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Rotational Motion Blurred Images Restoration Based on the Prior Mixed-Order Fuzzy Kernel

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ABSTRACT

Rotational blur is a common form of motion blur. To solve the problem that the differential autocorrelation method has a large estimation error on the rotational fuzzy kernel, a novel rotational blur restoration algorithm based on the prior mixed-order fuzzy kernel is proposed. In the proposed algorithm, the degradation principle of the image is analyzed and the regularization constraint of the prior mixed-order fuzzy kernel is used for the deconvolution operation of the one-dimensional vectors extracted along the fuzzy path of the image. Because of the diagonalization of the cyclic matrix, the restoration model of the blurred image is transformed in frequency domain. The simulation results show compared with the improved Wiener filtering and the gradient loading algorithm, the proposed algorithm has stronger ability to suppress noise and weaken the ringing-effect. The algorithm has a good real-time performance and certain engineering application value.

1. Introduction

Image restoration is mainly proposed for the ‘degradation’ in the image processing. The ‘degradation’ phenomenon in the image processing mainly refers to the image system is affected by various factors, such as the defocused image systems, equipment and relative movement between the object or the atmospheric effects, etc.. Image restoration or image reconstruction (e.g., Yao et al., 2019) techniques such as image dehazing (e.g., Yu et al., 2019) are used to improve the quality of image so as to better identify and track targets (e.g., Yang et al., 2019).

Image deblurring is an important research direction in the field of computer vision and image processing. In the military field, since the seeker of the image-guided weapon adopts a strapdown connection with the projectile, the target area obtained by the seeker will also be in rotation when the projectile itself rotates. When the rotation speed is too large, the image containing the target area information will inevitably be blurred, so that the target cannot be identified and located, which will cause great difficulties for the subsequent tracking and guidance of the target.

As a form of spatially variable motion blur, rotational motion blur has always been a difficult point in the field of image processing. Some scholars (e.g., Joshi et al., 2010; e.g., Cupta et al., 2010) decompose the motion path of the camera and construct the corresponding high-dimensional sparse matrix, and iteratively solve the maximum posterior probability distribution function of the sparse matrix and the original image, but due to the huge storage space and high-dimensional matrix operations in the process, the algorithm takes too long and is difficult to be applied to engineering practice. The literature (e.g., Hong, et al., 2003) proposes a fast recovery algorithm along the fuzzy path based on Bresenham circle method, the advantage of taking pixels in the fuzzy path is that it does not require a lot of interpolation, the result is prone to high-frequency oscillation and also involves matrix operations, which requires a large amount of computation. The literature (e.g., Yuan, et al., 2008) improves the Wiener filtering method by eliminating the ‘ill-conditioned’ component of the fuzzy matrix in the frequency domain analysis, which improves the image restoration effect, but the algorithm introduces a prior information of noise and is more sensitive to the changes of noise. The literature (e.g., Luo, et al., 2014) uses the minimum deviation circular interpolation method to extract the pixels along the fuzzy path, and improves the diagonal loading algorithm to increase the noise immunity of the algorithm by introducing the direction constraint, but there still is a certain ringing-effect at the edges of the image. The literature (e.g., Wang, et al., 2019) proposes a novel rotational blur restoration algorithm based on

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adaptive gradient prior regularization, the regularization term is used for the deconvolution operation of the one-dimensional vectors which is extracted along the fuzzy path in the frequency domain.

In this paper, when the center of rotation in the strapdown missile-borne imaging system is taken as the image center, Bresenham circle algorithm (e.g., Wang, et al., 2005) is used to convert the two-dimensional(2-D) rotational motion blur into one-dimensional(1-D) motion blur and obtain the gray values of the pixels along the fuzzy path. The blur angle of the rotary image is estimated by the differential autocorrelation method (e.g., Wang, et al., 2009). According to the estimated more accurate fuzzy parameters, the prior mixed-order fuzzy kernel is used to impose regularization constraints to effectively improve the anti-noise ability of the algorithm. The solution of the image restoration model in the frequency domain is derived to speed up the operation of the model and provide the possibility for its application in engineering.

2. Image Rotation Blur Principle and Degradation Model

2.1 Image Degradation Model

In the rotational motion blurred image model, assuming that the original image and the rotated blurred image are the pixel gray value of a certain arc path respectively. During the exposure time T, the scene energy will be abnormally accumulated on the imaging plane and the relationship between the blurred image $g(x,y)$ obtained by the blurred imaging system and the original image $f(x,y)$ is as follows:

$$g(x,y)=\frac{1}{T}\int_0^T f\left(x-r\cos(\omega\cdot t),y-r\sin(\omega\cdot t)\right)\mathrm{d}t \tag{1}$$

where r is the distance between the pixel and the center of rotation and $r=\sqrt{(x-x_0)^2+(y-y_0)^2}$, ω is the rotational angular velocity of the camera.

