Contents lists available at YXpublications

International Journal of Applied Mathematics in Control Engineering

Journal homepage: http://www.ijamce.com

# An Efficient Approach for the Optimization of Steelmaking-continuous Casting Tundish Batch Planning

Congxin Li[a,*], Tingwei Pan[b], Jiaxin Guo[c], Xinqi Hu[d], Liangliang Sun[e], Jiayu Peng[a], Jingjing Lou[a]

[a] *School of Mechanical Engineering, Shenyang Jianzhu University, Shenyang, Liaoning 110168, China*
[b] *School of Civil Engineering, Shenyang Jianzhu University, Shenyang, Liaoning 110168, China*
[c] *School of Transportation Engineering, Dalian Jiaotong University, Dalian 116045, China*
[d] *School of Electrical and Control Engineering, Shenyang Jianzhu University, Shenyang, Liaoning 110168, China*
[e] *School of Control Engineering, Northeastern University at Qinghuangdao, Qinghuangdao, Hebei 066004, China*

ARTICLE INFO

Article history:
Received 6 March 2023
Accepted 18 April 2023
Available online 28 April 2023

Keywords：
Tundish batch planning
Surrogate Subgradient Lagrangian Relaxation
Mixed integer programming

ABSTRACT

There is a major contradiction between the market demand of steel enterprises for multi-variety, small batch and just-in-time delivery, and the internal large-scale production of steel enterprises. Steelmaking-continuous casting batch planning can effectively solve this contradiction. Tundish batch planning is one of the important links of batch planning, so the optimization of tundish batch planning is of great significance for iron and steel enterprises to achieve batch production, cost reduction and efficiency increase. However, in the actual steelmaking-continuous casting process, the size of the data portfolio increases exponentially with the increase of the number of charges, which makes it difficult to prepare a high-quality cast planning within the time limit of production requirements. In order to solve the problems, the optimization of steelmaking - continuous casting tundish batch planning is studied in this paper. A mixed integer programming model is established to minimize the attribute difference between charges and minimize the remaining service life of tundish. A Surrogate Subgradient Lagrangian Relaxation model framework based on heuristic rules is proposed to optimize the solution. In the framework of this model, the approximate optimization can be obtained without solving all subproblems only if the optimal conditions of the Surrogate are satisfied. Therefore, the optimization efficiency is improved. Finally, the experiment verifies that the algorithm can efficiently compile a group of tundish batch planning with better quality.

## 1. Introduction

Typical integrated manufacturing system for steel production mainly consists of three continuous stages: ironmaking, steelmaking-continuous casting (SCC), rolling. Among them, SCC plays a connecting role in the process of steel manufacturing, which is an important stage and a short plate link in iron and steel production (Tang et al., 2011). It coordinates the supply of raw materials at the ironmaking stage on the upper side and supplies the required specification materials at the rolling stage on the lower side. Fig. 1. is a schematic diagram of SCC production process.

SCC process decision can be divided into batch planning and scheduling. Batch planning is a bridge between Enterprise Resource Planning (ERP) and Manufacturing Execution System (MES). ERP efficiently uses the resource allocation manufacturing by optimizing the batch planning so that production can be carried out in an orderly manner. Steelmaking-continuous casting batch planning include charge batch planning, tundish batch planning, and cast batch planning. The problem of optimization of the tundish batch planning is mainly to study how to make a reasonable combination of multiple varieties and small batch charges on the premise of considering the characteristics of the same tundish production process. And the combined tundish is provided to the subsequent cast batch planning Therefore, it can be seen that the tundish batch planning is the bridge for the optimization of the charge batch planning and the cast batch planning, and it is one of the important parts of the steelmaking-continuous cast batch planning. Reasonable preparation of tundish batch planning is of great significance to improve production efficiency and reduce production cost. Therefore, how to make efficient and high-quality compilation tundish batch planning is the main problem in this article.

There are the following difficulties in steelmaking-continuous casting tundish batch planning. The first difficulty is the optimization objectives need to be fully considered to meet the actual production process. The process of steelmaking is accompanied by chemical and physical changes of high temperature and high pressure, and it is carried out in the mode of assembly line, which also means that the production process is accompanied by huge energy changes. In the

* Corresponding author.
E-mail addresses: 1095038606@qq.com (C. Li)

optimization process of steelmaking-continuous casting tundish batch planning, it is necessary to consider the limitation of customer demand, the limitation of production process execution rules and the limitation of machine capacity, so as to make the "flow" of steel production process proceed dynamically and orderly, so as to make efficient use of energy and increase production efficiency. However, the complexity of mathematical optimization model is also increased. The second difficulty is the scale of the problem increases exponentially as orders increase. Due to the large number of orders to prepare the charge batch planning, The charge is planned to be redistributed in tundish for processing, each charge also includes the steel grade, width, composition, delivery time and many other factors. Therefore, in the process of reorganizing the diverse charge data of multi-variety and small-batch customers into large-scale production batches, the scale of the problem increases exponentially with the increase of the number of charges, which makes it difficult to optimize and solve the problem efficiently.

