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Fuzzy Adaptive Finite-Time Prescribed Control for Robotic Manipulator

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ABSTRACT

Due to the unknown nonlinear friction, there was a barricade preventing the precision of manipulators from further improvement. To overcome this challenge, the dynamic model of 2-degrees of freedom (DOF) robot manipulator based on LuGre friction is established in this paper. The adaptive sliding mode observer (ASMO) is used to estimate the immeasurable states, meanwhile the fuzzy logic system (FLS) to approximate the friction. On this foundation, an adaptive finite-time trajectory tracking control is designed to improve controlled system robustness. In particular, with the Lyapunov stability theory, the proposed scheme can be proved to make the errors be finite-time stable. Finally, simulation results show that the control scheme has a good control effect of the manipulators.

1. Introduction

In the past several decades, the emergence of robot manipulators has greatly improved productivity. However, due to the existence of nonlinear elements including friction in the system, and the difficulty in obtaining accurate system parameters, it is difficult to control the manipulator with high accuracy. Therefore, a controller with targeted performance is design to suppress or eliminate the impact of the above conditions.

The control of uncertain robotic manipulators is challenging due to their friction. It is necessary to establish an accurate friction model of the manipulator before compensating the nonlinear components of the system. Actually, most of the model-based friction compensation methods use typical friction models, such as viscous and Coulomb friction. Therefore, Canudas. [1-2] propose a new dynamic model for friction, this model has included most of the friction phenomena observed in experiments.

Technique based on FLS [3-5] has been widely employed to approximate and compensate for the unknown function. In [6], a robust positioning control scheme has been developed using friction parameter observer and recurrent fuzzy neural networks based on the sliding mode control. Deng YP. [7] proposed an adaptive fractional fuzzy control that guarantees tracking errors tend to an arbitrary small region is established for robotic manipulators, The main idea of this work consists in using fractional input to control complex integer order nonlinear systems.

In the practical applications design of the robot manipulator control scheme, the velocity and acceleration signals are used which must be known for practical implementation of the controllers, but only the position signal measurement is available. Therefore, with the SMO based on second-order exact differentiation (SOED) [8] is introduced and used as an accelerometer and velocity estimator [9]. However, SMO ignore the important influence of the sliding mode gain on the sliding mode motion. in [10] proposed a novel adaptive-gain sliding mode observer.

In order to further improve the tracking performance, many nonlinear control methods have been utilized to control robot manipulator systems, such as iterative learning control for time-varying systems subject to variable pass lengths [11], a new nonlinear self-tuning PID control [12], reconfigurable tolerant control with actuator faults [13], a stable robust impedance control with time-delay compensation [14], under the control of these methods, the robot manipulator performance from different aspects was improved to a certain extent. In the robust control schemes, due to its employment of a variable structure control concept sliding mode control (SMC) has been successfully applied to the control of robot manipulator [15,16].

Among the typical non-linear control methods, SMC [17-18], widely used in complex linear and non-linear system control with uncertain parameters and unknown disturbances, is developed for tracking control problems. However, classical SMC guarantees asymptotic convergence of system state but cannot obtain convergence to the equilibrium point in finite time. Therefore, terminal sliding mode control (TSMC) strategy [19,20] has become a hot topic in non-linear control system design. It can obtain a

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finite-time convergence with robustness control performance. Li et al. [21] proposed A new design approach of a multiple input multiple output adaptive fuzzy terminal sliding mode controller (AFTSMC) for robotic manipulators is described in this article. The AFTSMC incorporating the fuzzy logic controller (FLC), the TSMC, and an adaptive scheme, is designed to retain the advantages of the TSMC while reducing the chattering. Esmaeili *et al*. [22] aims to design a data driven observer-based model free adaptive terminal sliding mode controller for rigid robot manipulators whose models are unknown in advance, and tracking accuracy of the controller is improved. Furthermore, fast terminal sliding mode controller (FTSMC) scheme can not only ensure the trajectory of tracking-error converge to a small neighborhood of equilibrium point in finite-time but also obtain some superior properties such as fast response, better disturbances rejection and improvement of control accuracy [23]. Truong et al. [24] propose a backstepping global fast terminal sliding mode control for trajectory tracking control of industrial robotic manipulators in this article to improve the dynamic performance and fast convergence of TSMC, which also obtains a finite-time convergence. Doan et al. [25] to enhance the response, fast convergence time, against uncertainties, and accuracy of the tracking position, the novel fast terminal sliding mode manifold (FTSMM) is developed. Then, a supper-twisting control law (STCL) is applied to combat the unknown nonlinear functions in the control system. Finally, the proposed controller is launched from the proposed sliding mode manifold and the STCL to provide the desired performance. In addition, the chattering problem has been alleviated by replacing *sgn* function to *fal* function [26].