The above formula is expressed in polar coordinates as:

$$g(r,\theta)=\frac{1}{T}\int_0^T f(r,\theta-\omega t)\mathrm{d}t \tag{2}$$

Represent r in subscript form, and make $s=r\omega t$, $l=r\theta$.

$$g_r(l)=\frac{1}{N_r}\int_0^{N_r} f_r(l-s)\mathrm{d}s \tag{3}$$

To describe the model of rotational motion blurred images in the discrete case. Discretize the equation (3) as:

$$g_r(i)=\frac{1}{N_r}\sum_{x=0}^{N_r} f_r(i-x) \tag{4}$$

where N_r is the fuzzy scale, which is a positive integer. The equation (4) shows during the exposure time, the gray value of any pixel in the fuzzy path is the weighted accumulation for the gray values of this pixel and following multiple pixels, and the sum of the weights is one.

The fuzzy kernel, also known as point-spread function(PSF) is expressed as:

$$h_r(i)=\begin{cases}1/N_r & 1\le i\le N_r-1\\ 0 & N_r\le i\le N\end{cases}$$

Substitute the PSF into the equation (4) , and the convolutional form is expressed as:

$$g(i)=\sum_{x=0}^{N} h(i-x)f(x)=h(i)\otimes f(i) \tag{5}$$

where i is a pixel in the blur path and the value range of i is $[1,N]$, N is the total number of pixels in the blurred path, the symbol '$\otimes$' denotes as convolution.

Represent equation (5) in matrix form and introduce noise:

$$g=Hf+\mathbf{N} \tag{6}$$

where f and g are the matrix forms of the pixel gray value of the original image and the blurred image, $\mathbf{N}$ is the noise matrix, H is a cyclic matrix composed of PSF.

$$H=\frac{1}{N_r}\begin{bmatrix} h(1) & h(2) & h(3) & \cdots & h(N)\\ h(N) & h(1) & h(2) & \cdots & h(N-1)\\ h(N-1) & h(N) & h(1) & \cdots & h(N-2)\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ h(2) & h(3) & h(4) & \cdots & h(1)\end{bmatrix}_{N\times N}$$

where N_r is the fuzzy scale and it is a positive integer,$h(1)$~$h(N_r)$ are all one and $h(N_r+1)$~$h(N)$ are all zero.

The equation (5) and equation (6) transform the rotational blurred image restoration problem in 2-D space into a 1-D image restoration problem along the fuzzy path. For a uniformly rotated blurred image, the pixels in the same fuzzy path have the same PSF, so the blurred image can be restored by deconvolution along the fuzzy path or inverting the matrix.

2.2 Using Bresenham Circle Algorithm to Extract Fuzzy Path

Bresenham circle algorithm is an efficient circle algorithm suitable for computer applications. It only needs to judge the sign of the error term and the operation of addition and subtraction to obtain the set of pixels more accurately which are approximate the arc in the first quadrant. Taking the first quadrant as an example, assuming that the center of the image is the origin, and the coordinate of a pixel approximates the arc is (x,y) . The following arc candidate pixels are H($x+1,y$), V($x,y-1$) and D($x+1,y-1$). By solving the distances of these three points relative to the center of circle, the pixel whose distance is closest to the arc will be the desired pixel. Let the difference between the distance from the pixel to the center of the circle and the radius be $\Delta d=(x+1)^2+y^2-R^2$, so the distances need to be judged are $\Delta d'_{\mathrm{H}}=d+2x'+1$, $\Delta d'_{\mathrm{D}}=d+2x'-2y'+2$ and $\Delta d'_{\mathrm{V}}=d-2y'+1$. Finally, the first quadrant arc pixels are quickly extracted, and then combined with the symmetry of the circle, the set of pixels along the entire fuzzy path can be obtained as shown in Figure 1.

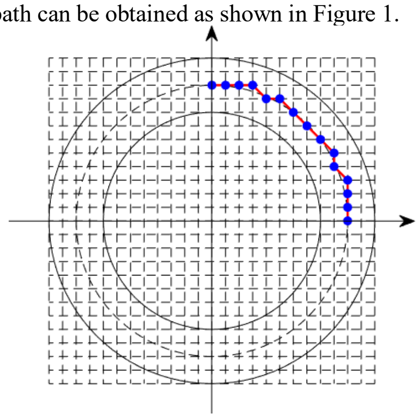


Fig.1. Trajectory diagram of Bresenham circle algorithm

Since there also are many pixels in the whole image that have not been extracted by Bresenham circle algorithm, the bilinear interpolation algorithm can be used to calculate the pixels in the image that have not been restored in order to eliminate the 'hole' points in the blurred image.

$$f(i+u,j+v)=(1-u)(1-v)\cdot f(i,j)+(1-u)v\cdot f(i,j+1) + u(1-v)\cdot f(i+1,j)+uv\cdot f(i+1,j+1)$$

where i and j both are positive integers, u and v both are floating point numbers in the interval [0, 1). The above formula shows the pixel gray value of $f(i+u,j+v)$ is determined by the surrounding four pixels such as $f(i,j)$, $f(i+1,j)$, $f(i,j+1)$ and $f(i+1,j+1)$. Since the pixels are represented by the coordinates of the upper left corner point, the pixels obtained by Bresenham circle algorithm and the bilinear interpolation algorithm are processed to be suitable for processing of the image pixel coordinates. For a specific size image, the interpolation pixels can be stored in advance, and the relevant data can be read to improve the operation speed.

