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Research and Improvement of Adaptive Filtering Algorithm

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ABSTRACT

Based on the adaptive filtering principles, several typical adaptive filtering algorithms and its application were introduced. The performance characteristics of the algorithm was analyzed and compared, and these performance characteristics were analyzed based on adaptive filtering algorithm. In response to noise processing method, the LMS adaptive filtering algorithm under the minimum mean square criterion was carried out. The adaptive filter can meet some best requirements of the guidelines by automatically adjusting the parameters of itself. In order to further improve the quality of stereo sound reproduction system, the new algorithm improved the convergence factor on the basis of traditional least mean square algorithm to make it change depending on the input signal, and avoid the contradiction between convergence and steady state error. At last, verifies the effectiveness of the algorithm by MATLAB simulation.

1. Introduction

Adaptive filtering refers to a filter that automatically adjusts system parameters according to changes in the external environment and uses the characteristics of the adaptive algorithm itself to make the entire system work in the best state. It can be seen from this feature that the advantage of adaptive filtering is that it can automatically adjust the coefficients of the system itself without manual control and adjustment without knowing the statistical characteristics of the input, so that the system can reach the best state.

When designing a filter, it is necessary to consider the selection of the structure, and to determine whether the filtering algorithm is suitable. Adaptive filtering algorithm is a search process in which the filter automatically adjusts its own coefficients to make the result reach the best state. In the search process, the objective function, error function and search method need to be defined. The selection of the search method is the process of selecting the algorithm, choosing the appropriate algorithm so that the whole system can achieve the effect of not only fast convergence speed but also low complexity.

A basic and important technology in signal processing is filtering, and digital filtering is more widely used than analog filtering, so the research on digital filtering has never stopped, and it has attracted much attention at home and abroad. There are many types of filtering, but they can be divided into traditional filtering and modern filtering according to the chronological order of research. The disadvantage of traditional filtering compared with modern filtering is that it requires prior knowledge of noise and known signals, such as Wiener filtering and Kalman filtering, and modern filtering does not need to know their statistical characteristics in advance after being improved. Adaptive filtering, its research began in the 1950s, compared with Wiener filtering and Kalman filtering, it has many advantages. When the statistical properties of unknown noise and known signals or their statistical properties change, the adaptive filter can automatically adjust its parameters to meet some criteria and make the system reach the optimal state. Adaptive filtering has relatively strong self-tracking and self-learning capabilities, and is suitable for estimating or detecting stationary and non-stationary random signals. Under a certain criterion, the adaptive filtering is meaningful, and the selection of the criterion is different, and the adaptive algorithm will be different. The LMS adaptive filtering algorithm is studied under the least mean square criterion, and the research scope is very wide.

2. Introduction to Adaptive Filters

The original concept of self-adaptation is derived from the survival of organisms in the environment, and the process of adapting to the environment in an effective way, thus making the vitality stronger is called self-adaptation. The concept of self-adaptation mentioned now mainly refers to the field of signal processing. In a system with

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variable structure or adjustable structure, change the relationship between itself and the external system to change the way it processes signals. This system is often nonlinear and time-varying, and can adapt to the signal transmission environment by adjusting its own weight coefficients. It is not necessary to know the structure and characteristics of the input signal, and it is not necessary to spend too much effort on the design of the system itself. The adaptive system can realize its nonlinear characteristics by adjusting its own parameters in different environments, and realize the time-varying characteristics of the adaptive system through its adaptive response or adaptive learning process. The linear adaptive system is formed when the adaptive process is no longer carried out. This type of system is simple and convenient in mathematical processing, so it is widely used. The most widely used and fastest growing digital filter is the adaptive filter, which is simple in design and best in performance, and is the most practical and meaningful in the field of digital filters.

Wiener was the first to use the least mean square criterion to design the optimal linear filter, which is mainly used to eliminate noise, predict or smooth stationary random signals. Kalman et al. developed and reasoned the theory of optimal time-varying linear filter design for non-stationary random signals. Predicting the statistical characteristics of signal and noise is the basis of Kalman and Wiener filter research. Therefore, an ideal filter can only be designed when the statistical characteristics based on the filter design coincide with the statistical characteristics of the actual input signal of the system. Otherwise, the best performance of the filter cannot be obtained. Widrow and others proposed adaptive filter and its algorithm, and developed the optimal filter design theory.

