from 2 to 0 during iteration. According to equation (15), the random number A can also describe the attacking behavior. A is a random number in the interval $[-a, a]$. In GWO algorithm, wolves attack the prey when the $|A| < 1$.

Step5: Searching for prey

The grey wolves search the prey based on the positions of the α wolf, β wolf and δ wolf. In step 4, when $|A| < 1$ the wolves attack the prey; in this step, when $|A| > 1$ the wolves move away from each other in search of the prey. The coefficient C is a random number which makes the searching behavior more random and enables the algorithm to avoid local optimal solutions (Mirjalili, S. et al., 2014).

There are two important parameters in GWO required to be initialized: the number of wolves in grey wolf group (populations) and the number of iterations. These two parameters vary in different optimization problems. The pseudocode of GWO algorithm is presented in Fig.8.

```
Initialize the two important parameters: populations m and maximum iterations n
Set the upper and lower limits of the wolf position: u_b and l_b
Randomly generate the first group of wolves' position information: P(0) ∈ [l_b, u_b]
While (t < maximum iterations n)
    for each wolf
        guarantee and limit the position of wolf P to interval [l_b, u_b]
        calculate the objective function value of each wolf
        update P_α, P_β, P_δ according to objective function value
    end for
    for each wolf
        calculate a and randomly generate A and C
        update the current position P(t) to P(t+1) according to equations (17)~(19)
    end for
    t = t+1
end while
Return the best result P_α
```

Fig. 8. Pseudocode of GWO algorithm

5. Simulation Results

In this paper, the parameters need to be tuned include: $k_P^\theta, k_I^\theta, k_D^\theta; k_P^\psi, k_I^\psi, k_D^\psi; k_P^\phi, k_I^\phi, k_D^\phi$. The parameter $s = 1.5$, s decides the convergence rate of boundary in equation (4).

The upper limit $\vec{u}_b = [30,10,10,10,1,3,10,1,3]$, the lower limit $\vec{l}_b = [10,0,0,0,0,0,0,0,0]$. The weight $\omega = 0.05$. The prescribed performance functions of pitch, yaw and roll channels are shown in Tab. 1. The parameters of helicopter plant are shown in Tab. 2.

Tab. 1. Performance functions

Channels	Performance functions
Pitch	$(0.8-0.04)e^{-1.5t}+0.04$
Yaw	$(0.3-0.04)e^{-1.5t}+0.04$
Roll	$(0.4-0.04)e^{-1.5t}+0.04$

Tab. 2. The parameters used in the helicopter modeling

Symbols	Values	Explanations
M_c	2.2kg	Mass of counterweight
M_h	2.5kg	Mass of motor and roll axis
l_c	0.4m	Distance from counterweight to yaw axis
l_h	0.6m	Distance from roll axis to yaw axis
l_r	0.2m	Distance from roll axis to motor
J_x	0.05kgm^2	Inertia of roll channel
J_y	1.25kgm^2	Inertia of pitch channel
J_z	1.25kgm^2	Inertia of yaw channel
g	9.8m/s^2	Gravity acceleration

Two other optimization algorithms: Sparrow Search Algorithm (SSA) (Xue, J. and Shen, B., 2020), Artificial Bee Colony (ABC) algorithm (Karaboga, D. and Basturk, B., 2008) are also used to optimize the parameters of PPC to compare to GWO. According to references, SSA and ABC are also excellent optimization algorithms in parameters tuning. The parameters of the three algorithms are shown in Tab. 3. All three algorithms are stochastic, each algorithm was run 20 times and the best result were selected. The tuning results are presented in Tab. 4. The GWO has the minimum parameters and highest operational efficiency (minimum running time). The objective function value of GWO, ABC and SSA are compared in Fig. 9. From Fig. 9, we can see that the GWO converges faster and gets the minimum objective function value in the PPC parameter tuning. This suggests that better overall performance can be obtained by using GWO.