3. Determine Fuzzy Scale by Autocorrelation

The fuzzy scale refers to the number of pixels in PSF that need to be weighted. Because rotational motion blur is a spatially variable blur along concentric paths, different fuzzy paths correspond to different fuzzy scales. Using Bresenham circle algorithm to extract the gray values of pixels along the fuzzy path quickly. This also means that the dimension of PSF or the order of the fuzzy circulant matrix is obtained. It is necessary to get the rotational blur angle to determine PSF or blur circulant matrix fully.

In signals and systems, the autocorrelation-function(AF) shows the degree of correlation between a random signal $x(t)$ at time t_1 and t_2.

$$R_{xx}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)x(t+\tau)\mathrm{d}t \quad (7)$$

For periodic signals, the integral averaging time T is the signal period. For finite-time signals like a single pulse, the average will tend to zero as T tends to infinity. AF can be transformed into:

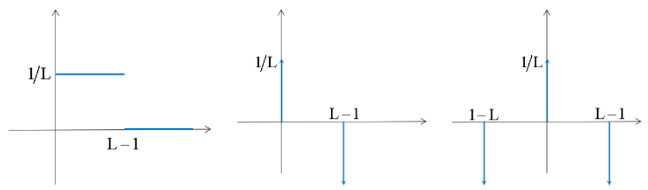
$$R_{xx}(\tau)=\int_{-\infty}^{\infty} x(t)x(t+\tau)\mathrm{d}t \quad (8)$$

The equation (8) is applied to the image to study the degree of autocorrelation of pixel gray value in the fuzzy path. Discretize it as:

$$R(i)=\sum_{x=-M}^{M} g(x)g(i+x) \quad (9)$$

where $R(*)$ is the AF, $g(*)$ is the gray values of pixels in the fuzzy path, i is the pixel in the fuzzy path and $i\in[-M,M]$. When $x\notin[1,M]$, $g(x)=0$.

The PSF, the difference function(DF) of PSF and the AF of difference function are shown in Figure 2.



(a) PSF (b) DF (c) AF

Fig.2. Function related graphics

As shown in Figure 2, the AF is even function, $R(i)=R(-i)$. When $i=0$, the AF has a maximum. Theoretically, in the curve of the image AF, the distance between the maximum point and the minimum point is the fuzzy scale. But for rotational blurred images, the blur path is a series of concentric circles. The pixels gray values of the image are integers from 0 to 255, and there also are many factors such as noise in practice. The fuzzy scale of a single circular path cannot be accurately identified by applying the AF. Therefore, in order to obtain the rotational blur scale more accurately, the blur path of the entire image is autocorrelated, and finally the overall blur scale is obtained based on statistical features.

With processing a group of random grayscale information sequences related to the image based on the fuzzy principle, 1000 simulations of fuzzy scale information are estimated. The dimension of the array, fuzzy scale and additive noise are studied on the autocorrelation method in order to estimate the effect of fuzzy scale accuracy as shown in Figure 3.