Along with the rapid development of the mobile communication industry, the application fields of adaptive filtering are becoming wider and wider. Wiener filtering is to design a filter based on the correlation characteristics and power spectrum of the input signal and the noise signal, and the principle based on it is the minimum mean square error estimation criterion. The Wiener filter can effectively extract the useful signal in the mixed signal and eliminate the interference signal, but when the characteristics of the input signal change and deviate from the design conditions, this method is not optimal. In practice applications, this limitation cannot be ignored. The Kalman filtering method is researched with the development of space technology, and it is widely used in many fields. Because it uses the model to make the optimal estimation of the sequence, it can be used for non-stationary, stationary, multi-input and multi-output The signal is used for nonlinear and linear filtering. In fact, Kalman filtering includes Wiener filtering, and Wiener filtering is just a special kind of Kalman filtering. As mentioned earlier, Kalman filtering also requires the statistical characteristics of the input signal and the interference signal to be known, that is, to understand the state equation of the input signal, but in practical applications, this situation is difficult to achieve, without the method counts the prior knowledge of the two signals. Therefore, this method is gradually eliminated, and another filtering method is adopted. Adaptive filtering in modern filtering can improve the parameters of the filter itself according to the environment in which it is located, so that the system can reach the best state, which overcomes the disadvantages of needing to know the statistical characteristics of the input signal and noise signal. However, adaptive filtering is more difficult to implement than Wiener filtering and Kalman filtering, but in terms of filter performance, it is not worse than classical filtering. Therefore, these two points determine that the application range of adaptive filtering is getting wider and wider.

Compared with Wiener filtering and Kalman filtering, adaptive filtering retains their advantages and overcomes their shortcomings. It is the best filtering method developed on the basis of Kalman filtering and Wiener filtering. It is better than Kalman filtering and Wiener filtering in terms of filtering performance, and has strong adaptability. In practical applications, it is more meaningful. It is widely used in the field of engineering and information processing. The characteristic that adaptive filtering can automatically adjust the filter weight coefficients makes it possible to study systems with uncertain information. The so-called uncertainty means that the prior knowledge of the signal cannot be known, the specific process of information processing cannot be determined, the structure of the model cannot be determined, and the unknown factors and random factors cannot be determined. Uncertainty is inevitable in every actual system, and the uncertainty of the information process may sometimes appear inside the system, or it may appear outside the system. If it is reflected in the system, it means that the mathematical model of the information process, system parameters, and mathematical structure are uncertain; if it is reflected in the outside of the system, it means that there are disturbances outside the system, and these disturbances can be random. It can also be unmeasurable. There are also external noises that will affect the information process of the system in different forms. Often the prior knowledge of these noises and disturbances cannot be measured. These objective uncertainties are very complicated to deal with. How to remove these disturbances and how to make the system The index is ideal and optimal, which is the next step of adaptive filtering.

Because adaptive filtering has many advantages compared with traditional Kalman filtering and Wiener filtering, such as excellent performance, simple implementation, good adaptability to the external environment, no need to know the statistical characteristics of the input signal and noise signal, strong Self-learning, self-tracking ability, algorithm is not complicated and easy to implement, etc. These advantages make adaptive filtering develop rapidly in the field of practical engineering. Adaptive filtering can be applied in signal enhancement, system identification, signal prediction, noise cancellation, signal equalization, etc. The following is a brief introduction to its application:

(1)Signal Enhancer Signal enhancement is the simplest application of adaptive filter. When narrowband signal and broadband noise are mixed together, the function of signal enhancer is to detect or enhance the narrowband signal. The useful signal is interfered by the noise signal, and a noise signal is known, the noise signal and the noise mixed in the useful signal are correlated and can be measured. If the noise signal is used as the input of the adaptive filter, and the useful signal polluted by the noise is used as the expected value of the system, then when the filter converges, the output error of the filter is the enhanced form of the signal. In addition to the estimation module, the signal booster should also include a delay module. The role of the estimation module is to use its output terminal to output the enhanced signal. Its principle is a finite impulse response filter, and the coefficient of this filter is adjustable. The function of the delay is to remove the noise signal in the input signal. The best predictive coefficients can be solved by the least mean method.

(2) An important application of the system identifier for adaptive filtering is system identification. The system is unknown. When the unknown system is stimulated by a broadband signal, it generates an output. This output is the desired signal. The broadband signal can sometimes be used as an adaptive filter. device input. In the actual

application system, we do not need to know much about the internal structure of the system, but only need to know the input characteristics and output characteristics of the system. The input of the system can be multiple or one, and the output is also the same. System identification is an extremely important issue in communication systems. The essence of system identification is the process of judging the internal characteristics of the system or the transfer function of the system through the input characteristics and output characteristics of the system.