Furthermore, featured with a selected performance function, it can be guaranteed that tracking errors remain in prescribed performance region, transient performance is a more prior index in comparison with the steady-state one. To achieve presented transient response and steady-state bounds on tracking errors, Prescribed performance control (PPC) has been proved to be a powerful tool [27].The performance functions mentioned above contain various design parameters but the quantitative relationship between them is unclear. Therefore, in [28], a new performance function called finite-time performance function (FTPF) is defined for the first time. Further, a new finite-time form of performance function is defined to constrain the trajectory tracking errors in [29].

Several control techniques have been integrated with robot manipulator, such as SMC, fuzzy logic control, finite-time performance function control, adaptive control. Accordingly, this paper proposed a fuzzy adaptive finite-time prescribed controller of strong robustness and good performance to compensate for the friction and immeasurable states of the robotic manipulator system. The main works are summarized as follows:

- Friction force caused by bristles contact is considered in the input of manipulators system, which effectively describes the nonlinear terms in the modeling process. By characteristic analysis, the friction is effectively estimated by FLS.
- Based on high-order sliding mode (HOSM) differentiator, an ASMO with SOED and highly robust performance is used to solve “only the position measurement and the velocity and acceleration unknown” of implement the controllers in practical applications.
- To reduce the chattering phenomena, a new design scheme for a fast terminal sliding surface is presented. Compared with the traditional sliding mode, it can achleves faster response. The closed-loop system is analyzed by the Lyapunov theory.

2. System description

The dynamics equation of 2-DOF robotic manipulator is described in [30] by

$$M(\theta)a + C(\theta, v)v + G(\theta) + F(v) = \tau \tag{1}$$

where $M(\theta)$, $C(\theta, v)$, $G(\theta)$ represents the positive definite moment of inertia, the Coriolis centripetal forces, and the gravitational forces, respectively. $\theta = [\theta_1, \theta_2]$, $v = [v_1, v_2]$ and $a = [a_1, a_2]$ stands for the angular positions, the angular velocities and the angular accelerations, respectively. $\tau \in R^2$ is the control input torque. $F(v) \in R^2$ represents the frictional force.

Assumption 1 [1]: Based on the actual working environment, it is assumed that the friction F exerted by people and the environment is bounded.

Property 1: There exists a parameter matrix D and $M(\theta)$, $C(\theta, v)$, $G(\theta)$ satisfies

$$M(\theta)\vartheta + C(\theta, v)\rho + G(\theta) + f = \Phi(\theta, v, \rho, \vartheta)D \tag{2}$$

where $\Phi(\theta, v, \rho, \vartheta) \in R$ is a regression matrix.

Lemma 1 [31]: Suppose that $f(x)$ be a Lipschitz continuous function defined on a compact set Ω . For anywhere always exists a FLS such that

$$\sup_{x\in\Omega} = \left| f(x) - \hat{f}(x) \right| \le \zeta \ . \tag{3}$$

Lemma 2 [32]: The observer state $z = [z_1, z_2, z_3] \in R^3$ converges to the vector of derivatives of the output $[\theta, v, a]$, for a finite-time

$$T \le \frac{V^{\sigma}(\zeta(\varepsilon))}{r\sigma} \tag{4}$$

where $0 \le \sigma \le 1$, $r = \lambda_{\min}(B) / \lambda_{\max}(D)$.

Lemma 3 [33]: Consider a scalar dynamic system

$$\dot{y} = -k_1 \operatorname{sgn}(y) - k_2(y),\ \mathrm{y}(0) = 0 \tag{5}$$

where, $k_1 > 0,\ k_2 > 0$. the Eq.5 in a finite-time T , which is stable. Otherwise, the settling time T for any $0 < q < 1$ of system with respect to x is limited by

$$T \le \left(\frac{1}{\varphi q(1-p)}\right)\left[V(x)^{(1-p)} - \left(\frac{g}{\varphi(1-q)}\right)^{(1-p)/p}\right]. \tag{6}$$

The control objective is to develop a fuzzy adaptive finite-time prescribed control scheme for 2-DOF robotic manipulator dynamics under immeasurable states and friction nonlinearity in this paper, such that

- Combining the developed ASMO and FLS, the precise trajectory tracking performance of θ is obtained, while the stability of whole closed-loop system is guaranteed. Based on finite-time stability theory, the errors of tracking signals, FLS approximations, and ASMO errors remain within the specified interval with faster response.
- Based on finite-time control theory and FTPF, using this method, finite-time stability of system is achieved. An adaptive fuzzy controller with prescribed performance is given to further reduces the complexity of calculation and

improve the transient performance and steady-state performance.

3. Control design

In Eq.1, the term $F(v)$ stands for nonlinear friction force. The LuGre friction model captures most of the friction behavior that has been observed experimentally[1], such as Stribeck velocity, viscous friction and Coulomb friction force. In this paper, the LuGre friction model is used to model the friction dynamics of the 2-DOF manipulators.