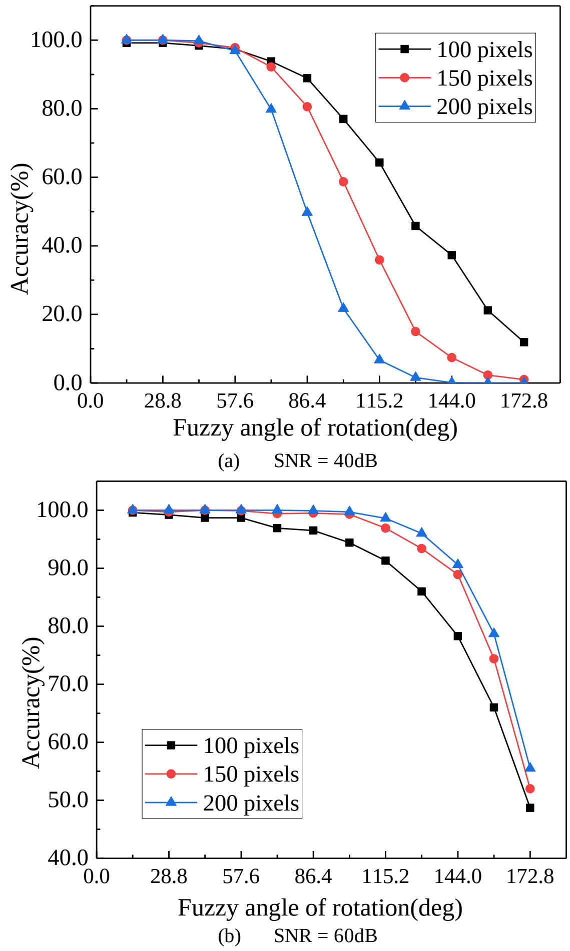


Fig.3. Accuracy of rotation blur angle determination by AF method

The randomly generated sequence of the gray values is regarded as a fuzzy path in the rotational blurred image, and the relationship between the fuzzy scale and the sequence dimension can be represented by the rotational fuzzy angle. Signal-to-noise ratio(SNR) refers to the ratio of signal energy to noise energy. Figure 3 shows that under the condition of the same SNR, the smaller the rotation blur angle is, the better the accuracy of applying the autocorrelation method is. When the rotation blur angle is not large, the more the total number of pixels are in the blur path, the better the accuracy is. For different SNR, the smaller the SNR is, the greater the impact on the discrimination of rotational blur angle is. Therefore, it is necessary to reasonably select the effective fuzzy path participating in the statistics to distinguish the blur angle of the actual rotated blurred image.

Assuming the actual rotation blur angle of the actual noise image is 30deg, convert it to radians about 0.5236, when the radius increases by two pixels, the blur scale in the blur path where it is located increases by one pixel. As the radius of the concentric circles increases, the fuzzy scale increases accordingly. At this time, the statistical information of the fuzzy scale based on the autocorrelation method becomes incomplete as shown in Table 1.

Tab.1. Autocorrelation method to determine fuzzy angle table

Actual rotation blur angle/(deg)	6	12	18	24	30	36
Actual blur radius	19.10	9.55	6.37	4.77	3.82	3.18
Image blur radius/(pixels)	20	10	7	5	4	4
Calculation/(deg)	5.73	11.87	17.73	24.29	30.72	36.12

It has been shown from Table 1 that when the blur angle is within 36deg, the fuzzy angle information obtained by the autocorrelation method is relatively accurate. Therefore, when the rotation angle does not exceed 36deg, statistical information about the rotation blur angle of the entire image can be obtained by performing autocorrelation on all blur paths of the rotated image. The error of the determined blur scale information is within 1deg.

4. Restoration Algorithm Based on Prior Mixed-Order Fuzzy Kernel

From the previous sections, the rotational blurred image restoration problem can be regarded as an inversion problem of circulant matrix. It is very strict to ensure the invertibility of the fuzzy cyclic matrix. On the contrary, as the radius increases, the ill-conditioned matrix will become more and more serious. Therefore, the key to the restoration of rotational blurred images lies in the solution of the ill-conditioned matrix.

The condition number is one of the indicators that distinguishes the ‘ill-conditioned’ of the matrix. It reflects the sensitivity of the pixel of the restored image to the blurred image in the image restoration problem. The larger the matrix condition number is, the more sensitive the restored image is. When the blurred image exists noise in practice, the image directly restored by inverse filtering will deviate far from the original image.

The method of adding a regularization term (e.g., SADA, et al., 2019) is used to improve the condition number of the circulant matrix, and the ill-posed problem of image is transformed into a well-posed problem. The circulant matrix determined by the autocorrelation method in the previous section constructs the optimal model based on the regularization method.

$$\hat{f} = \arg\min_{f}\left[\|Hf - g\|_2^2 + \lambda_1\varphi(f) + \lambda_2\rho(H)\right] \tag{10}$$

where $\hat{f}$ is the estimation of the original image, $\|Hf-g\|_2^2$ is the fidelity term for image data which is to ensure that the restored image can be consistent with the original image, $\varphi(f)$ is the regularization term for the restored image and is used to constrain the solution space of the restored image to ensure the well-posed of the problem, $\rho(H)$ is the regularization term for the circulant matrix in order to make the estimated circulant matrix more accurately reflect the degradation process of the image, λ_1 and λ_2 are the regularization parameter, all greater than 0.

The regularization term for restored image is $\varphi(f) = \|\mathrm{D}f - w\|_2^2$ and it introduces first-order gradient prior information of the image. The regularization term of circular matrix is $\rho(H) = \|\hat{H}f - Hf\|_2^2$ and it uses the prior information of the image to estimate the bias of the circulant matrix.

