Nonlinear Output Feedback Output Tracking of Mold Vibration Displacement System Driven by Servo Motor

Qiang Li,a,b, Xiancong Wu,a,b, Qing Lv,a,b, Jin Wang a,c,*

a College of Career Technology, Hebei Normal University, Shijiazhuang, Hebei, 050024, China
b Hebei Provincial Innovation Center for Wireless Sensor Network Data Application Technology, Hebei, 050024, China
c Hebei Provincial Key Laboratory of Information Fusion and Intelligent Control, Shijiazhuang, Hebei, 050024, China

ARTICLE INFO
Article history:
Received 15 June 2023
Accepted 21 July 2023
Available online 25 July 2023

Keywords:
continuous casting mold; non-sinusoidal vibration; prescribed performance; high order sliding mode differentiators; backstepping control

ABSTRACT
In this paper, a nonlinear output feedback output tracking control scheme based on prescribed performance is proposed for the tracking of mold vibration displacement driven by a servo motor. Firstly, asymptotic trajectories of internal states are conducted using the output and its derivatives, which obtained by the designed high-order sliding mode differentiators. Secondly, a performance function and an output error transformation are introduced to establish an equivalent system model which simplified the controller design. Thirdly, based on the Lyapunov stability theory, a backstepping controller is designed to guarantee the transient and steady state performance of the system. Simulation results show that the proposed scheme is effective.

1. Introduction

Continuous casting mold adopting non-sinusoidal vibration has been recognized as one of the key technologies in developing high efficient continuous casting. It is a new drive mode that servo motor as actuator drives mold to vibrate following the expected non-sinusoidal waveform[13]. In this drive mode, the servo motor rotates continuously in single direction and variable angular velocity to get the non-sinusoidal waveform. Compared with the existing drive mode, the new mode has many advantages such as simplified structure, long service life, saving energy, and so on[2]. It has high requirements on system performance while two issues should be considered: one is nonlinear relationship between servo motor speed and mold displacement. The other is uncertainties such as accuracy of gear reduction ratio, time-vary load and eccentric shaft mechanical zero offset. So, tracking control of nonlinear output under constraint conditions is the main research spot in this paper.

The control system, applied for nonlinear system in industry, aerospace, ship control and other fields has attracted considerable research effort during the past several years[3]. Various control approaches for tracking control have been presented, such as variable structure, adaptive backstepping, optimal trajectory with nonlinear feedforward and linear feedback and so on[4,5]. As to output feedback tracking control, many methods have some deficiencies in terms of system transient and steady state. Prescribed performance could guarantee output error converge to a predefined small residual set, with convergence rate no less than a certain pre-specified value and maximum overshoot less than a pre-assigned level. In recent years, the prescribed performance control (PPC) has attracted large attention in applying in actual field[6]. For example, the PPC is applied to the feedback linearized system for a class of SISO system[7]. In [8,9], PPC combines the robust control to guarantee system tracking performance for the strict feedback nonlinear systems with external disturbances. The adaptive control based on the PPC is adopted to solve the non-linear problem in institute systems[10,11]. In present, the PPC is promoted to the feedback linearized system for a class of MIMO system[12,13].

As to the characteristics and application requirements of mold non-sinusoidal vibration system driven by servo motor, we combine output feedback output tracking control with the PPC to improve system performance. Considering constraints of system states, using the output and its derivatives to conduct the asymptotic trajectories of internal states, and the output tracking problem is transformed to the solving of system stabilization. Then, a performance function and an output error transformation are introduced to transform the original system to an equivalent one. Based on the Lyapunov stability theory, a backstepping controller is designed and the system is stable according to stability analysis. Finally, the simulation results show that the proposed scheme is effective.

* Corresponding author.
E-mail addresses: 15933118610@163.com (J. Wang)
Digital Object Identifiers: https://doi.org/10.62953/IJAMCE.363406
2. System description

As to this system, the servo motor rotates continuously in single direction and variable angular velocity to get the non-sinusoidal waveform through the mechanical transmission parts. So the system mainly contains two parts: servo motor and mechanical transmission.

The structure drawing is shown in Fig. 1.