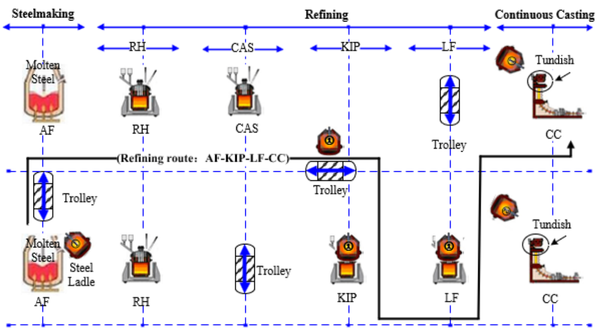


**Fig. 1.** schematic diagram of SCC production process.

Dong et al., (2009) have established a mathematical model with the minimum number of tundishes, the minimum additional process cost and the capacity balance of each flow direction as optimization objectives, and dealt with multi-objective optimization problems based on the strategy and weighted sum method. Yi et al., (2012a) have considered the utilization rate of tundish as the optimization objective and adopted heuristic and DNA evolution algorithm to solve the model. Zhu et al., (2021) have established a mathematical model to minimize the number of unselected charges, minimize the remaining service life of tundish and the unoptimized target of downstream productivity balance, and solved it based on mathematical programming method. In the above literature, the differences of charge components assigned to the same tundish are not considered, so it is difficult to meet the actual production demand. Tang et al., (2008) have summarized tundish planning as a vehicle routing problem, established a mixed integer programming model with minimum number of tundishes, minimum charge penalties not incorporated into tundish and downstream production processes as optimization objectives, and solved the problem based on tabu search and heuristic method. Yi et al., (2012b) have established a multi-travel salesman problem model with the optimization objectives of minimizing the total number of tundishes, minimizing the total number of adjustment width and the total number of times of different types of steel transfer, and proposed a hybrid optimization algorithm combining the heuristic k-opt neighborhood search and estimation of distribution algorithms (EDA) evolution. Dong et al., (2014) have established a mathematical model to minimize the number of tundishes, minimize the difference of tundish attributes and downstream processes, and solved the problem based on the improved variable neighborhood search algorithm. Ma et al., (2015) have decomposed the solving model into two sub-models in the process of establishing a mathematical model, and designed iterated local search (ILS) and variable neighborhood search (VNS) combined with the two-layer hybrid algorithm. The above literature studies consider the attribute difference between charges in the modeling process, but the optimization method based on neighborhood search is easy to lead to incomplete or inconsistent knowledge reserve of decision-making system in the engineering background of solving complex linkage relations, and it is difficult to efficiently respond to the production needs of practical problems.

On the basis of the existing optimization of tundish planning this paper establishes the weighted objective function to minimize the service life of tundish and the minimum difference of the charge components in the same tundish. At the same time, the service life capacity constraint, charge distribution constraint and downstream capacity balance constraint are considered. The mixed integer programming model is established. Firstly, by introducing a set of optimal solution adjustment coefficients, the complex coupling condition tundish service life capacity constraint was relaxed into the objective function, and the Surrogate Subgradient Lagrangian Relaxation (SSLR) model framework was constructed. In the framework of this model, the approximate optimization of subproblems can be obtained without solving all subproblems only if the optimal conditions of the Surrogate are satisfied. Compared with the Subgradient Lagrangian Relaxation (SLR) method, SSLR is less computational because it does not need to solve all the subproblems. Secondly, the original problem is decoupled into subproblems representing the optimal target value of each tundish, and solved based on backward dynamic programming algorithm. Finally, a two-stage heuristic optimization method is designed to adjust the feasible solutions that do not satisfy the relaxed coupling constraints and replace the non-optimal clustering centers to obtain the optimal target value of the original problem. The actual data of a large steel mill in China show that this method can guarantee the optimization efficiency and quality.

The rest of this paper is organized as follows. Section 2 provides the mathematical formulation of tundish batch planning. In Section 3 introduces the SSLR approach for tundish batch planning, solution methods for the subproblems and the construction method of feasible solution, etc. In section 4 demonstrates the computational experiments and the comparisons for SSLR and SLR. In Section 5 concludes our study.

## 2. Mathematical Formulation

### *2.1 Problem Description*

Tundish is a vessel used to hold molten steel in a continuous casting machine. Each tundish has a certain service life, and the different composition of molten steel will have a certain impact on the service life of tundish. The inside of each tundish contains a high temperature resistant layer, and whether it reaches its service life or not, it needs to be maintained, and the maintenance cost is high. Therefore, the optimization process is to minimize the remaining service life of the tundish as an objective function. Therefore, in the combination process, it is necessary to consider the production process restriction of the same tundish. These limitations include whether the difference of composition, width and delivery time of the same tundish charge is similar, and whether the sum of processing time of the same tundish charge exceeds the tundish life. The number of charges, the number of refining charges, the weight of ironing roller and the total weight of the downstream production line should be considered in the process of determining the number of charges and ranking.

The following requirements should be taken into account in the optimization process of tundish planning charges decision allocation (Sun, et al., 2018):

(1) The charges in the same tundish must be continuously cast.
(2) The total number of charges in the same tundish should not exceed the service life of tundish, which is usually 4-6 charges according to the different composition of molten steel.
(3) The width adjustment range of adjacent charge in the same tundish shall not exceed 100mm, and shall not exceed 2 times of each tundish.
(4) Because of the production process constraints of continuous casting machine, and each charge has steel grade properties. Therefore, in the same tundish processing charge steel must be similar or the same.
(5) Due to the production process constraints of continuous casting machine, the slab thickness carried by the charge processing in the same tundish must be the same.
(6) Balance the flow of each downstream production line.

*2.2 Parameters*

In order to better meet the actual optimization process of steelmaking and continuous casting tundish batch planning, four optimization objectives are designed in this paper, which are to minimize the composition difference between charges, minimize the delivery time difference, minimize the width difference between charges and minimize the remaining service life of tundish. The weight objectives are designed respectively, and the four optimization objectives are weighted and optimized. The constraints of charge distribution, tundish service life and downstream production process capacity balance were considered.