LuGre friction model is based on the average behavior of the bristles. And the average deformation of the bristles z is described as

$$\frac{dz}{dt} = v - \frac{|v|}{h(v)} z \tag{7}$$

where $h(v)$ is the Stribeck effect, and it is given by

$$\mu_0 h(v) = \tau_{fc} + \left(\tau_{fs} - \tau_{fc}\right) e^{-(v/v_s)^2} \tag{8}$$

where μ_0 is the stiffness, τ_{fc} and τ_{fs} represents the coulomb friction and the stiction force. v_s is used to describe the stribeck velocity.

Viscous friction can be added to friction, which is a term proportional to relative velocity. Thus, LuGre friction model defined as

$$F(v) = \kappa\left(\mu_0 z + \mu_1 \frac{dz}{dt} + \mu_2 v\right) \tag{9}$$

where μ_1 and μ_2 stands for the damping coefficient and the viscous friction coefficient, respectively. κ is the friction factor.

FLS is adopted to approximate continuous function $f(x)$ in robotic manipulator, the foundation of the FLS is composed of the following fuzzy If-Then rules:

$$R^{(i)}: IF\ x_1\ is\ \mathrm{A}_1^l \text{ and } \ldots \text{ and } x_n\ is\ \mathrm{A}_n^l \text{ THEN } \hat{f} \text{ is } \mathrm{B}^l \tag{10}$$

where $l = 1,2,...,N$, N is the number of fuzzy rules, $x = [x_1, x_2, \ldots, x_n]$ is a mapping from an input vector, $\hat{f}$ is the FLS output variable, A_1^l and B^l denote the fuzzy sets associated with the fuzzy membership functions $\mu_{\mathrm{A}_1^l}(x_i)$ and $\mu_{\mathrm{B}^l}(f)$, respectively.

By using singleton fuzzifier, center average defuzzification and product inference, the output of the FLS is given by

$$\hat{f}(x) = \frac{\sum_{l=1}^{N} \bar{f}_l(x) \prod_{i=1}^{n} \mu_{\mathrm{A}_1^l}(x_i)}{\sum_{l=1}^{N}\left[\prod_{i=1}^{n} \mu_{\mathrm{A}_1^l}(x_i)\right]} = \hat{\sigma}_i^T \varsigma(x) \tag{11}$$

where $\sigma_i = \left[\bar{f}_1, \bar{f}_2, \ldots, \bar{f}_N\right] = \left[\sigma_1, \sigma_2, \ldots, \sigma_N\right]$ is the weighting vector, $\bar{f}_l = \max_{f \in R} \mu_{\mathrm{B}^l}(f)$ being an adjustable value, and $\mu_{\mathrm{A}_1^l}(x_i)$ is the membership function value of the fuzzy variable, $\varsigma(x) = \left[\varsigma_1, \varsigma_2, \ldots, \varsigma_N\right]^T$ is a fuzzy basis function vector as

$$\varsigma_l(x) = \frac{\prod_{i=1}^{n} \mu_{\mathrm{A}_1^l}(x_i)}{\sum_{l=}^{N}\left[\prod_{i=1}^{n} \mu_{\mathrm{A}_1^l}(x_i)\right]} . \tag{12}$$

The ideal weight can be defined as

$$\sigma_i^* = \arg\min_{\theta_i \in \Omega}\left[\sup\left|\hat{f}(x) - f(x)\right|\right] \tag{13}$$

where $\Omega \subset R^N$ is a compact region for θ_i.

The design of manipulators controller requires the position, velocity, and acceleration to be used which must be known for practical implementation of the controller. However, in the robot manipulators system, only the position and velocity measurements are available. Thus, the acceleration estimator is required to implement controllers. Basically, the velocity and acceleration signals are calculated by SMO as

$$\begin{aligned} \dot{z}_0 &= -q_1 \left|z_0 - \theta\right|^{2/3} sgn(z_0 - \theta) + z_1 \\ \dot{z}_1 &= -q_2 \left|z_1 - \dot{z}_0\right|^{1/2} sgn(z_1 - \dot{z}_0) + z_2 . \\ \dot{z}_2 &= -q_3 sgn(z_2 - \dot{z}_1) \end{aligned} \tag{14}$$

In Eq.14, q_i is suitably chosen parameter and $z_0 = \theta$, $z_1 = v$, $z_2 = a$. However, the sgn function may cause a chattering problem. To alleviate chattering problem, replacing sign function to fal function

$$\begin{aligned} \dot{z}_0 &= -q_1 \left|z_0 - \theta\right|^{2/3} fal(z_0 - \theta) + z_1 \\ \dot{z}_1 &= -q_2 \left|z_1 - \dot{z}_0\right|^{1/2} fal(z_1 - \dot{z}_0) + z_2 . \\ \dot{z}_2 &= -q_3 fal(z_2 - \dot{z}_1) \end{aligned} \tag{15}$$

where nonlinear function fal is expressed as

$$fal(e, \gamma, \delta) = \begin{cases} \dfrac{e}{\delta^{1-\gamma}}, & |e| \le \delta \\ |e|^{\gamma} sgn(e), & |e| > \delta \end{cases} \tag{16}$$

where $0 < \gamma < 1$ is system adjustable positive parameter and $0 < \delta < 1$ is filtering factor. The current observer errors equation are $\varepsilon_1 = \theta - z_1$, $\varepsilon_2 = v - z_2$, $\varepsilon_3 = a - z_3$.