![Fig. 1 Drawing of continuous casting mold driven by servo motor](image)

The model of servo motor is expressed as follows:

\[
\begin{align*}
\dot{i}_q &= (-R_{i_q}i_q + L_p i_q)u_q / L \\
\dot{i}_d &= (-L_p i_d - R_{i_d}i_d - p\psi_q \omega + u_d) / L \\
\dot{\omega} &= (1.5 p \psi_q i_d - B\omega - T_s) / J \\
\dot{\theta} &= \omega
\end{align*}
\]

where: \( R_{i_d} \) is the stator resistance, \( i_d \) and \( i_q \) are the \( d \) and \( q \) axes stator currents, \( u_d \) and \( u_q \) are the \( d \) and \( q \) axes stator voltages, \( P \) is the pole pairs numbers, \( L \) is the stator inductance, \( \omega \) is the rotor angular velocity, \( \theta \) is the angular position of the rotor, \( \phi_1 \) is the flux linkage, \( J \) is the rotor inertia, \( B \) is the viscous friction coefficient, and \( T_s \) is the load torque.

Considering the gear reduction ratio accuracy and zero offset of eccentric shaft mechanical, the mechanical transmission parts is described as follows:

\[
x_h = h \sin \left( \frac{1}{i + \Delta i} \frac{2 \pi n}{60} \right)
\]

To simplify the controller design, define the state variable \( z \):

\[
z = \frac{2 \pi}{60} n + D(t)
\]

The integral nonlinear mathematical model is obtained as follows:

\[
\begin{align*}
\dot{\theta} &= \frac{2 \pi}{60} n + D(t) \\
\dot{n} &= \frac{1.5 p \psi_q i_d}{J} + \left( -\frac{B}{J} \right) - \frac{2 \pi}{J} \\
i_q &= \frac{2 \pi}{60} p \psi_q i_d - \frac{R_{i_q} i_q}{L} - \frac{p \psi_q}{L} + \frac{n + \frac{u_q}{L}}{L} \\
i_d &= \frac{R_{i_d} i_d}{L} + \frac{2 \pi}{60} p \psi_q i_d + \frac{u_d}{L}
\end{align*}
\]

where: \( D(t) = \int c(t) d\tau + d \) can be considered as the lumped mechanical disturbances, including uncertainties exist in reducer and original zero offset of eccentric shaft. The lumped mechanical disturbances \( D(t) \) is continuously differentiable.

Based on the framework of vector control, a structure of cascade control loop including a position tracking loop and two current tracking loops is employed. Here we adopt two PI controllers in two current loops. The reference current \( i_d^* = 0 \) is set to make the current states decoupled. The reference current \( i_q^* \) is determined by the position loop controller which is designed by the former two equations in (4):

\[
\begin{align*}
\dot{\theta} &= \frac{2 \pi}{60} n + D(t) \\
\dot{n} &= \frac{1.5 p \psi_q i_d}{J} + \left( -\frac{B}{J} \right) - \frac{2 \pi}{J} \\
i_q &= \frac{R_{i_q} i_q}{L} + \frac{2 \pi}{60} p \psi_q i_q + \frac{u_q}{L}
\end{align*}
\]

3 Output feedback controller design

**Define 1.** System (7) is assumed to be algebraically strongly observable with respect to output \( y(t) \) if there exists a function \( P(t) \) such that

\[
\begin{align*}
\dot{p}_i(t) &= f_i(y, \dot{y}, \cdots, y^{(i)}) \\
p_i(t) &= f_i(y, \dot{y}, \cdots, y^{(i)}) \\
\cdots \\
p_i(t) &= f_i(y, \dot{y}, \cdots, y^{(i)})
\end{align*}
\]

In actual system, mold displacement can be directly measured. Then system internal state trajectories could be construct through system output and its derivatives only. If the system states \( x_i(t) \) has asymptotic trajectories with \( p_i(t) \):

\[
x_i(t) - p_i(t) \to 0, \quad (t \to \infty), \quad i = 1, 2, \ldots, n.
\]
Then the system controller design is converted to the output-feedback stabilization about \( x(t) - p(t) \).