**Constant:** In the process of tundish batch planning, set the charges set as $\{n|n = 1,2,\dots,N\}$, The index number of the charges are $i,j(\forall i,j \in [1,N])$. The number set of tundishes is $\{k|k = 1,2,\dots,M\}$. $G_i$, $G_j$ are the component of charges $i$ and $j$. $W_i$, $W_j$ are the width of charges $i$ and $j$. $D_i$, $D_j$ are the delivery time of charges $i$ and $j$. $p_{ij}^{CD}$ is the penalty caused by different components of charges $i$ and $j$. $p_{ij}^{WD}$ is the penalty caused by different widths of charges $i$ and $j$. $p_{ij}^{DD}$ is the penalty caused by different delivery time of charges $i$ and $j$. $p_k^{TL}$ is the penalty for the tundish $m$ not being fully utilized. $TL$ is the tundish service life. $L_{chr}$ and $H_{chr}$ are the minimum and maximum values of the number of charges production requirements. $L_{rh}$ and $H_{rh}$ are the minimum and maximum values of the refining number of charges production requirements. $L_{pre}$ and $H_{pre}$ are the minimum and maximum values of the weight of hot roll materials production requirements. $L_f$ and $H_f$ are the minimum and maximum values of the weight requirements for downstream processes. $Q^{rh-i}$ is the refining mark of charge $i$. $Q^{pre-i}$ is the weight of hot roll materials in charge $i$. $Q^{f-i}$ is the slab weight required by the downstream production process f in charge $i$. $F$ is the total number of downstream processes, $f$ is the downstream process index and $f \in F$. $\varphi_1,\varphi_2,\varphi_3,\varphi_4$ respectively optimize the weight coefficients, $\varphi_1,\varphi_2,\varphi_3,\varphi_4 \in [0,1]$ and $\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 = 1$.

**Variable:** $y_{ijk}$ 0/1 decision variable. If the charge $i$ and $j$ in production in the tundish $k$, $y_{ijk} = 1$. Otherwise, $y_{ijk} = 0$. $y_{kk}$ 0/1 auxiliary variable. If the serial number for $k$ tundish is selected as the optimization object, $y_{kk} = 1$. Otherwise, $y_{kk} = 0$. $y_{ik}$ 0/1 auxiliary variable. If charge $i$ is produced in tundish $k$.

*2.3 Mathematical model*

Objective function

(1) Minimize the difference in charge composition within the same tundish.

$$O^{SG} = \sum_{k=1}^{M}\sum_{i=1}^{N}\sum_{j=2}^{N}|G_i - G_j| \cdot p_{ij}^{SG} \cdot y_{ijk} \tag{1}$$

(2) Minimize the difference in width between charges assigned to the same tundish.

$$O^{WD} = \sum_{k=1}^{M}\sum_{i=1}^{N}\sum_{j=1}^{N}|W_i - W_j| \cdot p_{ij}^{WD} \cdot y_{ijk} \tag{2}$$

(3) Minimize the difference in delivery time between charges assigned to the same tundish.

$$O^{DD} = \sum_{k=1}^{M}\sum_{i=1}^{N}\sum_{j=1}^{N}|D_i - D_j| \cdot p_{ij}^{DD} \cdot y_{ijk} \tag{3}$$

(4) Minimize the remaining service life of the tundish.

$$O^{TL} = \sum_{k=1}^{M} p_k^{TL}\left(TL \cdot y_{kk} - \sum_{i=1}^{N}\sum_{j=1}^{N} y_{ijk}\right) \tag{4}$$

The objective function is weighted and constraints are added to get the following form：

$$\min Z$$

$$\text{with } Z = \varphi_1 O^{SG} + \varphi_2 O^{WD} + \varphi_3 O^{DD} + \varphi_4 O^{TL}$$

Subject to:

$$\sum_{i=1}^{N}\sum_{j=1}^{N} y_{ijk} \le TL \cdot y_{kk} \tag{5}$$

$$\sum_{k=1}^{M} y_{ik} \le 1 \tag{6}$$

$$L_{chr} \le \sum_{k=1}^{M}\sum_{i=1}^{N} y_{ik} \le H_{chr} \tag{7}$$

$$L_{rh} \le \sum_{k=1}^{M}\sum_{i=1}^{N} Q^{rh-i} \cdot y_{ik} \le H_{rh} \tag{8}$$

$$L_{pre} \le \sum_{k=1}^{M}\sum_{i=1}^{N} Q^{pre-i} \cdot y_{ik} \le H_{pre} \tag{9}$$

$$L_f \le \sum_{k=1}^{M}\sum_{i=1}^{N} Q^{f-i} \cdot y_{ik} \le H_f \tag{10}$$

Constraint (5) tundish capacity constraint which means that the number of charges processing on tundish cannot exceed the tundish service life. Constraint (6) is the charge allocation constraint which means that each charge can only be allocated to one tundish for production. Constraint (7) is the capacity constraint means that the actual number of charges processed is within the upper and lower limits specified in the production. Constraint (8) indicates that the actual refining charges should be within the upper and lower limits of the refining number required by production. Constraint (9) means that the total weight of hot roll materials actually processed should be within the upper and lower limits of production requirements. Constraint (10) means that the total flow weight of slab in each charge must be within the upper and lower limits specified in the production.