The symmetric positive matrices D, B are

$$DA + A^T D = -B \tag{17}$$

where the Hurwitz matrix A is

$$A = \begin{bmatrix} -q_1 & 1 & 0 \\ -q_2 & 0 & 1 \\ -q_3 & 0 & 0 \end{bmatrix}. \tag{18}$$

When the observation errors is large, a larger adaptive gain should be used to make the system achleves faster response, and conversely. As a result, an adaptive sliding mode gain function is given by

$$w(t) = \begin{cases} w_1 - \upsilon, \varepsilon > \phi \\ w_2, \varepsilon < \phi \end{cases} \tag{19}$$

where ϕ is the max allowable error value. w_1 , w_2 represents the maximum gain and the minimum gain, respectively. adaptive parameter υ is Eq.19

$$\dot{\upsilon} = \upsilon_1 \max\left\{|\varepsilon_1| - \delta, |\varepsilon_2| - \delta, |\varepsilon_3| - \delta, 0\right\}. \tag{20}$$

To ensuring finite-time convergence and strong robustness. Define a manipulator tracking error as

$$\ddot{e} = a_d - a = \mathcal{P}^{-1}\left\{\mathcal{P} a_d + \mathcal{Z}(\theta, v, a) - \tau\right\} \tag{21}$$

where $\mathcal{Z}(\theta,v,a)=M(\theta)a-\mathcal{P}a+C(\theta,v)v+G(\theta)+f(v)$, $e\triangleq\theta_d-\theta$, $\mathcal{P}=diag[\mathcal{P}_1,\mathcal{P}_2]\in R^{2*2}$.

Assumption 2: The desired trajectory θ_d, v_d and a_d are continuous and bounded.

Assumption 3: $\mathcal{Z}(\theta,v,a)$ is bounded by the condition defined as

$$\left\|\mathcal{Z}(\theta,v,a)\right\|\le\Phi(\theta,v)\Theta \tag{22}$$

with $\Phi(\theta,v)=\left[1,\|\theta\|,\|v\|\right],\Theta=\left[D_1,D_2,D_3\right]^T$, $D_i>0$.

In order to characterize the convergence rate and steady-state error, the following positive decreasing function is constructed as the prescribed performance function

$$\sigma(t)=\begin{cases}(\sigma_0)e^{-\omega\left(1-\frac{t_f}{t_f-t}\right)}+\omega_\infty, & 0\le t<t_f\\ \sigma_\infty & t_f<t\end{cases} \tag{23}$$

where $\sigma_0>0$ and $\varsigma_\infty>0$ are the initial and ultimate error boundaries, respectively. $\omega>0$ determines the convergence rate.

The prescribed bound of the tracking error $e_i(t)$ is retained by the following constrained condition

$$e_i(t)=\sigma(t)E(\varepsilon_i). \tag{24}$$

Subquently, design the error transformation function as

$$E(\varepsilon_i)=\frac{e^{\varepsilon_i}-e^{-\varepsilon_i}}{e^{\varepsilon_i}+e^{-\varepsilon_i}}. \tag{25}$$

From the Eq.23, ε_i are transformed to

$$\varepsilon_i=\frac{1}{2}\log\frac{1+E}{1-E}. \tag{26}$$

In order to improved tracking accuracy and reduced chattering. Therefore, a sliding surface is described by

$$s=\dot{E}+\alpha_1E+\alpha_2sig^r(E) \tag{27}$$

where $sig^{r_1}(e)=|e|^{r_1}\cdot\mathrm{sgn}(e)$.

To guarantee finite-time convergence, a reaching law is expressed as

$$\dot{s}=-k_1\,\mathrm{sgn}(s)-k_2(s) \tag{28}$$

with $k>0$.

Considering that friction of system are influenced by environmental factors, the complete control law τ is given as

$$\tau=\mathcal{P}u_{eq}+u_{est} \tag{29}$$

where u_{eq} is equivalent control used for feedforward compensation of nominal dynamics. While estimation control u_{est} is used to estimate friction force and uncertainties dynamics under disturbance.