### 3.1 Construct internal states asymptotic trajectories

According to (7) and assumption (A1), (A2), we can get:

\[
p_2 = \frac{\dot{y} \text{sgn}(y)}{\sqrt{h^2 - y^2}} y \neq h, \sin \theta \neq \pm 1
= \frac{\dot{y}}{h} y = h, \sin \theta = \pm 1
\text{sgn}(y) = \text{sgn}(\cos \theta)
\]  

(9)

Form (9) and (10), the following relationship under condition \( p_2 \in [0, \infty) \) can be obtained:

\[
p_i = \dot{\hat{h}} = \int \dot{\hat{y}} \text{sgn}(\hat{y}) \frac{d\hat{y}}{h \sqrt{1 - \sin^2 \theta}} dt = -\int \frac{\text{sgn}(\hat{y})}{h \sqrt{1 - \sin^2 \theta}} dy
= \frac{\text{sgn}(\hat{y})}{h} \text{cos} \theta d\theta = \frac{\text{sgn}(\hat{y})}{h} \text{cos} \theta d\theta
= k \pi + \text{sgn}(\cos \theta) \arcsin(\sin \theta) + C
= k \pi + \text{sgn}(\hat{y}) \arcsin(\frac{\hat{y}}{h}) + C
\]

where: \( k=0,1,2,3, \ldots \) and \( C = 0 \) at the initial state. The results of (11) is consistent with those in literature [16].

### 3.2 Estimate the derivatives

In this paper, high-order sliding mode (HOMS) differentiator are used to estimate the required derivatives of the measured output.

Let \( y(t) \) be a component of the measured output to be differentiated \( k \) times. The \( k \)-th order HOMS differentiator takes the form

\[
\begin{align*}
\dot{\eta}_0 &= \eta_0 = -\lambda_i \dot{L}^\eta_i \eta_0 - \dot{y} \text{sgn}(\eta_0 - y) + \eta_1 \\
\dot{\eta}_1 &= \eta_1 = -\lambda_i \dot{L}^\eta_i \eta_1 - \eta_0 \text{sgn}(\eta_1 - \eta_0) + \eta_2 \\
\vdots & \\
\dot{\eta}_{k-1} &= \eta_{k-1} = -\lambda_i \dot{L}^\eta_i \eta_{k-1} - \eta_{k-2} \text{sgn}(\eta_{k-1} - \eta_{k-2}) + \eta_k \\
\dot{\eta}_k &= -\lambda_i \dot{L}^\eta_i \eta_k - \eta_{k-1} \text{sgn}(\eta_k - \eta_{k-1})
\end{align*}
\]

where \( \eta_k \) is the estimation of the true derivative \( y^{(k)} \). The differentiators provides finite-time exact differentiation under ideal conditions in continuous time. The only needed information is a priori known upper bound \( L \) for \( \{y^{(k)} \} \). Then an infinite parametric sequence \( \{ \lambda_i \} > 0, i = 0,1, \ldots, k \), is recursively built, which provides the convergence of differentiators for each order.

### 3.3 Prescribed performance

The mathematical expression of prescribed performance is given by the following inequalities:

\[ -\rho(t) < e_i(t) < \rho(t), \quad e_i(t) \geq 0 \]  

(8)

where \( \rho(t) \) is a function with continuously differentiable, bounded, strictly positive and \( \lim_{t \to \infty} \rho(t) > 0, 0 \leq \sigma \leq 1 \).

In this paper, we select the following exponential performance function \( \rho(t) = (\rho_i - \rho_0) e^\sigma + \rho_0, \quad \forall t \geq 0 \) with \( \rho_0, \rho_\sigma, \sigma \) chosen strictly positive constants respectively. Given any initial condition, the constant \( \rho_0 = \rho(0) \) is selected such that \( 0 < \rho_i(0) < \rho_0 \). The constant \( \rho_\sigma \) represents the maximum allowable size of tracking errors in steady state, which may even be set arbitrarily small to a value reflecting the resolution of the measurement device. And then the tracking error convergences to zero. Moreover, the decreasing rate of \( \rho \), which is affected by the constant \( \sigma \) in this case, introduces a low bound on the required speed. Therefore, selection of the performance function \( \rho \) imposes performance characteristics on the tracking error.