## 3. Solution Methodology

*3.1 Lagrangian Relaxation Frame*

By introducing the optimal solution adjustment coefficient $u_i$, the constraint condition of "tundish service life constraints". The mathematical model can be expressed in the following form:

$$min\ Z_{SSLR}(u_i)$$

$$with\ Z_{SSLR}(u_i) = Z + \sum_{i=1}^{N} u_i(TL \cdot y_{kk} - \sum_{i=1}^{N}\sum_{j=1}^{N} y_{ijk})$$

$$= \sum_{k=1}^{M}\sum_{i=1}^{N}\sum_{j=2}^{N} P_{ij} \cdot y_{ijk} + \varphi_4 \sum_{k=1}^{M} p_k^{TL}\left(TL \cdot y_{kk} - \sum_{i=1}^{N}\sum_{j=1}^{N} y_{ijk}\right)$$

$$+ \sum_{i=1}^{N} u_i(\sum_{i=1}^{N}\sum_{j=1}^{N} y_{ijk} - TL \cdot y_{kk})$$

$$i = 1,2,\dots,N. j = 1,2,\dots,N. k = 1,2,\dots,M. \quad (11)$$

$$P_{ij} = \varphi_1|G_i - G_j| \cdot p_{ij}^{SG} + \varphi_2|W_i - W_j| \cdot p_{ij}^{WD} + \varphi_3|D_i - D_j| \cdot p_{ij}^{DD}$$

$$i = 1,2,\dots,N. j = 1,2,\dots,N. \quad (12)$$

The constraints are (6) - (10). $i = 1,2,\dots,N. j = 1,2,\dots,N. k = 1,2,\dots,M.$

*3.2 Lagrangian Dual Problem*

Since the constraint (5) is relaxed, the optimal solution generated by the SSLR problem is not the optimal solution to the original problem. In order to be able to get closer to the optimal solution to the original problem, the solution to the relaxation problem is replaced by the maximum value of the dual problem. Lagrangian dual problem can be expressed as:

$$max\ Z_{SSLR}^{D}(u_i)$$

$$with\ Z_{SSLR}^{D}(u_i) = min\ Z_{SSLR}(u_i)$$

$$with\ Z_{SSLR}^{D}(u_i) = \sum_{k=1}^{M}\sum_{i=1}^{N}\sum_{j=2}^{N}(P_{ij} - \varphi_4 p_k^{TL} + u_i) \cdot y_{ijk}$$

$$+TL \cdot y_{kk}(\varphi_4 \sum_{k=1}^{M} p_k^{TL} - \sum_{i=1}^{N} u_i)$$

$$i = 1,2,\dots,N. j = 1,2,\dots,N. k = 1,2,\dots,M. \quad (13)$$

The constraints are (6) - (10). $i = 1,2,\dots,N. j = 1,2,\dots,N. k = 1,2,\dots,M.$ The relationship between the original problem, the relaxation problem and the dual problem (Han, et al., 2016) is that $Z_{SSLR}(u_i) \le Z_{SSLR}^{D}(u_i) \le Z.$

*3.3 Backward Dynamic Programming Solves Subproblems*

Based on formula (13), the capacity constraint of the device can be decoupled into a structure that takes the optimization objective of a single tundish as a subproblem for solving. To facilitate the calculation of this problem, let $M = N$ and $y_{kk} = 1$, $k = 1,2,\dots,N$. By optimizing each charge as a clustering center of a tundish, formula (13) can be converted to the following form:

$$max \cdot min\ Z_{SSLR}(u_i)$$

$$with\ Z_{SSLR}(u_i) = \sum_{j=1,k=1}^{N} t_j(y_{ijk}) + TL \cdot y_{kk}(\sum_{k=1}^{M} p_k^{TL} - \sum_{i=1}^{N} u_i)$$

$$j = 1,2,\dots,N. \quad (14)$$

The constraints are (6) - (10). $i = 1,2,\dots,N. j = 1,2,\dots,N. k = 1,2,\dots,M.$

$$min\ t_j(y_{ijk})$$

$$with\ t_j(y_{ijk}) = \sum_{i=1}^{N}(P_{ij} - \varphi_4 p_k^{TL} + u_i) \cdot y_{ijk} \quad (15)$$

The constraints are (6) - (10). $i = 1,2,\dots,N. j = 1,2,\dots,N. k = 1,2,\dots,M.$

Formula (15) satisfies constraint conditions (6) - (10), which can be expressed as a 0-1 knapsack problem and solved based on backward dynamic programming method (Li et al., 2019).

Each tundish is optimized as a backpack. The penalty value generated by the combination of the charges and the cluster center due to the difference in its physical properties is considered as the value of the item. Charge $i$ is assigned to the cluster center for processing $y_{ijk} = 1$, while the charge $i$ is not assigned to the cluster center for processing $y_{ijk} = 0$. The problem can be abstracted as follows: under the premise that the service life of tundish is certain, the decision set can maximize the utilization rate of tundish (the tundish life can be fully utilized) and at the same time make the target optimization value of the tundish best.

Let $\delta = P_{ij} - \varphi_4 p_k^{TL} + u_i$. The tundish service life is divided into $TL_B$ intervals, $TL_b$ is the index of the interval, and $TL_b \in TL_B$. $\delta[i, TL_b]$ is the optimal value for the first $i$ cycles in $TL_b$, $TL_{(i)}$ is the tundish life occupied by secondary $i$ in this interval. $\delta[i]$ is the penalty value generated by the combination of charge $i$ and clustering center.

The state transition formula of tundish batch planning 0-1 decision problem is as follows:

$$\delta[i, TL_b] = \min\{\delta[i-1, TL_b - TL_{(i)}] + \delta[i];\ \delta[i-1, TL_b]\} \quad (16)$$

This algorithm means that each backward state is optimized based on the previous state in order to judge the current charge is selected or not under the tundish lifetime constraint.