System identification and modeling have a wide range of applications and are of great significance in many fields, such as communication technology, control engineering, digital signal processing, and so on. In addition to these domestic engineering fields, it can also be applied to biological, economic, and social systems. Closely related to this paper are modeling and system identification problems in the fields of communication and digital signal processing. The model of the communication channel uses a filter to identify the communication channel with an adaptive system identification method, so that the entire channel can also be equalized.

The communication channel is understood as a box whose interior is invisible, the input and output of the box can be measured, and the model of the box is replaced by an adaptive filter, so that the input and output of the filter are the same as the box. The adaptive filter can adjust its own parameters so that its output matches the output of the box, so that the characteristics of the unknown system can be approximated by using the characteristics of the filter. Although, in a strict sense, the structure of the filter is not exactly the same as the real system communication channel, but from the perspective of input characteristics and output response, it is consistent, so we can regard the adaptive filter as this unknown box system model. Since the adaptive filter has many adjustable parameters, it can simulate this unknown box to any degree.

When building a model for an unknown system, you can mainly consider these three aspects: first, choose the appropriate model structure and order; second, estimate the parameters in the model; finally, check whether the performance of the built model is as expected Same. If not, re-select the model and order.

(3)When the signal predictor predicts the system, the desired signal can be used as the front-end input of the adaptive filter (or as the back-end input). When the filter converges, the signal model of the input signal can be used by the adaptive filter To represent. When the predictor is applied to speech signal coding prediction, its function is to estimate the parameters in the signal. Adaptive line spectrum enhancement is also an application of signal prediction. The predicted narrowband signal and wideband signal are mixed and used as the input of the signal. After adaptive processing, the enhanced narrowband signal is output. Signal predictors can also be applied in suppressing noise.

(4)Noise canceller In the field of digital signal processing and communication applications, the required signal is often doped with noise and interference signals, which will affect the output of the signal and the reliability of the signal, which will cause unnecessary errors. The adaptive filter is to remove the noise and interference in the original signal, detect and estimate the original signal, and restore the original signal as much as possible. For example, Wiener filtering and Kalman filtering mentioned in the previous section. Wiener filtering and Kalman filtering belong to the category of classical filtering, while adaptive filtering belongs to the category of modern filtering. As mentioned earlier, classical filtering requires prior knowledge of known signals and noise signals, while adaptive filtering does not need to know or need to know little about the statistical characteristics of signals and noise.

The adaptive noise cancellation system is transformed from the adaptive optimal filter, which was originally the research result of Stanford University in the United States. The principle of adaptive noise cancellation is very simple. It is to compare the input signal (including useful signal and interference signal) with the expected signal, and remove the interference signal in the input signal through the adaptive cancellation method. But the limitation is that the correlation between the reference signal and the noise in the signal needs to be known, and it cannot be correlated with the signal to be detected. In most cases, it is more difficult to eliminate the noise doped in the useful signal, because while eliminating the noise, the useful signal may also be eliminated. However, the adaptive noise cancellation system can ideally eliminate the noise mixed with the useful signal through the parameter adjustment and control of the adaptive system.

(5)The application of the signal equalizer adaptive filter can also be reflected in the signal equalization. The signal equalizer can generate a transfer function, which is the inverse function of the channel transfer function, and the gain outside the passband is zero. Therefore, the system including the channel and the equalizer has a uniform amplitude within the passband and zero outside the passband. When applied to channel equalization, the original signal is affected by channel distortion before being input to the adaptive filter, and the delayed original signal is used as the expected signal of the adaptive filter, and the delayed original signal can be received at the receiving end measured out. When the minimum mean square error is minimized, this adaptive filter can replace the equalizer of the channel.

3. Principle of Adaptive Filtering

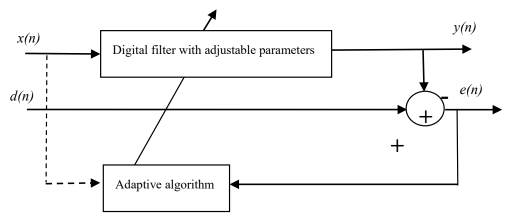

Fig. 1. General structure of adaptive filter.