### 3.4 Error transformation

An error transformation is incorporated to modulate the tracking error element with required performance bounds imposed by \( \rho(t) \). So we can get the following equation:

\[ e(t) = \rho(t) \bar{S}(e) \]  

(10)

where: \( \varepsilon \) is transformation error and \( S(e) \) is a smooth and strictly increasing function, defined as follows:

\[
\begin{align*}
-\sigma < S(e) &< 1, e(0) > 0 \\
-1 < S(e) &< \sigma, e(0) < 0
\end{align*}
\]

\[
\begin{align*}
\lim_{t \to +\infty} S(e) &< -\sigma \quad e(0) > 0 \\
\lim_{t \to +\infty} S(e) &< 1 \quad e(0) < 0
\end{align*}
\]

When \( e(0) > 0 \), \( -\sigma < S(e) < 1 \), \( \rho(t) > 0 \), then

\[
-\sigma \rho(t) < \rho(t) \bar{S}(e) < \rho(t)
\]  

(11)

According to (5), the following inequality will be obtained:

\[
-\sigma \rho(t) < e(t) < \rho(t)
\]  

(12)

In the similar way, when \( e(0) < 0 \), then it can get that

\[
-\rho(t) < e(t) < \sigma \rho(t)
\]  

(13)

The following equation is obtained by the inverse transformation:

\[
\varepsilon = T \left( \frac{e(t)}{\rho(t)} \right)
\]  

(14)

where: \( T = S^{-1} \). If \( \varepsilon(t) \in L_1, t \in [0, \infty) \), according to the strictly deceasing character of \( \rho(t) \), the tracking error will be bounded in the following area:

\[
E = \{ \varepsilon \in R^1, |\varepsilon(t)| \leq \rho_\sigma \}
\]  

(20)

### 3.5 Control law design with backstepping mode

The error variable is defined as follows:

\[
\begin{align*}
e_i(t) &= \dot{\hat{h}} - \theta' \\
e_i(t) &= \dot{\hat{z}} - \hat{\theta}'
\end{align*}
\]

(15)

Transform the original model to the error model:

\[
\begin{align*}
\dot{\hat{e}}_i &= \dot{e}_i \\
\dot{\hat{e}}_i &= \frac{1.5 \rho(t)}{J} i_k + \frac{B}{J} \ddot{z} + \dot{N}(t) - \hat{\theta}'
\end{align*}
\]

(16)
The error transformation is found from the first equation in (22)

\[ e_i(t) = \rho(t)S(x) \]

and its derivative can be get as follows:

\[ \dot{e}_i(t) = \dot{\rho}(t)S(x) + \rho(t)\frac{\partial S}{\partial \dot{e}} \dot{e}(t) \]

Substituting (24) into (22)

\[ \dot{e}(t) = F(x) + G(x)e_z(t) \]

where: \( F(x) = -\rho(t)S(x) \frac{\partial S}{\partial \dot{e}} \dot{e} \), \( G(x) = \frac{\partial S}{\partial \dot{e}} \).

Using \( F, G \) indicate \( F(x), G(x) \) for convenience.

**Step 1:** Transform the first equation in (22) as follows:

\[ \dot{e}(t) = F(x) + G(x)e_z(t) \]

Define the stabilizing function

\[ \alpha_i = (-k_i e - F)/G \]

Define \( z_2 = e_2 - \alpha_i \), then the derivative of \( V_z \) is

\[ V_z = e_2^2 = -k_i e^2 + Ge_z \]

**Step 2:**

From (19)

\[ \theta(t), \dot{\theta}(t), \dot{\omega} \]

The servo motor parameters: rated power \( P_s = 20.4kW \), rated current \( I_n = 45A \), rated speed \( n_n = 1500r/min \), stator inductance \( L = 4.6m\Omega \), stator resistance \( R = 0.1452 \), number of pole pairs \( p = 3 \), rotor flux linkage \( \psi_r = 0.96Wb \), rotor inertia \( J = 0.0547N\cdotm^2 \), viscous friction coefficient \( B = 0.004Nms/rad \).

The mechanical transmission parts parameters: reduction ratio of the reducer \( i = 5 \).