*3.4 SSLR Method is Used to Solve the Dual Problem*

In order to calculate the formula conveniently, the quantity unrelated to the decision variable in formula (13) is temporarily excluded from the calculation process and converted into the following form:

$$max\ Z_{SSLR}^{D}(u_i)$$

$$with\ Z_{SSLR}^{D}(u_i) = \sum_{k=1}^{M}\sum_{i=1}^{N}\sum_{j=2}^{N}(P_{ij} - \varphi_4 p_k^{TL} + u_i) y_{ijk}$$

$$- y_{kk} TL \sum_{i=1}^{N} u_i$$

$$i = 1,2,\dots,N. j = 1,2,\dots,N. k = 1,2,\dots,M. \qquad (17)$$

The Lagrangian surrogate subgradient method does not need to solve all the subproblems and can greatly improve the solving efficiency (Zhao et al., 1999; Cui et al., 2017; Pang et al., 2017). And can get a proper subgradient direction, with less computation to solve large-scale problems, is a good optimization method. In this paper, a surrogate subgradient optimization method based on heuristic rules is designed. For formula (17), the optimization steps are as follows：

**Step 1:** Initialization:

$m = 1. \varepsilon_1 > 0, \varepsilon_2 > 0$ . $u_i^{(0)} = 0, (i = 1,2,\dots,N)$ . ${y_{ijk}}^{(0)} = 0, (i = 1,2,\dots,N. j = 1,2,\dots,N)$.

**Step 2:** It is judged whether or not $Z_{SSLR}^{D}(u_i) \le Z$, if so, a subgradient direction of the dual problem is obtained.

**Step 3:** The subproblem is solved based on the adjustment coefficient $u_i$ of the given optimal solution:

$${y_{ijk}}^{(m)}\left({u_i}^{(m)}\right) = arg\ min\ \left(Z_{SSLR}^{D}\left({u_i}^{(m)}\right)\right), i = 1,2,\dots,N. j = 1,2,\dots,N.$$

**Step 4:** Set the direction of the subgradient required for updating the optimal solution adjustment coefficient:

$$\tilde{g}\left({u_i}^{(m)}\right) = Ay_{ijk}\left({u_i}^{(m)}\right) - b = \sum_{i=1}^{N} a_i\, {y_{ijk}}^{(m)}\left({u_i}^{(m)}\right) - b$$

**Step 5:** Calculate the gradient step size required for updating the Lagrangian multiplier:

Step size $d^{(m)}$ satisfied the following equation:

$$0 < d^{(m)} < (Z - Z_{SSLR}^{D}\left({u_i}^{(m)}\right))/\|\tilde{g}^{(m)}\|^2$$

Where $\tilde{g}^{(m)} = \tilde{g}({y_{ijk}}^{(m)})$ is $Z_{LR}^{D}(u_i)$ in ${u_i}^{(m)}$ position gradient.

**Step 6:** Based on the gradient direction and step size, the Lagrangian multiplier is updated:

$${u_i}^{(m+1)} = [{u_i}^{(m)} + \tilde{g}\left({u_i}^{(m)}\right)d^{(m)}]^{+}$$

**Step 7:** Perform approximate optimization:

According to ${u_i}^{(m+1)}$, approximate optimization is performed to obtain ${y_{ijk}}^{(m)}$ so that ${y_{ijk}}^{(m)}$ is satisfied:

$$Z_{SSLR}^{D(m+1)}\left({u_i}^{(m+1)}, {y_{ijk}}^{(m+1)}\right) < Z_{SSLR}^{D(m+1)}\left({u_i}^{(m+1)}, {y_{ijk}}^{(m)}\right)$$

$$Z_{SSLR}^{D(m+1)}\left({u_i}^{(m+1)}, {y_{ijk}}^{(m)}\right) = \sum_{i=1}^{N} Z\ \left(y_{ijk}\right) + ({u_i}^{(m+1)})^{T}(A{y_{ijk}}^{(m)} - b)$$

If ${y_{ijk}}^{(m+1)}$ cannot be obtained, let ${y_{ijk}}^{(m+1)} = {y_{ijk}}^{(m)}$.

**Step 8:** Check whether the stop criteria are met:

If the stop criteria are met, the multiplier update will be stopped. Otherwise, the next multiplier iteration update will be performed by turning to step 2.

$$\| {u_i}^{(m+1)} - {u_i}^{(m)} \| < \varepsilon_1 \text{ or } \| {y_{ijk}}^{(m+1)} - {y_{ijk}}^{(m)} \| < \varepsilon_2.$$

Where $m$ is the number of iterations, $\varepsilon_1$, $\varepsilon_2$ are infinitesimal natural numbers. In Step 4, this term of $\sum_{i=1}^{N} a_i\, {y_{ijk}}^{(m)}\left({u_i}^{(m)}\right) - b$ is the coupling constraint relaxed by the SSLR problem, and $\sum_{i=1}^{N} a_i\, {y_{ijk}}^{(m)}\left({u_i}^{(m)}\right) - b \le 0$. SSLR does not need to solve all the optimal values of subproblems in the optimization process, which is reflected in Step 7. As long as the inequalities mentioned in Step 7 are satisfied, the optimal agent condition is satisfied. The stopping criteria in Step 8 stop the iteration as long as one of them is met. Fig. 2 shows the flow chart of the proposed algorithm.