The structure diagram of the adaptive filter is shown in Figure 1 above. It can be seen from the figure that the most important part is the parameter adjustable digital filter and adaptive algorithm. There are two types of digital filters, one is a finite impulse response filter and the other is an infinite impulse response filter. The input signal is represented by x(n) , and the input signal enters the digital filter with adjustable parameters to obtain the output response, which is represented by y(n) . The desired signal is represented by d(n) , the desired signal and the output signal are superimposed, and the difference between the two is obtained, that is, the error signal is represented by e(n) . If the value of the error signal is not ideal, its value can be adjusted through the adaptive algorithm of the input signal, and the ultimate goal is to minimize the value of the error signal. This is what we mentioned earlier. The adaptive filter can automatically adjust its own parameters. It is a special Wiener filter. The Wiener filter has to know the statistical characteristics of the input signal and the interference signal in advance, while the adaptive filter does not need to know in advance, it estimates the statistical characteristics it needs in the process of adjusting its own parameters, and continues to adjust Its own parameters in order to achieve the purpose of the minimum mean square value. When the statistical characteristics of the input signal or the interference signal change, it can automatically adjust the weight coefficient to make the effect of the filter reach the best.

3.1 Least Mean Square Algorithm Principle

Among the adaptive filters, the FIR transversal filter is more commonly used. The structure diagram of the transversal filter is shown in Figure 2 below.

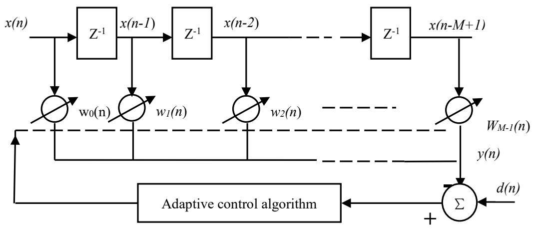

Fig. 2. Structure block diagram of transversal filter.

In the figure

$x(n) = [x(n) \quad x(n-1) \quad \cdots \quad x(n-M+1)]^T$—the vector of the input signal at time n;

$y(n)$ ——vector of output signals;

$d(n)$ ——expected signal;

$e(n)$ ——error signal;

M——filter;

$w(n) = [w_1(n) \quad w_2(n) \quad \cdots \quad w_{M-1}(n)]^T$- vector of filter weighting coefficients.

$$\begin{aligned} y(n) &= w^T(n) * x(n) \\ &= \sum_{i=0}^{M-1} w_i(n) x(n-i) \end{aligned} \tag{1}$$

The difference between the desired signal and the output signal is the error signal, expressed as:

$$e(n) = d(n) - y(n) \tag{2}$$

Express the mean square error as:

$$\alpha(n) = E[e^2(n)] = E[d^2(n) - 2d(n)y(n) + y^2(n)] \tag{3}$$

The above formula (3) is one of the most commonly used objective functions, the mean square error (MSE), and the error expression is brought into the mean square error expression as:

$$\begin{aligned} \alpha(n) &= E[e^2(n)] \\ &= E[d^2(n)] - 2E[d(n) w^T(n) * x(n)] \\ &\quad + E[w^T(n) x(n) x^T(n) w(n)] \end{aligned} \tag{4}$$

For expressive convenience, set:

$$R = E[x(n)x^T(n)] \; R = \mathrm{E}[x(n) x^T(n)] \tag{5}$$

Expressed as an input signal autocorrelation matrix, defined as:

$$R = \mathrm{E}[x(n) x^T(n)]$$

$$= \begin{bmatrix} m_{xx}(0) & m_{xx}(1) & \cdots & m_{xx}(M-1) \\ m_{xx}(1) & m_{xx}(0) & \cdots & m_{xx}(M-2) \\ \vdots & \vdots & & \vdots \\ m_{xx}(M-1) & m_{xx}(M-2) & \cdots & m_{xx}(0) \end{bmatrix} \tag{6}$$

The elements in the matrix are autocorrelation functions. set up:

$$P = \mathrm{E}[d(n)x(n)] = \left[m_{dx}(0) \quad m_{dx}(1) \quad \cdots \quad m_{dx}(M-1) \right]^T \tag{7}$$

The elements in the matrix are autocorrelation functions. From the mean square error expression, it can be seen that for Quadratic function, which means that the mean square error is a quadratic function of the filter weighting coefficients. When the filter coefficient is a certain value, the mean square error expression can be simplified as follows:

$$\alpha(n) = [d^2(n)] - 2w^T(n)P + w^T(n)Rw(n) \tag{8}$$

If the parameters R and P of the matrix are known vectors, then the mean square error can be obtained from the weighting coefficients of the filter. From the extreme value theorem in mathematics, it can be known that the mean square error function is derived, the independent variable is taken, and the first derivative is zero. If R is a non-singular matrix, the optimal value when the minimum value can be obtained can be obtained. At this time:

$$\omega_0 = R^{-1}P \tag{9}$$

Equation (9) is called the optimal (Wiener) solution.

