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Adaptive Neural Network Tracking Control for Nonlinear Systems with Multiple Actuator Constraints via Command Filter

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ABSTRACT

In real physical systems, too large control inputs can easily cause serious accidents, so it is of great practical significance to study the control problems of nonlinear systems with input dead zones and saturations. In this paper, an adaptive NNs command filter tracking control algorithm for multi-actuator constrained nonlinear systems based on backstepping method is designed based on finite time stability theory. By introducing a second-order command filter, the complexity explosion problem of the traditional backstepping method for designing controllers is solved. The designed control algorithm ensures good tracking control of the controlled system under the corresponding saturation and dead zone input environments. With the neural network command filtering control scheme, all variables in the controlled system are ensured to be bounded and the output tracking error fluctuates within a small domain of the equilibrium point. Simulations demonstrate the feasibility of the designed control scheme.

1. Introduction

In reality, there are many real physical systems that can be characterized as uncertain nonlinear systems, which has led to their extensive development in the last two decades, and tracking control of nonlinear systems is one of the interesting research works[1-3].The presence of uncertainty makes the construction of controllers for nonlinear systems a difficult and significant task[4-6].The backstepping method is a practical approach to nonlinear control problems, was proposed by Krstic, Kanellakopoulos and Kokotovic at the end of the last century[7].Combining the backstepping method with the fuzzy or neural adaptive technique yields an effective control tool for solving uncertain nonlinear systems[8]. Due to the Characteristics of the adaptive backstepping method, it is possible to achieve asymptotic sedimentation of nonlinear systems and guarantee boundedness of the signal under parameter uncertainty, which has led to many fruitful results[9-11]. In the literature[12], adaptive tracking control of nonlinear systems with unknown input constraint and unpredictable variables is studied. In the literature[13] a controller design strategy based on separation of variables is designed for non-strict feedback nonlinear systems. The neural adaptive FTC technique allows the controlled system to achieve good tracking performance in finite time and the whole variables of the closed-loop system are bounded[14-16].

Despite the fact that the design of controllers for nonlinear systems using adaptive inversion algorithms has solved many problems in the field of control, the algorithm still suffers from many problems[17].It should be noted that, since the virtual control inputs in the controller design process need to be differentiated and iterated repeatedly, the design of controllers for nonlinear systems using adaptive inversion algorithms will become more and more computationally intensive along with the increase in the order of the system, a phenomenon we refer to as ”complexity explosion”[18].In order to solve the defect of ”complexity explosion” of traditional backstepping algorithms, two methods of dynamic surface control (DSC) and command filter backstepping control have emerged[19-20]. However, as DSC ignores the errors introduced by the filter, it affects the control accuracy of the controlled system. Since then, the backstepping command filtering method has been combined with the adaptive technique to achieve significant results in eliminating the filter errors[21-22].

Compared with asymptotic sedimentation methods, finite-time control methods have the advantages of fast convergence, high accuracy, good performance, and robustness to uncertainty by, and have achieved fruitful results[23].In practical engineering should be applied, nonlinear problems such as hysteresis, dead band,

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saturation and external disturbances often occur. Deadband and saturation, as a non-smooth function, have a large impact on system control performance. Therefore, special attention should be paid in the controller design process of nonlinear systems. Li et al. combined the obstacle Lyapunov function with an adaptive backstepping control method to solve a FTC problem for nonlinear systems with dead zones[24].In recent years, many effective results have emerged for nonlinear systems with different input constraints[25-26].However, to our knowledge, few research results that use a combination of neural adaption and command filtering to solve simultaneous input deadband, saturation, and nonlinear disturbances. Therefore, it is an interesting task to study finite-time stabilized controllers for nonlinear systems with simultaneous multiple input constraints, and Non-linear disturbances.