Substitute $\varphi(f) = \|\mathrm{D}f - w\|_2^2$ and $\rho(H) = \|\hat{H}f - Hf\|_2^2 = \|\hat{H}f - g\|_2^2$ into the equation (10).

$$\begin{aligned} T(f) = {} & f^{\mathrm{T}}\left(H^{\mathrm{T}}H + \lambda_1\mathrm{D}^{\mathrm{T}}\mathrm{D} + \lambda_2\hat{H}^{\mathrm{T}}\hat{H}\right)f - 2\left(g^{\mathrm{T}}H + \lambda_2 g^{\mathrm{T}}\hat{H}\right)f \\ & - 2\lambda_1 w^{\mathrm{T}}\mathrm{D}f + \left(g^{\mathrm{T}}g + \lambda_1 w^{\mathrm{T}}w + \lambda_2 g^{\mathrm{T}}g\right) \end{aligned} \tag{11}$$

To obtain the optimal value of the equation (11) in the constraint space, take the partial derivative of the equation (11) with respect to f, and make it equal to zero. According to the equation $\frac{\partial}{\partial x}\left(y^{\mathrm{T}}x\right) = y$ and $\frac{\partial}{\partial x}\left(x^{\mathrm{T}}\mathrm{A}x\right) = \left(\mathrm{A}^{\mathrm{T}} + \mathrm{A}\right)x$:

$$\begin{aligned} \frac{\partial T(f)}{\partial f} = {} & 2\left(H^{\mathrm{T}}H + \lambda_1\mathrm{D}^{\mathrm{T}}\mathrm{D} + \lambda_2\hat{H}^{\mathrm{T}}\hat{H}\right)f \\ & - 2\left(H^{\mathrm{T}}g + \lambda_1\mathrm{D}^{\mathrm{T}}w + \lambda_2\hat{H}^{\mathrm{T}}g\right) = 0 \end{aligned}$$

Simplify the above formula to:

$$\hat{f} = \frac{\left(H^{\mathrm{T}} + \lambda_2\hat{H}^{\mathrm{T}}\right)g + \lambda_1\mathrm{D}^{\mathrm{T}}w}{H^{\mathrm{T}}H + \lambda_1\mathrm{D}^{\mathrm{T}}\mathrm{D} + \lambda_2\hat{H}^{\mathrm{T}}\hat{H}} \tag{12}$$

where $\hat{f}$ is the estimation of the original image, g is the blurred image, w is the image first-order gradient prior, H is the fuzzy circular matrix, $\hat{H}$ is the estimation matrix, D is the first-order gradient matrix.

The first-order gradient matrix of the image is:

$$\mathrm{D} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & 0 & 0 & \cdots & 1 \end{bmatrix}_{N\times N}$$

The circulant matrix of the image prior information can be approximated by calculating the partial derivative of the objective function with respect to H and make its value zero. The objective function is expressed as:

$$J(H) = \arg\min_{H}\left[\|Hf - g\|_2^2 + \gamma_1\|H\|_2^2 + \gamma_2\|\nabla H\|_2^2\right] \tag{13}$$

where the symbol ‘ ∇ ’ denotes the first-order gradient operator, γ_1 and γ_2 are the weight coefficient of fuzzy circulant matrix estimation.

The partial derivative of $J(H)$ with respect to H is:

$$\frac{\partial J(H)}{\partial H} = \frac{\partial}{\partial H}\left(f_-^{\mathrm{T}} H^{\mathrm{T}} H f_- - 2g^{\mathrm{T}} H f_- + g^{\mathrm{T}} g\right) + \frac{\partial}{\partial H}\left[\gamma_1 H^{\mathrm{T}} H + \gamma_2 (\nabla H)^{\mathrm{T}} (\nabla H)\right] = 2 f_-^{\mathrm{T}} f_- \cdot H - 2g^{\mathrm{T}} f_- + 2\gamma_1 H + 2\gamma_2 \nabla^{\mathrm{T}} \nabla H = 0$$

The estimated bias of the fuzzy circulant matrix is:

$$\hat{H} = \frac{g^{\mathrm{T}} f_-}{f_-^{\mathrm{T}} f_- + \gamma_1 + \gamma_2 \nabla^{\mathrm{T}} \nabla} \tag{14}$$

The initial restoration model of blurred image is set as:

$$f_- = \frac{H^{\mathrm{T}} g}{H^{\mathrm{T}} H + \lambda \mathrm{D}^{\mathrm{T}} \mathrm{D}} \tag{15}$$

where λ is a parameter greater than zero. And it can be seen from the above that the blur angle estimated by the autocorrelation method has an error of ± 1 deg. H in the equation(15) is taken as the average.

Since the numerator in the equation (14) is actually a number, it can be known that $\hat{H}$ is a circulant matrix. So in the equation (12), H, $\hat{H}$ and D are circulant matrices. Due to the property that the circulant matrix can be diagonalized, the Fourier transform is performed on both sides of the equation (12), and the image deconvolution problem is turned into a frequency domain filtering problem(e.g., Wang, et al., 2021).

$$\hat{F}(u,v) = \frac{\left[\overline{H(u,v)} + \lambda_2 \overline{\hat{H}(u,v)}\right] * G(u,v) + \lambda_1 \overline{D(u,v)} * W(u,v)}{\overline{H(u,v)} * H(u,v) + \lambda_1 \overline{D(u,v)} * D(u,v) + \lambda_2 \overline{\hat{H}(u,v)} * \hat{H}(u,v)} \tag{16}$$

where $H(*)$ is the Fourier transform of H, $\hat{H}(*)$ is the Fourier transform of $\hat{H}$, $D(*)$ is the Fourier transform of D, $\hat{F}(*)$ is the Fourier transform of the restored image, $G(*)$ is the Fourier transform of the blurred image, $W(*)$ is the Fourier transform of the image prior, $\overline{(*)}$ is the complex conjugate corresponding to (*), the symbol '*' means element-wise multiplication.