Tab. 3. Parameters of three metaheuristic algorithms

Algorithms	Parameters	Explanation
GWO	*Search Agent* =12	Population of grey wolves
ABC	*Colony Size* = 16	Population of artificial bees
	n_o = 8	Onlooker bees
	n_e = 8	Employed bees
	n_s = 1	Scout bee
	Limit = 20	Control parameter
SSA	n = 20	The number of sparrows
	PD = 0.2	Number of producers
	SD = 0.2	Early warning sparrows
	R_2 = 0.8	The alarm value

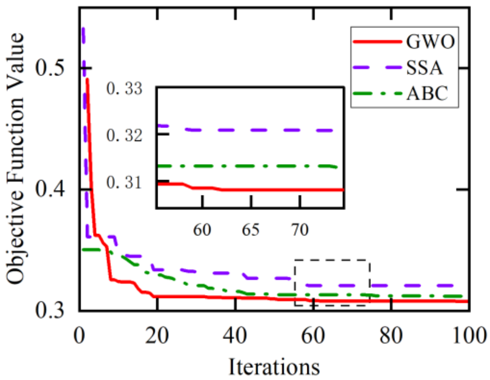


Fig. 9. Comparison of 100 iterations: GWO, SSA, AB

Tab. 4. Tuning result

Algorithms	Results
GWO	$k_P^\theta = 24.11, k_I^\theta = 10.00, k_D^\theta = 5.66$
	$k_P^\psi = 3.71, k_D^\psi = 0.06, k_D^\psi = 1.39$
	$k_P^\phi = 0.77, k_I^\phi = 0.07, k_D^\phi = 1.88$
	Running time = 121.59 s
ABC	$k_P^\theta = 19.06, k_I^\theta = 5.63, k_D^\theta = 4.24$
	$k_P^\psi = 4.66, k_I^\psi = 0.10, k_D^\psi = 1.73$
	$k_P^\phi = 0.80, k_I^\phi = 0.36, k_D^\phi = 1.47$
	Running time = 139.88 s
SSA	$k_P^\theta = 19.61, k_I^\theta = 1.29, k_D^\theta = 3.18$

$k_P^{\psi}=1.83, k_D^{\psi}=0.65, k_D^{\psi}=1.25$
$k_P^{\phi}=5.25, k_I^{\phi}=0.52, k_D^{\phi}=2.07$
Running time = 154.59 s