Expressed in matrix as:

$$\begin{bmatrix} w_0^* \\ w_1^* \\ w_2^* \\ w_3^* \end{bmatrix} = \begin{bmatrix} m_{xx}(0) & m_{xx}(1) & \cdots & m_{xx}(M-1) \\ m_{xx}(1) & m_{xx}(0) & \cdots & m_{xx}(M-2) \\ \vdots & \vdots & & \vdots \\ m_{xx}(M-1) & m_{xx}(M-2) & \cdots & m_{xx}(0) \end{bmatrix}^{-1} \begin{bmatrix} m_{dx}(0) \\ m_{dx}(1) \\ \vdots \\ m_{dx}(M-1) \end{bmatrix} \tag{10}$$

When the optimal solution is obtained, the mean square error is:

$$\begin{aligned} \alpha_{\min} &= \mathrm{E}[d^2(n)] - 2w^T(n)P + w^T(n)Rw(n) \\ &= d^2(n) - 2w_0^{\ T}(n)Rw_0(n) + w_0^{\ T}(n)Rw_0(n) \\ &= d^2(n) - w_0^{\ T}(n)Rw_0(n) \end{aligned} \tag{11}$$

The general form of the steepest descent algorithm is:

$$w(k+1) = w(k) - ug(k) \tag{12}$$

where $g(k)$ is the gradient vector at time n .

According to formula (12) to adjust the coefficient of the filter, $n+1$ the full vector of the filter at any time $w(n+1)$can be expressed as:

$$w(n+1) = w(n) - ug(n) \tag{13}$$

Among them, u is a positive real number, called the step size. And because the gradient can be expressed as follows:

$$g(k) = \frac{\partial E[e^2(n)]}{\partial w(n)} = E[2e(n)\frac{\partial e(n)}{\partial w(n)}] = -E[2e(n)x(n)] \tag{14}$$

From the formula (4), it can be seen $\alpha(n)$that $w(n)$the quadratic function of is, for $w(n)$partial derivative:

$$g(k) = -2P + 2Rw(n) \tag{15}$$

Put formula (15) into formula (13) to get:

$$w(n+1) = w(n) + 2u[P - Rw(n)] \tag{16}$$

3.2 An Improvement to the Least Mean Square Algorithm

In most cases, the exact values of the input signal and the desired signal are difficult to obtain, and the calculation is also relatively complicated. Therefore, we can introduce the estimated value instead, so that the least mean square algorithm can be obtained. $w(n+1)$is to adjust the corresponding increment $w(n)$according to $\alpha(n)$the negative slope of the performance plane:

$$w(k+1) = w(k) - u\hat{g}(k) \tag{17}$$

Formula (17) can be expressed as $\hat{g}(k)$:

$$\hat{g}(k) = \frac{\partial E[e^2(n)]}{\partial w(n)} = -2E[e(n)x(n)] \tag{18}$$

Substituting the estimated value in formula (18), we can get:

$$w(n+1) = w(n) + 2ue(n)x(n) \tag{19}$$

Since the convergence and stability of the least mean square algorithm are closely related to the input of the filter, considering this weakness, a new step size algorithm is proposed, and the step size is expressed as:

$$u^* = \frac{u}{\delta^2}\, u^* = \frac{u}{\delta^2} \tag{20}$$

$\delta^2$is the variance of $x(n)$ in Formula (20). Estimate the above formula (20) with the average value of timeδ^2, then:

$$\hat{\delta}^2 = \sum_{t=0}^{M} x^2(n-i) = x^T(n)x(n) \tag{21}$$

Then the iterative equation of the least mean square algorithm is updated as:

$$\begin{aligned} w(n+1) &= w(n) + 2\frac{u}{\delta^2} e(n)x(n) \\ &= w(n) + 2\frac{u}{x^T(n)x(n)} e(n)x(n) \end{aligned} \tag{22}$$