The inverse Fourier transform of the equation (16) can be used to obtain the pixel grayscale sequence of the restored image:

$$\hat{f} = \mathbb{F}^{-1}\left[\hat{F}(u,v)\right] \tag{17}$$

5. Simulation Analysis

5.1 Image Restoration Evaluation Metrics

In this paper, the objective image quality evaluation method is adopted. The mean square error (MSE), peak signal-to-noise ratio (PSNR) and signal-to-noise ratio improvement factor(ISNR) are used as objective evaluation indicators to evaluate the contrast effect of blurred images before and after restoration. The definition of MSE, PSNR and ISNR are:

$$\mathrm{MSE} = \frac{1}{\mathrm{MN}} \sum_{i=1}^{\mathrm{M}} \sum_{j=1}^{\mathrm{N}} \left[f(i,j) - \hat{f}(i,j)\right]^2 \tag{18}$$

$$\mathrm{PSNR} = 10\log_{10}\left(\frac{255^2}{\mathrm{MSE}}\right) = 10\log_{10}\left\{\frac{255^2 \times \mathrm{M} \times \mathrm{N}}{\sum_{i=1}^{\mathrm{M}} \sum_{j=1}^{\mathrm{N}} \left[f(i,j) - \hat{f}(i,j)\right]^2}\right\} \tag{19}$$

$$\mathrm{ISNR} = 10\log_{10}\left\{\frac{\sum_{i=1}^{\mathrm{M}} \sum_{j=1}^{\mathrm{N}} \left[g(i,j) - f(i,j)\right]^2}{\sum_{i=1}^{\mathrm{M}} \sum_{j=1}^{\mathrm{N}} \left[f(i,j) - \hat{f}(i,j)\right]^2}\right\} = \mathrm{PSNR}_{\hat{f}} - \mathrm{PSNR}_g \tag{20}$$

where M is the number of pixels in the width and N is the height of the image, $f(i,j)$ is the grayscale value of the original image and $\hat{f}(i,j)$ is the grayscale value of the restored image at pixel coordinate (i,j).

Generally speaking, the larger the value of PSNR and ISNR (when it is greater than 0), and the smaller the value of MSE, the better the restoration effect of blurred image is. On the contrary, the image restoration effect is poor.

5.2 Simulation Results and Analysis

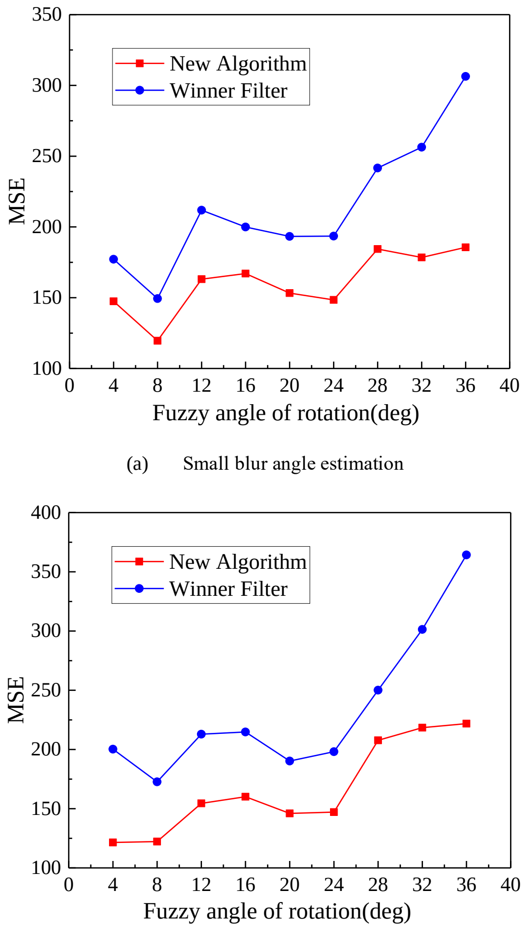
The restoration algorithm based on image prior information proposed in this paper is an image restoration method based on fuzzy path. The algorithm aims to improve the image restoration effect when the rotation blur angle parameter cannot be accurately obtained by the autocorrelation method. Experiments with the running speed of the gradient loading algorithm, Wiener filtering and the algorithm in this paper are studied. The image restoration results of three algorithms are compared under the conditions of different noise, blur scales and their inability to be accurately estimated. The above-mentioned image restoration index is used to evaluate the effect before and after image restoration. The image size used in the test is unified as 200*200 pixels, and the computer configuration is Intel(R) Core(TM) i5-11400H @ 2.70GHz, RAM16.00GB.