The HOSM differentiators parameters: \( L = 800 \), \( \lambda_0 = 1.1 \), \( \lambda_1 = 1.5 \), \( \lambda_2 = 2 \).

The prescribed performance function: \( \rho(t) = (1 - 10^{-4})e^{-5t} + 10^{-3} \), the error transmission function \( S(x) = e^x - e^{-x} \).

The controller parameters \( k_1 = 1 \times 10^6 \), \( k_2 = 10 \).

The given signal for the mold is Demark non-sinusoidal function:

\[ x_m = h \sin(\alpha t - A \sin(\alpha t)) \]

and the given displacement of eccentric shaft is:

\[ \dot{\theta} = \alpha t - A \sin(\alpha t) \]

where: \( h \) is amplitude of the mold, \( \alpha = 2\pi f / 60 \), \( f \) is the non-sinusoidal vibration frequency of the mold.

\[ A = \frac{\pi \alpha}{2 \sin^2(\alpha)} \]

The load disturbance (including mechanical parts of the mold friction disturbance) is adopt the identification data available in literature [15]

\[ T_i = (5.1335 + 6.4985 \sin(\alpha t - A \sin(\alpha t)))N \]

The reduction ratio uncertainty is mainly caused by machining precision. According to the common requirements of machining precision \( \pm 3\% \), this paper take the worst situation: \( \Delta i = 3\%i \).

The initial zero offset is: \( d = -0.2rad \). So the lumped mechanical disturbances in this paper is chosen as follows:
\[ D(t) = \int \frac{-\Delta i}{i(i+\Delta i)} \frac{2\pi}{60} n \Delta t - 0.2 \text{ rad} \]

Fig. 3 expresses the relationship between tracking error and the prescribed performance. It obviously shows that the tracking error remains within the prescribed performance bounds and illustrates the robustness of the proposed control scheme.

Fig. 4 shows the position tracking of the mold, the curve indicated that the actual mold displacement can keep up with the desired signal and stable tracking in a very short period, and the tracking effect is good.

As shown in Fig. 5, it shows that tracking error is bigger in initial time, but this is precisely confirm to the system inertia characteristics and ensure the smooth start. Finally, the system could track the given displacement in short period. Compared with the single backstepping method, the design controller can make the mold tracking accuracy further improved after the system stability.

Fig. 6 and Fig. 7 show the motor angular velocity and d-q axial current respectively. The curves indicate that the motor rotates in single direction and variable angular velocity. Meanwhile, it ensures the motor speed conversion rate would not be too large under the action of the current limiter. The d-q axial current is also in line with expectations.

5 Conclusion

In view of the mold non-sinusoidal vibration system driven by servo motor, the integrated model is established that considering the uncertain factors such as the gear reduction ratio accuracy and zero offset of eccentric shaft. As to the system structure characteristics and state-parameter constraints, the output feedback control and high order sliding mode differentiator are used to build the system internal state asymptotic trajectories. An output feedback controller which adopts backstepping method is proposed based on the prescribed performance in this paper. Through the prescribed performance, the proposed controller can restrain the uncertain disturbance of the system and certain robust interference suppression is effective for the presence of unknown external disturbance. The simulation results show that the designed output feedback controller can satisfy the system control requirements.

Acknowledgements

This work was supported by the Doctoral research project of Hebei Normal University in Hebei Province (No. L2022B25)

References


Qiang Li received the B.S. degree in control science and engineering from Shandong University in 2005. He is currently pursuing the Ph.D. degree with Yanshan University. His research interests include nonlinear control, adaptive control, and system identification.

Xiancong Wu was born in Shijiazhuang, Hebei, P.R.China, in1987. She received the master degree from North China Electric Power University, P.R.China. Now, she works in College of engineering, Hebei Normal University, her research interests include intelligent control and intelligent algorithms.

Qing Lv received her Ph.D. degrees from the LILLE 1 University - Science and Technology, France. She is an associate professor of Hebei Normal University, China. Currently, she is involved in research dealing with microwave characterization of materials and optical communication technology.

Jin Wang was born in Huanghua, Hebei, P.R.China, in1989. She received the master degree from North China Electric Power University, P.R.China. Now, she works in College of engineering, Hebei Normal University, her research interests include intelligent control and intelligent algorithms.