The equivalent control term u_{eq} of manipulators is developed as

$$u_{eq}=a_d+\left(\alpha_1+\alpha_2r|E|^{r-1}\right)\dot{E}. \tag{30}$$

The estimation control term u_{est} is expressed as

$$u_{est}=\hat{\mathcal{Z}}(z_0,z_1,z_2)+\mathcal{P}\left(k_1sgn(s)+k_2(s)\right) \tag{31}$$

where $\hat{\mathcal{Z}}(z_0,z_1,z_2)=M(\theta)z_2-\mathcal{P}z_2+C(\theta,v)z_1+G(\theta)+\hat{\sigma}^T\varsigma(x)$, $\tilde{W}$ is the estimation error of weight vector and $\tilde{W}=W^*-\hat{W}$.

Theorem 3: Considering robotic manipulator dynamics (1), complete sliding mode control law τ is established dynamically and obtains finite-time convergence under Lemma 2.

Proof: To obtain the stability of the robotic manipulator system, choose a Lyapunov function as

$$V=\frac{1}{2}s^2+\frac{1}{2\gamma_i}\tilde{\sigma}^T\tilde{\sigma} \tag{32}$$

then the derivative of Eq.32 is obtain as

$$\dot{V}=s\dot{s}+\frac{1}{\gamma_i}\tilde{\sigma}^T\dot{\hat{\sigma}}. \tag{33}$$

Substituting the derivative of Eq.27 into Eq.35, as

$$\dot{V}=s\left(\ddot{e}+\left(\alpha_1+\alpha_2r|E|^{r-1}\right)\dot{E}\right)+\frac{1}{\gamma_i}\tilde{\sigma}^T\dot{\hat{\sigma}}. \tag{34}$$

According to Eq. 25, Eq.34 can be realized as

$$\begin{aligned}\dot{V}&=s(\mathcal{P}^{-1}\left\{\mathcal{P}a_d+\mathcal{Z}(\theta,v,a)-\tau\right\}\\&+\left(\alpha_1+\alpha_2r|e|^{r-1}\right)\dot{e})+\frac{1}{\gamma_i}\tilde{\sigma}^T\dot{\hat{\sigma}}\end{aligned}. \tag{35}$$

Substituting the control law τ into Eq.35, one gets

$$\begin{aligned}\dot{V}&=s(\mathcal{P}^{-1}\left\{\mathcal{Z}(\theta,v,a)-\hat{\mathcal{Z}}(z_0,z_1,z_2)\right\}\\&-s\left(k_1sgn(s)+k_2(s)\right))+\frac{1}{\gamma_i}\tilde{\sigma}^T\dot{\hat{\sigma}}\\&=s(\mathcal{P}^{-1}\left\{-\tilde{\sigma}_i^T\varsigma(x)+\zeta\right\}\\&-s\left(ksgn(s)+k(s)\right))+\frac{1}{\gamma_i}\tilde{\sigma}^T\dot{\hat{\sigma}}\end{aligned}. \tag{36}$$

The adaptation law for $\dot{\hat{\sigma}}$ is given as

$$\dot{\hat{\sigma}}=\gamma_is\mathcal{P}^{-1}\varsigma(x). \tag{37}$$

According to adaptive law $\dot{\hat{\sigma}}$, reaching law $\dot{s}$ and Assumption 3, Eq.36 is realized as

$$\dot{V}\le-s\left(k_1sgn(s)+k_2(s)\right)). \tag{38}$$

The equilibrium of Eq.38 is finite-time stable with the settling time T bounded by

$$T\le T_{\max}=\left(\frac{1}{-k_2q}\right)\left[V(x)^{\frac{1}{2}}+\left(\frac{c}{2k_2(1-q)}\right)\right] \tag{39}$$

where $c>0$. Based on the abovementioned analysis, the structure of the closed-loop system is depicted in Fig. 1.

Remark: The settling time function Eq.39 is rely on the design parameters. It is clearly observed that the $T_{\max}$ is inversely proportional to k_i, while increasing of k_i increases response speed but worse chattering reduction of whole closed-loop system. Consequently, to obtain finite-time convergence and reduce the chattering problems should be considered in the choice of the parameter k_i simultaneously.

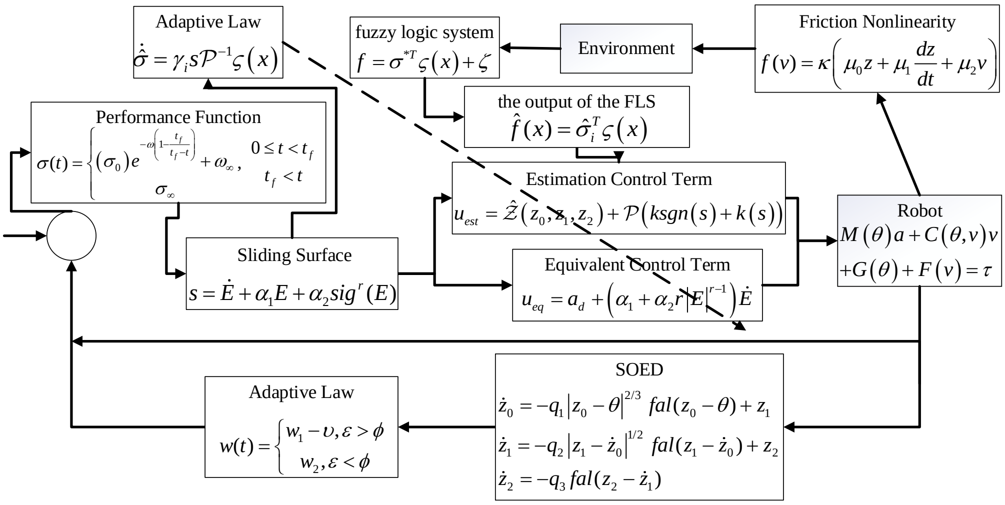

Fig. 1. Block diagram of the closed-loop system.