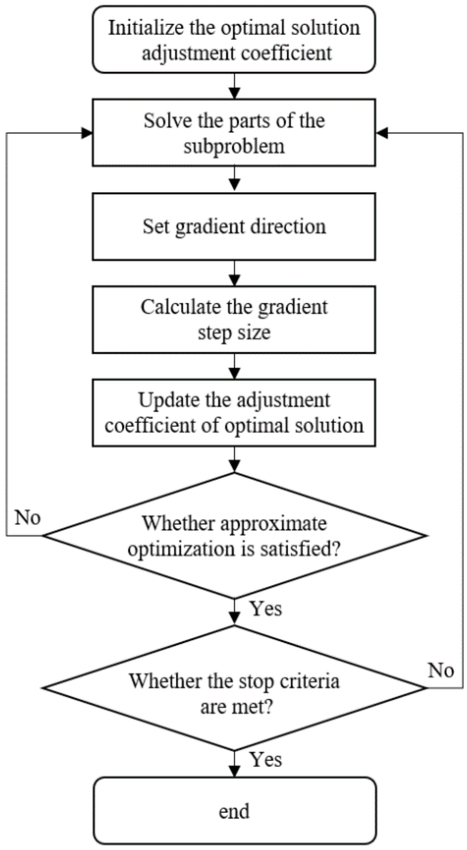


**Fig. 2.** Algorithm flow chart.

*3.5 Construct feasible solution of original problem*

In order to solve the tundish batch planning optimization problem, the service life capacity constraint of tundish is relaxed, resulting in the approximate optimal solution is not satisfied with the feasible solution of the original problem. Two stage heuristic rules are designed to adjust the approximate optimal solution, so that the adjusted decision charge allocation satisfies the tundish lifetime capacity constraint and the objective function value is more ideal.

The first stage: decoupling constraints. Determine whether each charge $i$ meets the constraint condition of being relaxed. If so, enter the second stage. If not, it will be compared with the charge sequences with conflicting constraints, and the clustering center selected by the optimal solution will be regarded as the standard solution to optimize the second stage. The second stage: Adjust the clustering center. The selection of the clustering center to obtain each group of feasible solutions is related to the quality of the optimization of the objective function value. Each charge of each group of solutions is respectively used as the clustering center for optimization, and the other penalty values are replaced with more ideal penalty values. At the same time, the clustering center is replaced as the better one.

## 4. Results

*4.1 Parameters*

The method was implemented by using MATLAB，the experiment was carried out on an Intel Core i5 5200 CPU 4GB Windows 11/64 bit operating system PC. Through the simulation of the actual production data in China, the following simulation cases and results are obtained. In Example 1, the service life utilization of tundish rate of SSLR and SLR in the optimization process was compared through the tundish batch planning optimization experiment of 20 charges. In Example 2, the CPU times and duality gap of 100 charges are compared based on SSLR and SLR. In Example 3, the control variable method optimizes 100 sets of data based on SSLR and SLR, each set of 50 charges. First, the optimization time of SSLR and SLR under the same duality gap is compared. Then, the duality gap of SSLR and SLR are compared under the same CPU time.

**Example 1.** Optimize 20 charges based on SSLR and SLR, and the utilization rate of tundish service life are compared.

Tab. 1. shows the optimization results based on the tundish service life. It can be seen that the total number of tundishes generated by SLR optimization method is 6, and the average utilization rate of tundish service life is 66.67%. The total number of tundishes generated by SSLR optimization method is 5, and the average usage of tundishes reaches 80.00%. In conclusion, the SSLR algorithm based tundish batch planning optimization method has better production resource utilization effect in the production process.

**Tab. 1.** 20 charges are optimized based on SSLR and SLR, and the service life consumption of tundish is compared

| SLR | | | | SSLR | | | |
|---|---|---|---|---|---|---|---|
| Tundish | Charges | *TL* Consumption | Utilization Rate of TL (%) | Tundish | Charges | TL Consumption | Utilization Rate of TL (%) |
| 1 | 2,7,11 | 3 | 60 | 1 | 1,9,5,16 | 4 | 80 |
| 2 | 15,14,17 | 3 | 60 | 2 | 3,18,7,2,19 | 5 | 100 |
| 3 | 4,12,1,5 | 4 | 80 | 3 | 13,10,12,11 | 4 | 80 |
| 4 | 20,6,16 | 3 | 60 | 4 | 15,17,6,14 | 4 | 80 |
| 5 | 3,18,9 | 3 | 60 | 5 | 8,4,20 | 3 | 60 |
| 6 | 8,10,13,19 | 4 | 80 | 6 | — | — | — |
| Average Value | | | 66.67 | Average Value | | | 80.00 |

**In Example 2.** In the optimization process of 100 charges, the CPU time and duality gap of SSLR and SLR algorithms were compared.

**In Tab. 2.** The convergence time and duality gap of SSLR and SLR

| Cases | Number of tundish | CPU Time (seconds) | | Duality Gap (%) | |
|---|---|---|---|---|---|
| | | SSLR | SLR | SSLR | SLR |
| 1 | 5 | 15.73 | 18.92 | 3.38 | 4.01 |
| 2 | 10 | 39.21 | 54.86 | 2.14 | 2.23 |
| 3 | 15 | 56.13 | 67.45 | 2.35 | 2.84 |
| 4 | 20 | 76.98 | 89.43 | 1.89 | 2.07 |
| 5 | 25 | 95.22 | 129.47 | 1.36 | 1.51 |

In Tab. 2, the CPU time and duality gap of two optimization algorithms SSLR and SLR were recorded when 100 charges were optimized to 5, 10, 15, 20 and 25 tundishes, and it can be seen that SSLR has better effect.

In Fig. 3, 100 charges were optimized, and the stopping condition was set when the dual gap was equal to 10%. It can be seen from the experimental results that when the number of charges is less than 30, the running time used by the two algorithms is not much different. When the number of problems reaches a certain scale, the optimization time of SSLR algorithm is significantly less than the running time of SLR algorithm. As can be seen from the box graph, with the increase of data scale, the advantage of SSLR algorithm in optimization time becomes more obvious.