In summary, it can be seen that external disturbances and the presence of input constraints on the actuator can have a significant impact on system control performance and safety, especially in control processes such as Mars drones far from Earth and chemical reactions with major safety incidents. Therefore, in this paper, for a class of uncertain nonlinear systems with multiple actuator constraints and external disturbances, a practical adaptive neural network FTC method is designed to reduce the effects of actuators and disturbances on the tracking performance of the system. Compared with the current research results, the main contributions can be summarized as follows:

1. In this paper, we study the problem of adaptive NNs backstepping control for strictly feedback nonlinear systems with multiple actuator constraints and external disturbances by combining neural adaptive and command filter techniques.

2. The introduction of command filter and compensation mechanisms solves the problem of exploding complexity arising from the traditional backstepping method of designing controllers. The designed control algorithm can explode the controlled system to achieve good tracking control performance while suffering from saturation and dead zone inputs.

3. The control algorithm designed in this paper ensures that all signals of the controlled system are bounded, and the tracking error can quickly converge in finite time to the bounded adjustable tight set range.

2. Problem formulation and preliminaries

2.1 System Model

Consider the following strictly feedback nonlinear system.

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\overline{x}_i) + d_i(t), 1 \le i \le n-1, \\ \dot{x}_n = u(t) + f_n(\overline{x}_n) + d_n(t), \\ y = x_1 \end{cases} \tag{1}$$

in which $\overline{x}_i = [x_1(t),\ldots,x_i(t)] \in \mathbb{R}^n, i = 1,\ldots,n$ expresses the system state variables, and $y \in \mathbb{R}$ indicates the system output; $f_i(\overline{x}), i = 1,\ldots,n$ displays the unknown smooth nonlinear functions. $d_i(t)$ represents unknown bounded external interference with $x_{i+1} + f_i(x) \ne 0$. $u(t)$ represents control inputs subject to nonlinearities of multiple actuator constraints and is described as

$$u = s(v)v + t(v) \tag{2}$$

where v represents the input signal for dead zone and saturation nonlinear models. Select l_+, l_-, u_+, u_- are positive design constants, $u_H > 0, u_L > 0$ denote the normal number to be designed. $m(v)$ called the dead zone slope, $m(v), \pi(v)$ is description as follows

$$s(v) = \begin{cases} \dfrac{u_H}{v}, v > u_H \\ l_+, \quad u_+ < v < u_H \\ l_+, \quad -u_- < v < u_+ \\ l_-, \quad -u_L < v < u_- \\ -\dfrac{u_L}{v}, v < -u_L \end{cases} \tag{3}$$

$$t(v) = \begin{cases} 0, v > u_H \\ -l_+ u_+, u_+ < v < u_H \\ -l_+ v, \quad -u_- < v < u_+ \\ -l_- u_-, -u_L < v < u_- \\ 0, v < -u_L \end{cases} \tag{4}$$

2.2 Mathematical Preparation

The objective of this paper is to design a new finite-time tracking control algorithm for non-linear systems with tight feedback so that the system output can trace the wanted trajectory signal in finite time and all the variables of the considered system are well bounded, so the below assumptions and lemmas are implemented without loss of generality.

Assumption 1:[27] The positive and negative slope of the dead zone and saturation nonlinear models are equal, i.e $l_+ = l_- = l$.

Assumption 2:[27] Dead-zone parameters of the controller u_+, u_- and l are bounded , that is, there are known parameters $u_{+\max}, u_{-\max}$ and $l_{\max}$ that $|u_+| < u_{+\max}, |u_-| < u_{-\max}$ and $|l| < l_{\max}$.

Assumption 3:[28] The anticipated tracking trace signals y_d and their nth-order derivatives $y_d^{(n)}$ considered in this paper are continuous and bounded.