It can be seen from the above formula (22) that if the denominator in the step size is $x^T(n)x(n)$very small, the step size will become very large. Therefore, in order to avoid this situation, a normal constant is added to the denominatorθ. Then the iterative equation is updated as:

$$w(n+1) = w(n) + 2\frac{u}{\theta + x^T(n)x(n)} e(n)x(n) \tag{23}$$

3.3 Least mean square algorithm convergence

For example, suppose an unknown filter coefficient is , use an unknown adaptive filter to identify, this filter has the same order as the unknown filter, and adopts the least mean method. White noise is added to the unknown output.

difference from ideal : w_0

$$\Delta w(k) = w(k) - w_0 \tag{24}$$

The least mean method in the previous section can be expressed as:

$$\Delta w(k+1) = \Delta w(k) + 2ue(k)x(k) \tag{25}$$

in:

$$e(k) = x^T(k)w_0 + n(k) - x^T(k)w(k) \tag{26}$$

The optimal error corresponding to the optimal solution is: w_0

$$\begin{aligned} e_0(k) &= d(k) - w_0^{\ T} x(k) \\ &= w_0^{\ T} x(k) + n(k) - w_0^{\ T} x(k) \\ &= n(k) \end{aligned} \tag{27}$$

Bring formula (26), formula (27) into formula (25):

$$\begin{aligned} \Delta w(k+1) &= \Delta w(k) + 2ux(k)[e_0(k) - x^T(k)\Delta w(k)] \\ &= [I - 2ux(k)x^T(k)]\Delta w(k) + 2ue_0(k)x(k) \end{aligned} \tag{28}$$

Find the expectation of formula (28):

$$\begin{aligned} E[\Delta w(k+1)] &= E\{[I - 2ux(k)x^T(k)]\Delta w(k)\} \\ &\quad +2uE[e_0(k)x(k)] \end{aligned} \tag{29}$$

If$e_0(k)$and$x(k)$are independent of each other, then$E[e_0(k)x(k)] = 0$. Formula (29) can be simplified as:

$$\begin{aligned} E[\Delta w(k+1)] &= \{I - 2uE[x(k)x^T(k)]\}E[\Delta w(k)] \\ &= (I - 2uR)E[\Delta w(k)] \end{aligned} \tag{30}$$

It can be known from the assumption that the elements of the optimal error signal and the input signal vector are orthogonal. Then formula (30) can be expressed as:

$$E[\Delta w(k+1)] = (I - 2uR)^{k+1}E[\Delta w(0)] \tag{31}$$

Multiply the formula (31) to the leftG^T, here$G$is a unitary matrix, which can be transformed R into a diagonal matrix after similar transformation:

$$E[G^T\Delta w(k+1)] = (I - 2uG^TRG)E[G^T\Delta w(k)] \tag{32}$$

Order$\Delta w'(k+1) = G^T\Delta w(k+1)\ \Delta w'(k+1) = G^T\Delta w(k+1)$,

the above formula is expressed as:

$$\begin{aligned} \mathrm{E}[G^T\Delta w(k+1)] &= (I - 2u\Lambda)\mathrm{E}[\Delta w'(k)] \\ &= \begin{bmatrix} I-2u\lambda_0 & 0 & \cdots & 0 \\ 0 & I-2u\lambda_1 & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I-2u\lambda_N \end{bmatrix} \mathrm{E}[\Delta w'(k)] \end{aligned} \tag{33}$$

In order for the coefficients to converge, then the absolute value of the coefficients in the matrix$I - 2u\lambda_i$should be less than 1. The convergence factor of the least mean square algorithm should be guaranteed to be within this range:

$$0 < u < \frac{1}{\lambda_{max}}\ 0 < u < \frac{1}{\lambda_{\max}} \tag{34}$$

where$\lambda_{max}$is the largest eigenvalue of R.

But for formula (20), its convergence range will be different from the above formula.

When the iteration equation is:

$$w(n+1) = w(n) + 2u^*e(n)x(n) \tag{35}$$

it can be understood that the change of the weight coefficient is$\Delta\widetilde{w}(n)$given, then the iterative equation can be written as:

$$w(n+1) = \widetilde{w}(n) = w(n) + \Delta\widetilde{w}(n) \tag{36}$$

The square of the error for the instantaneous value is:

$$e^2(n) = [d(n) - w^T(n)x(n)]^2 \tag{37}$$

After changing, the square of the error is:

$$\begin{aligned}\tilde{e}^2(n) &= [d(n) - \tilde{w}^T(n)x(n)]^2 \\ &= \{d(n) - [w(n) + \Delta\tilde{w}(n)]x(n)\}^2\end{aligned} \tag{38}$$