Table 2 in appendix 1 shows the running time of gradient loading, Wiener filtering and the new algorithm to restore the image when the blur angle is 30deg. Since the circulant matrix in the image restoration model is Fourier transformed, the image restoration in the space domain is transformed into the frequency domain. Therefore, it greatly avoids a large number of large-scale matrix operations, and improves operating efficiency, also provides a prerequisite for engineering applications.

When different rotational blur angles are accurately known, the image restoration results of each algorithm are shown in Table 3 which is in appendix 1. When the noise is different and the blur angle is 12deg, the image restoration results are shown in Table 4 that is in appendix 2. It can be seen from Table 3 and Table 4 that as the blur angle and the proportion of noise energy in the image increases, the ringing-effect of the gradient loading algorithm becomes more obvious, and the image restoration effect becomes worse. When SNR is 20dB, the image restoration effect is extremely poor, and it is almost impossible to accurately identify the target. When the rotation blur angle is accurately known, the image restoration effect of the Wiener filtering algorithm and new algorithm is not much different, and the ringing-effect is weakened. Compared with the Wiener filtering algorithm, the new algorithm is insensitive to the changes of noise.

In appendix 3, When the actual rotation blur angle of the image is

32deg, the image restoration results for three possible blur angle estimates obtained by autocorrelation are shown in Table 5. From the intuitive visual effect of the image, it can be seen that the algorithm in this paper has better visual effect after image restoration, and there is no obvious arc-shaped trajectory near the center of the image. From the objective evaluation index of the image, it can also be obtained that the algorithm in this paper is generally better than the Wiener filtering. When the estimated blur angle does not match the actual angle, there is an error of the cyclic matrix in the restoration model, which will cause the restoration effect of each blur path to deteriorate, the closer it is to the center of the image, the more obvious it is. Because the order of the image blur matrix will change with the number of pixels in the blur path, the smaller the number of pixels is, the larger the element error is in the circulant matrix caused by the blur angle estimation error. Moreover, when the blur angle estimation is inaccurate, there will be obvious ringing-effect at the edges of the image.

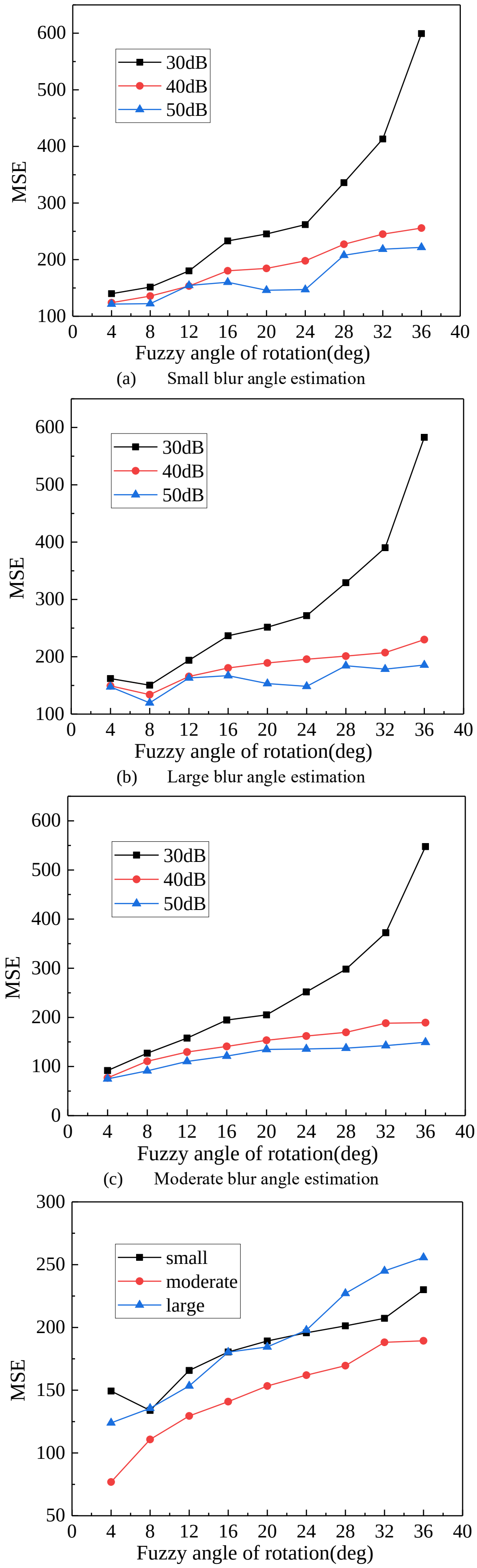


(a) Small blur angle estimation

(b) Large blur angle estimation

Fig.4. MSE values of two algorithms for estimation of different blur angles

As shown in Figure 4, in the case of different actual rotation fuzzy angles, no matter the blur angle estimated by the autocorrelation method is larger or smaller than the exact blur angle. Under the premise of a given filter coefficient, compared with the image restoration effect of the Wiener filtering algorithm, the image restoration result of the new algorithm is better. Comprehensively comparing Figures (a) and (b), it can be seen that when the actual blur angle is less than 24deg, it is better to estimate a larger blur angle than a smaller one. Conversely, when the actual blur angle is greater than 24deg, a smaller blur angle is estimated to get a better restoration image than a larger one.