Remark 1: In real production process environments, there are usually special requirements for system actuator inputs, such as controlling the maximum amplitude of the inputs within limits and trying to avoid fluctuations around zero to minimize consumption. Deadband and saturation are typically used to address the above requirements, and constraints on the control inputs can be accomplished by setting the actuator constraint-related parameters in advance according to actual requirements. Therefore, assumptions 1 and 2 above are reasonable and simplify the complexity of the subsequent controller design process. In real physical systems, we always want the control process of the actuator to be smooth, which is beneficial to the actuator. Therefore, assumption 3 is also reasonable. From Assumptions 1 and 2, it is not difficult to obtain that $\pi(v)$ is bounded, and $|\pi(v)| \le D$,in which D represents the upper limit value.

Remark 2: Consider the actual situation, the control input signal w cannot be infinite. Therefore, consider that $m(v)$ satisfies the following inequality

$$0<\gamma \le \min\left\{\frac{u_H}{v_{\max}}, l\right\} \le m(v) \le \max\{1, l\} \tag{5}$$

in which $v_{\max}$ represents the maximum value of the designed controller.

An RBF neural network is a three-layer forward network containing input layer (signal source node), hidden layer (layer of neurons), and output layer (linear combination of the outputs of the neurons in the hidden layer). Noteworthy is that neural networks have good parallel processing capability, approximation of arbitrary smooth nonlinear functions and self-organized learning.

Lemma 1:[29] During the design of the system controller, RBF neural networks will be utilized to address the uncertainty terms in the uncertain nonlinear system. Specifically as follows：

Supposing $q(\Lambda)$ is a continuous function defined on $\Omega_Z \subset R^n$, for $\forall \varepsilon > 0$, $|\sigma(\Lambda)| \le \varepsilon$,and $l>0$ being the NN node number, there exists a RBF NN satisfying:

$$q(\Lambda) = \Psi^{*T} P(Z) + \sigma(\Lambda) \tag{6}$$

Where

$$\begin{cases} \Psi^* = \arg\min_{\Psi \in R^n} \left\{ \sup_{\Lambda \in \Omega_\Lambda} \left| q(\Lambda) - \Psi^{*T} P(Z) \right| \right\} \\ \sup_{Z \in \Omega} \left| q(\Lambda) - \Psi^T P(Z) \right| \le \varepsilon \end{cases} \tag{7}$$

in which $\Lambda = [\lambda_1 \;\; \lambda_2 \;\; \ldots \;\; \lambda_l]^T \in \Omega_\Lambda$ is the input of the NN, $\Psi = [\psi_1, \ldots, \psi_l]^T \in R^l$ is the desired weight vector, and $P(\Lambda) = [p_1(\Lambda), p_2(\Lambda), \ldots, p_l(\Lambda)]^T$ represents the basis function vector selected as

$$p_i(\Lambda) = \exp\left[-\frac{(\Lambda - \iota_i)^T (\Lambda - \iota_i)}{\kappa^2} \right], i = 1, 2, \cdots, l \tag{8}$$

Where ι_i, κ represents the center of the receptive field and width of the Gaussian function, respectively.

Lemma 2:[30] The filters used in this paper are described as follows

$$\begin{cases} \dot{\omega}_i = \omega_\tau \omega_{i,2} \\ \dot{\omega}_{i,2} = -2\varsigma \omega_\tau \omega_{i,2} - \omega_\tau (\dot{\omega}_i - \alpha_{i-1}) \end{cases} \tag{9}$$

where $\varsigma \in (0,1]$ and $\omega_\tau > 0$ indicates positive filter parameters. Virtual Controller α_{i-1} and ω_i as input and output of filter, respectively, in which $\omega_i(0) = \alpha_{i-1}(0), \omega_{i,2}(0) = 0$.