In Fig. 4, 100 charges are optimized, and adjustment coefficient of optimal solution is set to iterate until no more changes is the stop condition. It can be seen from the experimental results that the dual gap of SSLR algorithm optimization problem is smaller than that of SLR algorithm optimization problem. As can be seen from the box graph, with the increase of data scale, SSLR algorithm has more obvious advantages in optimizing quality.

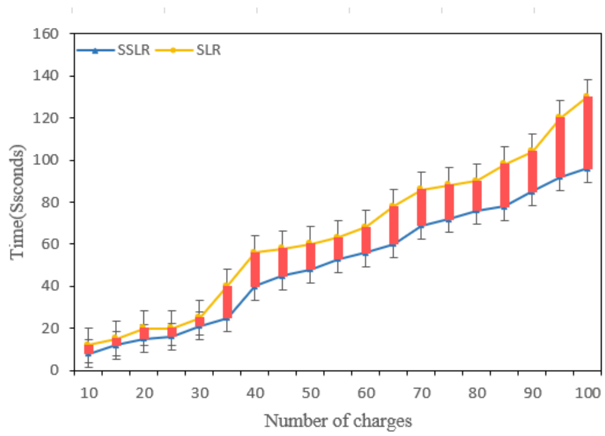


**Fig. 3.** Comparison of CPU Times

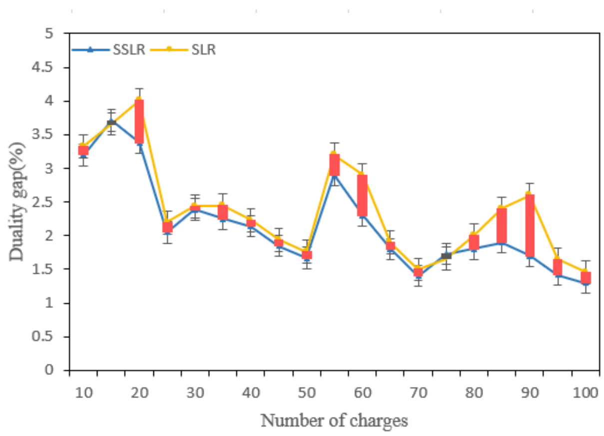


**Fig. 4.** Comparison of Duality Gaps

**Example 3.** Based on the control variable method, the CPU time under the same duality gap is compared, and the duality gap under the same CPU time is compared.

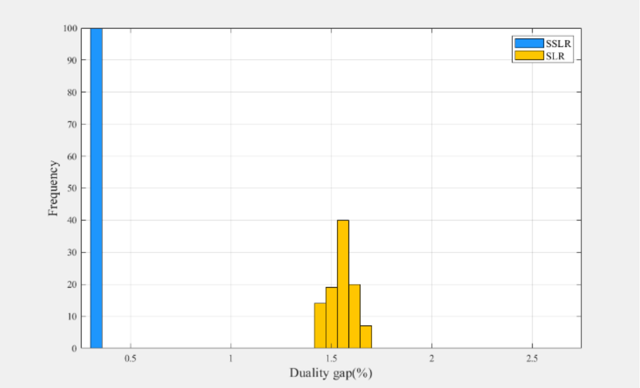


**Fig. 5.** Setting the Same Duality Gap (0.83%), Comparison of CPU Times

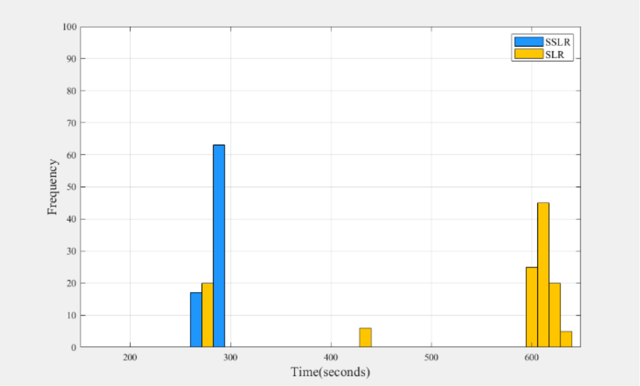


**Fig. 6.** Setting the Same CPU Time (230 Seconds), Comparison of Duality Gaps

In Fig. 5, the same duality gap (0.83%) is set to stop conditions, and the two algorithms optimize 100 sets of data (50 charges in each group of data). Compare the CPU time used. From the histogram, it can be seen that the SSLR algorithm is 100% completed 300 seconds ago, while the SLR completion is only 20%; the SLR completes 8% in the 400 to 500 seconds, and the remaining 72% will be completed after 590 seconds. It can be seen that when a large number of optimization charges batch planning, the SSLR algorithm optimization efficiency is higher than SLR algorithm.

In Fig. 6, the same optimization time (230 seconds) is set to stop conditions, and two algorithms optimize 100 sets of data (50 charges in each group). Compare the two algorithms to the duality gap. From the histogram, it can be seen that the SSLR algorithm can control 100% of the data to below 0.3% within 450 seconds; while the LR algorithm controls the gap between 1.35%-1.7% at the same time. It can be seen that when a large number of optimization charges batch planning, the SSLR algorithm optimization results are better.