4. Simulation

As shown in Fig.2, a robotic manipulator with 2-DOF freedom is constructed.

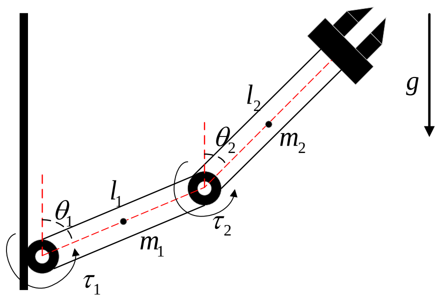

Fig. 2. 2-DOF robot manipulators Schematic Diagram.

In order to show the ability of the presented control, simulations of a 2-DOF robotic manipulator dynamics system are carried out. We define the positive definite inertia matrix $M_0(\theta)$, the centrifugal and Coriolis matrix $C_0(\theta, v)$, and the vector of gravitational torques $G_0(\theta)$ are expressed as

$$M(\theta) = \begin{bmatrix} a + 2c\,c_2 + 2d\,s_2 & b + c\,c_2 + d\,s_2 \\ b + c\,c_2 + d\,s_2 & b \end{bmatrix} \tag{40}$$

$$C(\theta, v) = \begin{bmatrix} (-2c\,s_2 + 2d\,c_2)v_2 & (-c\,s_2 + d\,c_2)v_2 \\ (c\,s_2 - d\,c_2)v_1 & 0 \end{bmatrix} \tag{41}$$

$$G(\theta) = \begin{bmatrix} ce_2\,c_{12} + de_2\,s_{12} + (a - b + e_1)e_2\,c_1 \\ ce_2\,c_{12} + de_2\,s_{12} \end{bmatrix} \tag{42}$$

where $a = 6.7$, $b = 3.4$, $c = 3$, $d = 0$, $\cos(\theta_2) = c_2$, $\sin(\theta_2) = s_2$, $\cos(\theta_1 + \theta_2) = c_{12}$, $\sin(\theta_1 + \theta_2) = s_{12}$, $\cos(\theta_1) = c_1$, $e_1 = m_1 * m_2 * l_1 - l_2 * l_1^2$, $e_2 = g/11$, $g = 9.8$, $m_1 = 1$, $m_2 = 1/2$ are masses of the link; $l_1 = 1$, $l_2 = \frac{1}{12}$ are lengths of the links; τ_1, τ_2 represents the control input; g is accelerations of gravity; define $s_i = \sin(\theta_i)$, $c_i = \cos(\theta_i)$, $\tau = \tau_1 + \tau_2$.

Further, the desired inputs are selected as

$$\begin{cases} \theta_{d1} = 0.2\sin(\pi t/2) + 0.2\sin(\pi t) \\ \theta_{d2} = -0.2\cos(0.5t - 0.2) + 0.2\cos(0.7t) \end{cases}. \tag{43}$$

The control parameters are selected as $\alpha_1 = 50$, $\alpha_2 = 5$, $\mathcal{P} = diag(5,7)$, $\gamma = 0.2$, $\delta = 0.3$. The parameters defined in the reaching law $\dot{s}$ are selected as $k_1 = 5$, $k_2 = 20$. And the parameters of ASMO are selected as $q_1 = 4$, $q_2 = 4$, $q_3 = 2$, $\phi = 1$, $w1 = 1$, $w2 = 2$ .The parameters defined in the prescribed performance function are chosen as $\sigma_0 = 2/3$, $\sigma_\infty = 0.05$, $\omega = 0.08$. In this scenario, the parameters of friction force is shown in Table I.

Tab. 1. The parameters of friction force

Parameters and Value	Unit
$\tau_{fc} = 1$	$[N/m]$
$\tau_{fs} = 1.5$	$[Ns/m]$
$\mu_0 = 10^5$	$[Ns/m]$
$\mu_1 = \sqrt{10^5}$	$[N]$
$\mu_2 = 0.4$	$[N]$
$v_s = 0.001$	$[m/s]$

The Gaussian membership functions is selected as

$$\mu_{A_i^l}(x_i) = \exp\left(-\left(\frac{x_i - \bar{x}_i^l}{\pi/24}\right)^2\right) \tag{44}$$

where $\bar{x}_i^l = [-\pi/6, -\pi/12, 0, \pi/12, \pi/6]^T$, $i = 1,2,3,4,5$, A_i

are NB, NS, NO, PS, PB, respectively.