Lemma 3:[31] For any positive constant b_1, b_2, b_3 and a real variable x, y , the below inequality holds

$$|x|^{b_1} |y|^{b_2} \le \frac{b_1}{b_1+b_2} b_3 |x|^{b_1+b_2} + \frac{b_2}{b_1+b_2} b_3^{-\frac{b_1}{b_2}} |y|^{b_1+b_2} \tag{10}$$

Lemma 4:[32] Consider a dynamic system $\dot{x} = f(x,u)$, $x(0) = x_0$,in which $f: R^n \to R^n$ represents a smooth mapping. If there exists a Radically unbounded and deterministic positive scalar function $V(x)$, $\mu_1 > 0, \mu_1 > 0$ and $0 < q < 1, 0 < \delta < \infty$ such that

$$\dot{V}(x,t) \le -\mu_1 V(x) - \mu_2 V^q(x) + \delta \tag{11}$$

then the output of this system $\dot{x} = f(x,u)$ is practical finite-time stable, the set of residuals of the system solution is shown below

$$\lim_{t \to T_r} V(x) \le \min\left\{ \frac{\sigma}{(1-\varphi)\alpha}, \left(\frac{\sigma}{(1-\varphi)\beta}\right)^{\frac{1}{q}} \right\} \tag{12}$$

in which $\varphi \in (0,1)$. Then one can obtain the settling time bounded by

$$\begin{aligned} T_r \le \max\Bigg\{ & t_0 + \frac{1}{\varphi\mu_1(1-q)} \ln \frac{\varphi\mu_1 V^{1-q}(t_0) + \mu_2}{\mu_2}, \\ & t_0 + \frac{1}{\mu_1(1-q)} \ln \frac{\mu_1 V^{1-q}(t_0) + \varphi\mu_2}{\varphi\mu_2} \Bigg\} \end{aligned} \tag{13}$$

Remark 3: In the above equation, we choose the parameter as: $q = 0.75$. We also found this operation in the literature[33] , which facilitates the subsequent controller design process and satisfies the citation conditions.

3. Controller design and stability analysis

3.1 Finite-time Controller Design

In this subsection, an adaptive neural network controller will be designed for system 1 by backstepping algorithm. The controller will handle both the tracking performance of the system and the boundedness of the variables. Moreover, the complexity explosion of the classical backstepping algorithm is conquered by the command filtering method. The controller consists of some basic stages and the design is built on the following coordinate transformation

$$\begin{cases} z_1 = x_1 - y_d \\ z_i = x_i - \omega_i \end{cases} \tag{14}$$

in which y_d expresses the desired reference trajectory signal, ω_i with $i = 2, 3, \ldots, n$ represents the introduced command filter variable.

Remark 4: Note that the command filter will increase the operational burden of the actuator, due to the error defect introduced. To address this drawback, we will design a compensatory signal to compensate for the tracking fault $(\omega_i - \alpha_{i-1})$ incurred by the command filter.

In addition, to reduce the computational effort of controller design, we construct the compensation tracking error based on the idea of coordinate transformation as follows:

$$\xi_i = z_i - r_i, i = 1, \ldots, n \tag{15}$$

where r_i indicates the compensated signal.

Step 1: With the aid of (1) ,(12) and (13),one can get

$$\dot{\xi}_1 = \xi_2 + r_2 + \omega_2 + f_1 + d_1 - \dot{y}_d - \dot{r}_1 \tag{16}$$

Select a Lyapunov function V_1 as

$$V_1 = \frac{1}{2}\xi_1^2 + \frac{1}{2\lambda_1}\tilde{\theta}_1^2 \tag{17}$$

where $\lambda_1 = 2\rho_1/(2\rho_1 - 1), \rho_1 > 1/2$ denotes the parameters to be constructed. $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$,in which $\hat{\theta}_1$ denotes the estimate of the uncertain parameter θ_1 .