## 2. Summary

In this paper, a heuristic Surrogate Subgradient Lagrangian Relaxation (SSLR) algorithm is proposed to optimize the problem of steelmaking - continuous casting tundish batch planning, and the capacity constraint of tundish service life is relaxed by a set of optimal solution adjustment coefficients. The relaxation problem is decoupled into the subproblems aiming at the optimal value of a single tundish, and backward dynamic programming method is used to solve some subproblems satisfying the agent optimization based on the state transition equation. Finally, a two - stage heuristic method is designed to adjust the relaxed constraints. Experimental results show that the proposed method is effective.

## Acknowledgements

Congxin Li would like to acknowledge financial support provided by National Natural Science Foundation of China (61873174), Liaoning Provincial Natural Science Foundation (2020-KF-11-07), Ministry of Education Cooperative Education Project (201902233028), and China Postdoctoral Science Foundation Funded Project (2017M611261).

## References

Cui, H., Luo, X., 2017. An improved Lagrangian relaxation approach to scheduling steelmaking-continuous casting process. J. Computers & Chemical Engineering. 106, 133-146.

Dong, H.Y., Huang, M., Wang, X.W., et al., 2009. Model and algorithm for integrated tundish planning. J. Control and Decision. 24(11), 1729-1734.

Dong, H.Y., Huang, M., et al., 2012. Improved variable neighborhood search for integrated tundish planning in primary steelmaking processes. J. International Journal of Production Research. 50(20), 5747-5761.

Han, D., Tang, Q., Zhang, L., et al., 2016. Lagrangian lower bound solution based method for steelmaking-continuous casting production scheduling. J. Journal of Wuhan University of Science & Technology. 39(5), 354-360.

Li, C.X., Yuan, B.L., Ren, J.H., et al., Research on optimization of charge batch planning based on Augmented Lagrangian Relaxation algorithm. C. IFAC-PapersOnLine. 52(1). 814-820.

Ma, T.M., Luo, X.C., Chai, T.Y., 2015. Multi-objective tundish planning model and hybrid optimization algorithm. JOURNAL OF SYSTEMS ENGINEERING. 30(4), 451-465.

Pang, X.F., Liang, G., Pan, Q.K., et al. 2017. A novel Lagrangian relaxation level approach for scheduling steelmaking-refining-continuous casting production. J. Journal of Central South University. 24(2), 467-477.

Sun, J.Y., Sun, Z.Z., Chen, H.B., et al., 2018. Variable neighborhood search for integrated determination of charge batching and casting start time in steel plants. J. Journal of Intelligent & Fuzzy Systems. 34(6), 3821-3832.

Tang, L.X., Wang, G.S., 2008. Decision support system for the batching problems of steelmaking and continuous-casting production. J. Omega. 36(6), 976-991.

Tang, L.X., Wang, G.S., Liu, J.Y., et al., 2011. A combination of Lagrangian Relaxation and column generation for order batching in steelmaking and continuous-casting production. J. Naval Research Logistics. 58(4), 370-388.

Yi, J., Li, W.G., Zhu, J., et al., 2012a. Optimization method for tundish plan under new technique condition of steelmaking-continuous casting. J. Iron and Steel. 47(8), 31-35.

Yi, J., Tan, S.B., Li, W.G., et al., 2012b. Hybrid optimization algorithm for solving combining tundish MTSP model on continuous casting plan. J. Journal of Northeastern University. 33(9), 1235-1239.

Zhu, W.Y., Li, Y.P., Sun, L.L., et al., 2021. Research on batch planning method of steelmaking-continuous casting tundish. C. 3rd International Conference on Industrial Artificial Intelligence.

Zhao, X., Luh, P.B., Wang, J., 1999. Surrogate gradient algorithm for Lagrangian relaxation. J. Journal of Optimization Theory & Applications. 100(3), 699-712.

***Congxin Li*** is currently pursuing her PhD study at the School of Mechanical Engineering, Shenyang Jianzhu University, Shenyang, China. He obtained his BS degree from North University of China, China in 2017 and his MS degree from Shenyang Jianzhu University, China in 2020. His main research interests are in the areas of include industrial scheduling decision making and batch planning

***Tingwei Pan*** is currently pursuing his baccalaureate study at the School of Civil Engineering, Shenyang Jianzhu University, Shenyang, China. His main research interest is structural mechanics

***Jiaxin Guo*** is currently studying for a bachelor's degree in transportation at Dalian Jiaotong University, Dalian, China. His main research interest is transportation.

***Xinqi Hu*** is currently pursuing his baccalaureate study at the School of Electrical and Contral Engineering, Shenyang Jianzhu University, Shenyang, China. His main research interest is Electrical Engineering and Automation.

***Liangliang Sun*** obtained the B.S. degree in Information and communication engineering from Beijing Jiaotong University, Beijing, China, in 2003, and the M.S. and Ph.D. degree in control theory and control engineering form Northeastern University in 2007 and 2015, respectively. He is a professor with the School of Control Engineering, Northeastern University at Qinghuangdao. His research interests are intelligent manufacturing systems-planning, scheduling, and coordination of design, manufacturing, and service activities.

***Jiayu Peng*** is currently pursuing her PhD study at the School of Mechanical Engineering, Shenyang Jianzhu University, Shenyang, China. She obtained the BS degree and the MS degree in Automation and Control Engineering from Shenyang Jianzhu University in 2018 and 2021. Her main research interests are in decision making and optimization of complex industrial production scheduling.

***Jingjing Lou*** received the M. S. degree in Control Theory and Control Engineering from Shenyang Jianzhu University, Shenyang, China, in 2006. She is currently pursuing the Ph. D. degree in mechanical engineering at Shenyang Jianzhu University, Shenyang, China. She is currently a Professor with the School of Mechanical Information, Yiwu Industrial & Commercial College. Her research interests include computer graphic, computer aided design, geometric design, control engineering and computational geometry.