(a) Small blur angle estimation

(b) Large blur angle estimation

(c) Moderate blur angle estimation

(d) SNR=40dB

Fig.5. MSE values of image restoration under different blur angle estimation and noise conditions

The sub-figures (a), (b), and (c) in Figure 5 show the MSE values of different noise image restoration evaluation parameters when the new algorithm has errors in the estimated blur angle. The larger the proportion of noise is in the blurred image, the worse the image restoration effect is. When the noise is 40dB or 50dB, the MSE value of the image restoration result is not much different. When the blur angle estimation is accurate, as the blur angle increases, the image restoration MSE value also increases, and the image restoration effect becomes worse. Sub-figure (d) in Figure 5 shows the image restoration result when the blur angle of the 40dB noise image is inaccurate. The results show that the estimated value when the blur angle is 8~24deg under the condition of error ±1 deg has little effect on the image restoration. But the larger the estimated value is, the better the restoration effect is. Conversely, when the blur angle is greater than 24deg, the smaller the estimated value is, the better the image recovery is. From sub-figures (a) and (d), it can be seen that when the angle estimate is small, the image recovery is worse relative to the precise image blur angle for small rotation blur angles. Because of the small rotation blur angle, the blur path of the original clear image is farther away from the center of the image. If the estimated value of the angle is smaller, the blur path of applying the image restoration model will be farther, which will affect the restoration effect of the image instead.

6.Conclusion

In this paper, the restoration of spatially radially variable blurred images is studied. The Bresenham circle algorithm is used to extract the gray value of the pixels on the blur path of the image, and 2-D space variable blur is converted into 1-D space invariant blur. On the premise that the blur angle parameter cannot be accurately estimated by the autocorrelation method, an image prior regularization constraint restoration algorithm based on mixed-order fuzzy kernel estimation is proposed. Compared with the gradient loading restoration algorithm and the Wiener filtering algorithm, the comparative simulation experiments under the conditions of different noise and rotation blur angles are carried out. The results of the comparative analysis show that the rotational motion blurred image restoration algorithm proposed in this paper has good anti-noise ability, strong robustness, and can adapt to large rotational blur angles. The image deconvolution problem in the spatial domain is converted into an image filtering problem in the frequency domain, which greatly reduces the operation time. The algorithm has better real-time performance and restoration effect, and also has certain engineering application value.

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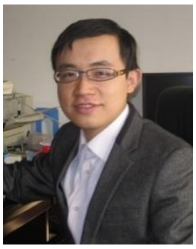

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Appendix 1

Tab.2. Image restoration running time of each algorithm

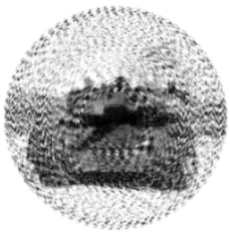
algorithm	Gradient loading	Wiener filter	New algorithm
Restore			
Time/(s)	0.0273	0.0289	0.0315

Tab.3.Image restoration effect of each algorithm under fifferent accurate blur angles

blur angle /(deg)	Image Restoration Algorithm Based on Fuzzy Path		
	Gradient loading	Wiener filter	New algorithm
12			
24			
36			

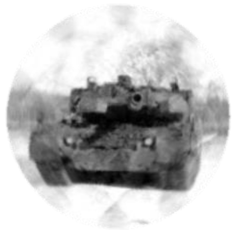
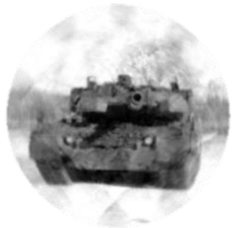
Appendix 2

Tab.4. Image restoration effect of each algorithm under different noise conditions

SNR /(dB)	Image Restoration Algorithm Based on Fuzzy Path		
	Gradient loading	Gradient loading	New algorithm
20			
30			
40			
50			

Appendix 3

Tab.5. Image Restoration Effect of Two Algorithms
in Different Blur Angle Estimation

blur angle	Exact	32deg		
	Estimated	31.2838deg	32.0deg	32.9139deg
Image restoration effect and quality evaluation	Wiener filter			
	PSNR/ISNR	24.04/7.13	24.69/7.77	23.34/6.42
	New algorithm			
	PSNR/ISNR	25.62/8.70	26.59/9.68	24.74/7.82