To counteract chattering problem, the sgn function in Eq.31 is replaced by a new gain function

$$u_{est} = \hat{\mathcal{Z}}(z_0, z_1, z_2) + \mathcal{P}\left(k_1 K(s) + k_2(s)\right) \tag{45}$$

where, based on fal function, the new gain function is defined as

$$K(s) = \frac{\alpha_1 fal(s,\gamma,\delta)}{s} = \begin{cases} \alpha_1 |s|^{\gamma-1} sgn(s), & |s| > \delta \\ \dfrac{\alpha_1}{\delta^{1-\gamma}}, & |s| \le \delta \end{cases} \tag{46}$$

Consider controller τ is designed for robotic manipulator dynamics (1) under reference trajectory θ_d and initial states $\theta_d(0)=0$, $\theta_d(0)=0.2$, $\theta_d(0)=0.5$.

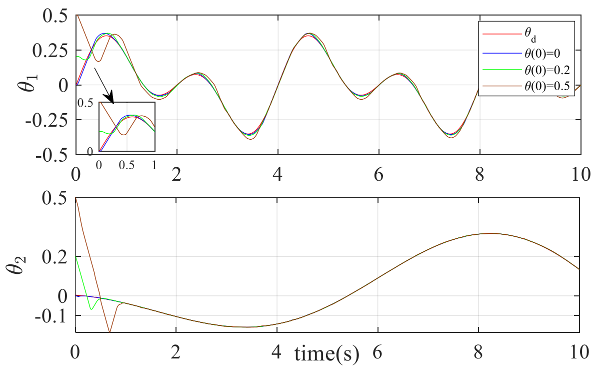

Fig. 3. Tracking performance.

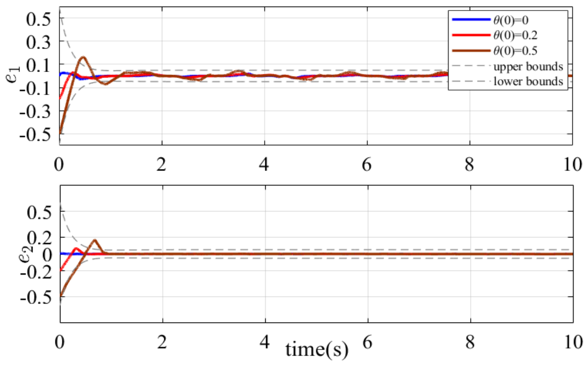

Fig. 4. Tracking errors.

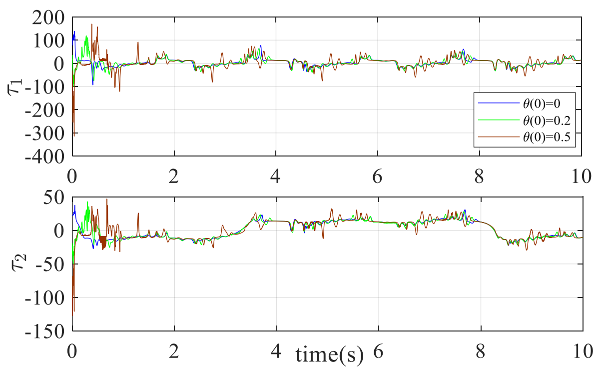

Fig. 5. Control input τ.

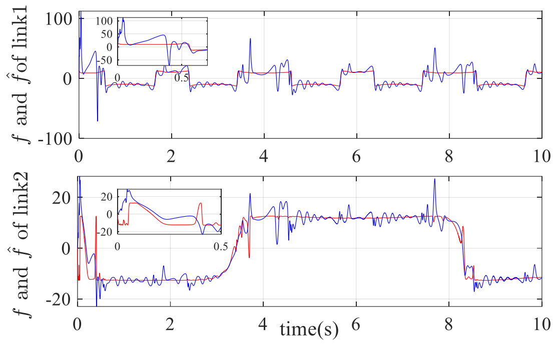

Fig. 6. Friction and compensation $\theta(0)=0$.

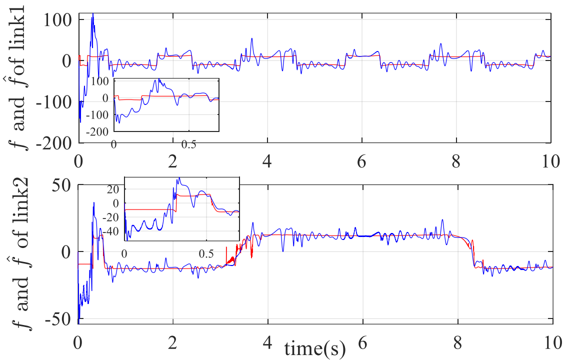

Fig. 7. Friction and compensation $\theta(0)=0.2$.