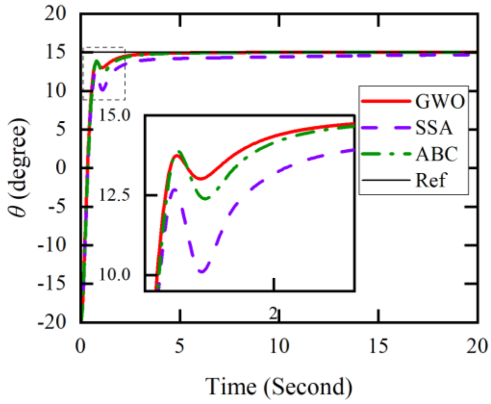


Fig. 10. Time response of the pitch channel

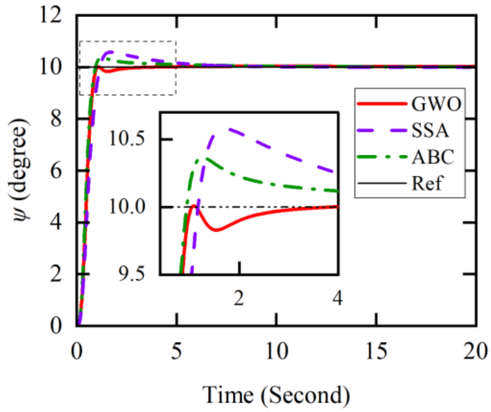


Fig. 11. Time response of the yaw channel

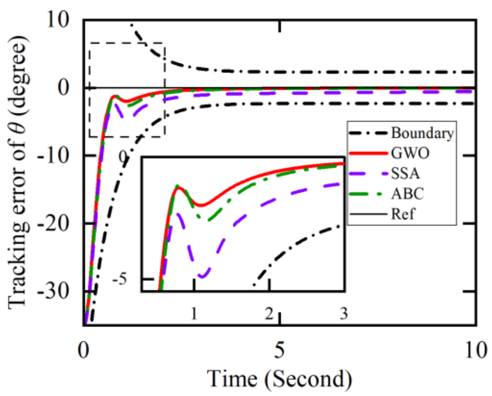


Fig. 12. Tracking error of the pitch channel

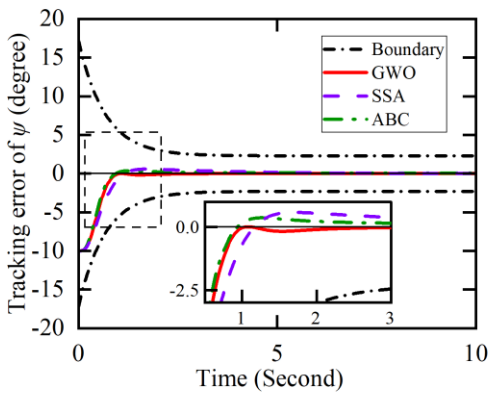


Fig. 13. Tracking error of the yaw channel

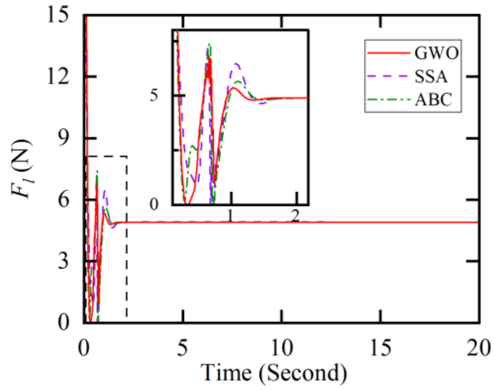


Fig. 14. The lifting force of left motor

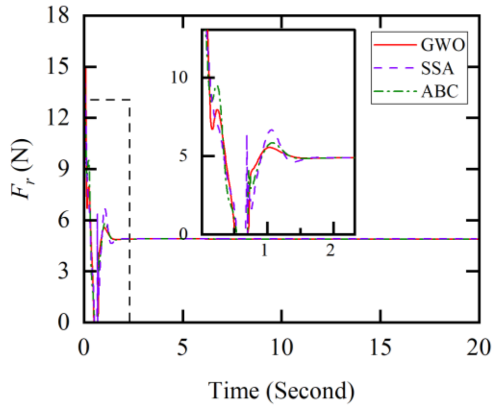


Fig. 15. The lifting force of right motor

Fig. 10 and Fig. 11 show the time response of the pitch channel and the yaw channel. It can be seen that the GWO-tuned system has the faster convergence rate and smaller steady-state tracking error in both pitch response and yaw response. From Fig. 11, it can be seen that the GWO-tuned system only has slight overshoot, while ABC-tuned system and SSA-tuned system have obvious maximum overshoot. That means by selecting a suitable weight ω in equation (13) the GWO algorithm effectively reduces the maximum overshoot. Fig. 12 and Fig. 13 show the tracking error of the pitch channel and the yaw channel. It can be seen that all three tuned systems have a undershoot at about 1s. The SSA-tuned system is closest to prescribed performance boundaries which means it has the risk of crossing the boundary. The GWO-tuned system is closest to the ideal curve, which proves the effectiveness and robustness of GWO. Fig. 14 and Fig. 15 show the lifting force of left motor and right motor. It can be seen that the GWO-tuned system's lifting force curve is under the ABC-tuned system's and the SSA-tuned system's. It means the GWO-tuned system can obtain better control performance with less power in PPC parameters tuning.

6. Summary

The three nature-inspired optimization algorithms GWO, ABC and SSA are used to tune the prescribed performance controller. Although ABC and SSA are excellent optimization algorithms in many engineering cases. The results show that the GWO can provide competitive results compared to SSA and ABC. GWO has the minimum number of parameters and the simplest optimization format. Due to the complexity of the 3-DOF helicopter plant control system which has 9 parameters to be tuned, GWO is more time efficient. Moreover, GWO can avoid local minima and converges faster, GWO-tuned system has smaller overshoot and faster convergence rate. GWO is also expected to be applied to other complex controller optimization problems.

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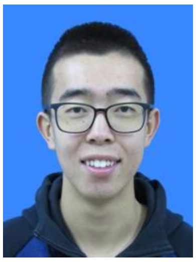

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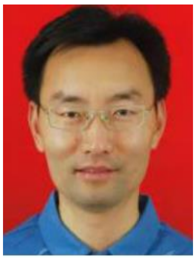

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