The goal is to minimize the square of the instantaneous error, so:

$$\begin{aligned}\Delta e^2(n) &= \tilde{e}^2(n) - e^2(n) \\ &= -2[d(n)\Delta\tilde{w}^T(n)x(n) \\ &\quad -w(n)\Delta\tilde{w}(n)x^2(n)] \\ &\quad +\Delta\tilde{w}^T(n)x(n)x^T(n)\Delta\tilde{w}(n) \\ &= -2\Delta\tilde{w}^T(n)x(n)e(n) \\ &\quad +\Delta\tilde{w}^T(n)x(n)x^T(n)\Delta\tilde{w}(n)\end{aligned} \tag{39}$$

because:

$$\Delta\tilde{w}(n) = 2u^*e(n)x(n)\ \Delta\tilde{w}(n) = 2u^*e(n)x(n) \tag{40}$$

Putting this formula into formula (39), we can get:

$$\Delta e^2(n) = -4u^*e^2(n)x^T(n)x(n) + 4u^{*2}e^2(n)[x^T(n)x(n)]^2 \tag{41}$$

It can be known from the extreme value theorem of mathematics that at that time, $u^* = \frac{1}{2x^T(n)x(n)}$ $u^* = \frac{1}{2x^T(n)x(n)}\Delta e^2(n)$, $\Delta e^2(n)$ the minimum point is obtained.

Due to the range of convergence discussed earlier u, this theory can be easily obtained:

$$0 < u = u^*/2 < 1 \tag{42}$$

$$0 < u^* < 2 \tag{43}$$

4. Algorithm simulation

Through the theoretical understanding of the least mean method, we use the algorithm in the application of adaptive noise removal, use Matlab as a simulation tool, compare the effects of the adaptive filter output and the expected output of the two algorithms, and express the error of the two In the picture.

(1) Input signal

$$x = \sin(2\pi t/lengt)$$

Input signal after adding noise:

$$d = awgn(x, snr)$$

(2) The noise can be easily removed from the input signal by using the least average method. In Figure 3, the upper waveform is the original input sinusoidal signal, and the lower waveform is the input signal waveform after adding noise interference. Figures 4 are graphs comparing the original algorithm filter output with the expected output. Figure 5 is a comparison graph between the filter output of the improved step size algorithm and the expected output, where the red line indicates the error between the two. From the red error curve, it can be clearly seen that the error of the algorithm after the improved step size is much smaller than that of the original algorithm.

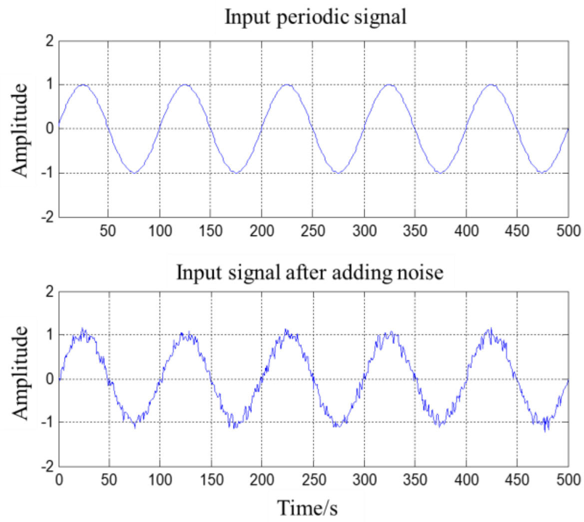

Fig. 3. Simulation diagram of input signal waveform and noise interference waveform

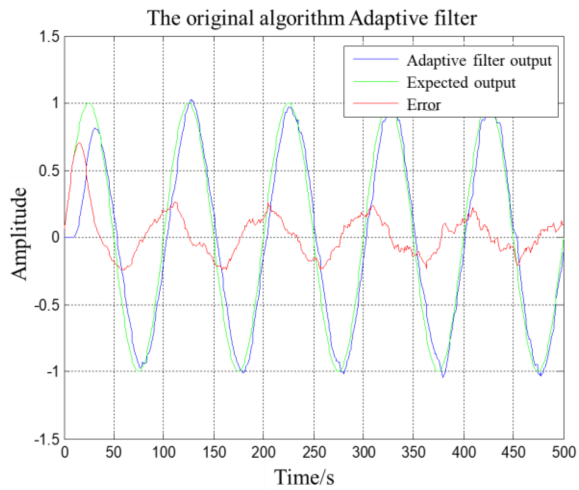

Fig. 4. Comparison chart of original algorithm adaptive filter output and expected output