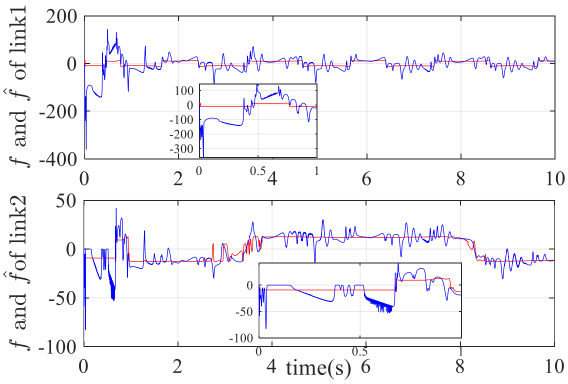

Fig. 8. Friction and compensation $\theta(0)=0.5$.

Simulation results are show in Figs. 3-8. Fig. 3 gives the tracking performance of joints 1 and 2. Fig. 4 indicates that the tracking errors. meanwhile, $\theta_d(0)=0$ and $\theta_d(0)=0.2$ are obviously repelled from the performance function bounds $\sigma(t)$, which is equivalent to that the output constraints are not violated. But it is obvious that the constraints of $\theta_d(0)=0.5$ is breached. The control inputs are presented in Fig. 5. From Figs. 6-8 indicates that the simulations about the approximation effect of the uncertainty term f under the action of FLS. We find that the simulation data of the proposed controller with model error and friction compensation of $\hat{f}$ can quickly estimate the desired term f, and the ASMO is capable of estimating for the effect of the immeasurable states, but the transient performance has been deteriorated without any compensation of the large initial error. From these results, desired performances of the proposed control law in terms of fast response, small steady-state errors and good position tracking.

In the section, we verify the effectiveness of the finite-time control. The initial value of $\theta_d(0)$ is $\theta_{d1}(0)=0.8$, $\theta_{d2}(0)=1$. The convergence rate of finite-time control is relative with the controller parameters, and in order to show this relationship, three cases of controller parameters k_1 and k_2 are carried out, i.e., case 1: $k_1=5, k_2=20$; case 2: $k_1=10, k_2=25$; and case 3: $k_1=15, k_2=30$. According to Fig. 9, it is easy to find that trajectories of Joint 1 and Joint 2 of case 3 can better track the desired trajectory θ_d compared with cases 1 and 2. In Fig. 10, controllers τ_1, τ_2 are plotted, and we can see that, A large k_i will converge quickly, but an excessively large k_i will cause chattering.

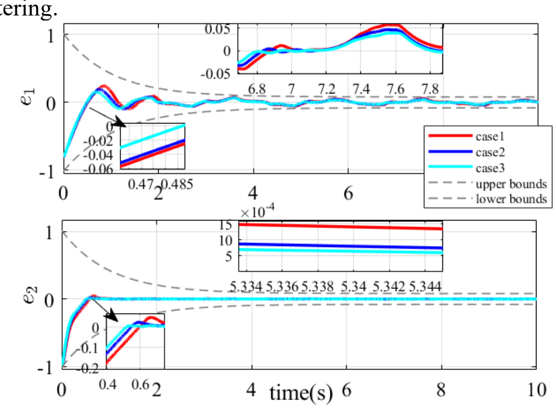

Fig. 9. Tracking errors.

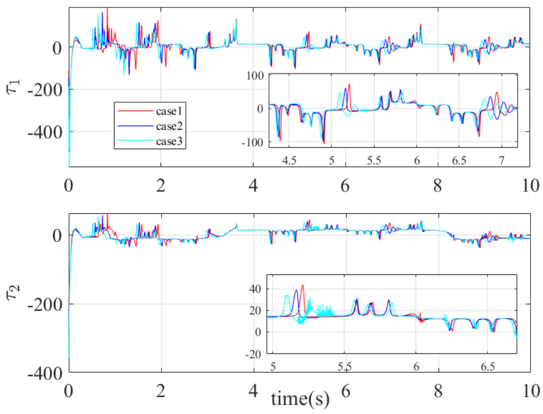

Fig. 10. Control input τ.

It is ready to know from Fig. 9 that as the gains k_1 and k_2 increase, the tracking error e1 decreases faster. As the gains k_1 and k_2 increase, it is found from Eq.39 that $T_{\max}$ also increases, therefore, the convergence rate will be greater, and both setting time and steady-state errors will be smaller.

5. Conclusion

A trajectory tracking problem with the help of ASMO and FLS are studied for robot manipulator in the presence of friction and immeasurable states in this paper. The simulation results for the presented control scheme of the robot manipulator that the proposed method has great capability to counteract chattering problem. Furthermore, The proposed control scheme can maintain high precision tracking of the desired joint angle under the condition of changing friction, and has good robustness.

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