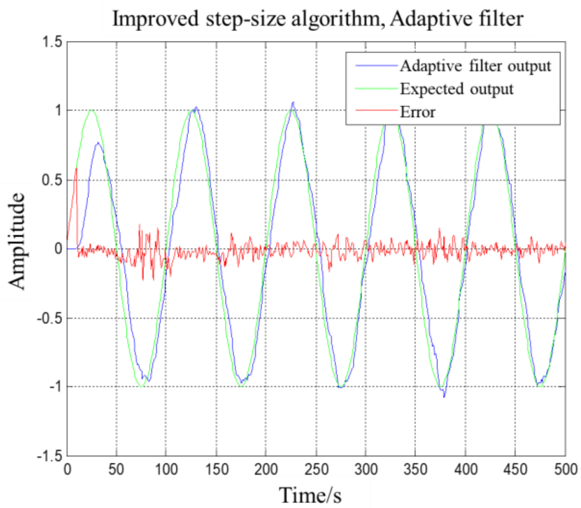

Fig. 5. The comparison chart of the adaptive filter output and the expected output of the improved step size algorithm

(3) Figure 6 is the mean square error curve of the original algorithm.

Figure 7 is the mean square error curve of the algorithm after changing the step size.

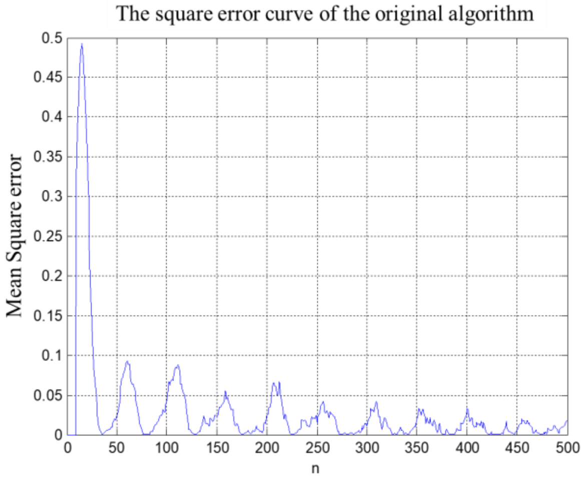

Fig. 6. Original algorithm error square curve

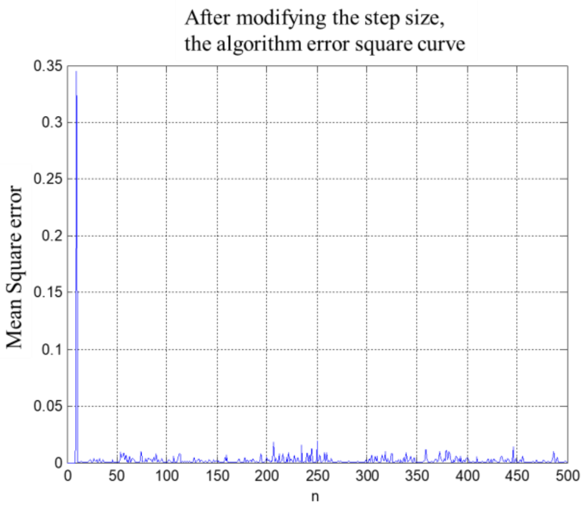

Fig. 7. Algorithm error square curve graph after modifying the step size

Figure 6 and Figure 7 describe the curves of the original algorithm and the improved new algorithm. Through the comparison of the two figures, we can see that the steady-state error and convergence speed have been significantly improved, and the performance of the new algorithm is better than that of the original algorithm. has been greatly improved.

5. Summary

In this chapter, based on the principle of adaptive filtering, the noise mixed in the input signal is filtered out. On this basis, an LMS adaptive filtering algorithm based on the least mean square criterion is proposed. After comparing with Wiener filter and Kalman filter, we found the disadvantages of these algorithms, which require prior knowledge of noise and known signals, while LMS adaptive filter algorithm can automatically adjust the parameters of the adaptive filter itself, in On the premise of giving full play to the advantages of the algorithm, on the basis of the traditional least mean method, the convergence factor is improved and applied to the audio noise removal system, which has the advantages of small amount of calculation and fast convergence speed, and the algorithm is verified by Matlab simulation effectiveness.

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