Taking the time derivative of V_1 ,one can get

$$\dot{V}_1 = \xi_1(-k_{1,2}\xi_1^{2q-1} + \xi_2 + r_2 + \omega_2 + h_1(Z_1) - \dot{r}_1) - \frac{1}{\lambda_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \quad (18)$$

where $h_1(Z_1) = f_1 - \dot{y}_d + d_1 + k_{1,2}\xi_1^{2q-1}$ is an unknown term that contains unknown terms. Then $h_1(Z_1)$ will be approximated by RBF NNs as

$$h_1(Z_1) = \phi_1^{*T} R_1(x) + \delta_1(x),, \quad |\delta_1(\zeta_1)| \le \varepsilon_1 \quad (19)$$

where $Z_1 = [x_1, y_d, \dot{y}_d]^T$,and $\delta_1(Z_1)$ is estimate error, arbitrary constants $\varepsilon_1 > 0$ .Applying Young's inequality, it produces

$$\xi_1 h_1(Z_1) \le \frac{1}{2a_1^2}\xi_1^2\theta_1 R_1^T R_1 + \frac{a_1^2}{2} + \frac{1}{2}\xi_1^2 + \frac{1}{2}\varepsilon_1^2 \quad (20)$$

where $\theta_1 = \|\phi_1^*\|^2, a_1 > 0$.

Next, the virtual controller is introduced as α_1 ,and design a compensating signal r_1 as

$$\alpha_1 = -k_{1,1}z_1 - \frac{1}{2a_1^2}\xi_1\hat{\theta}_1 R_1^T R_1 - \frac{1}{2}\xi_1 \quad (21)$$

$$\dot{r}_1 = -k_{1,1}r_1 + r_2 + \omega_2 - \alpha_1 \quad (22)$$

where $k_{1,1}$ indicates a parameter to be build.

With the aid of (17)-(20), one can get

$$\dot{V}_1 \le -k_{1,1}\xi_1^2 - k_{1,2}\xi_1^{2q} + \frac{\tilde{\theta}_1}{\lambda_1}\left(\frac{\lambda_1}{2a_1^2}\xi_1^2 R_1^T R_1 - \dot{\hat{\theta}}_1\right) + \xi_1\xi_2 + \frac{1}{2}a_1^2 + \frac{1}{2}\varepsilon_1^2 \quad (23)$$

Then, building an adaptive law

$$\dot{\hat{\theta}}_1 = \frac{\lambda_1}{2a_1^2}\xi_1^2 R_1^T R_1 - \eta_1\hat{\theta}_1 \quad (24)$$

Thus, on the basis of above equation, one has

$$\dot{V}_1 \le -k_{1,1}\xi_1^2 - k_{1,2}\xi_1^{2q} + \frac{\eta_1}{\lambda_1}\tilde{\theta}_1\hat{\theta}_1 + \xi_1\xi_2 + \frac{1}{2}a_1^2 + \frac{1}{2}\varepsilon_1^2 \quad (25)$$

Step i: With the aid of (1) ,(12) and (13),similar to step 1 as

$$\dot{\xi}_i = \xi_{i+1} + r_{i+1} + \omega_{i+1} + f_i + d_i - \dot{\omega}_i - \dot{r}_i \quad (26)$$

Select a Lyapunov function V_i as

$$V_i = V_{i-1} + \frac{1}{2}\xi_i^2 + \frac{1}{2\lambda_i}\tilde{\theta}_i^2 \quad (27)$$

where $\lambda_i = 2\rho_i / (2\rho_i - 1), \rho_i > 1/2$ represents a parameter to be constructed.

Then, Similarly to step 1, we get

$$\begin{aligned}\dot{V}_i \le & -\sum_{j=1}^{i-1} k_{j,1}\xi_1^2 - \sum_{j=1}^{i-1} k_{j,2}\xi_1^{2q} + \sum_{j=1}^{i-1}\frac{\eta_j}{\lambda_j}\tilde{\theta}_j\hat{\theta}_j \\ & + \xi_i(\xi_{i-1} - k_{i,2}\xi_i^{2q-1} + \xi_{i+1} + r_{i+1} + \omega_{i+1} \\ & + h_i(Z_i) - \dot{r}_i) - \frac{1}{\lambda_i}\tilde{\theta}_i\dot{\hat{\theta}}_i - \frac{1}{2}\xi_i^2 + \frac{1}{2}\sum_{j=1}^{i-1}(a_j^2 + \varepsilon_j^2)\end{aligned} \quad (28)$$

in which $h_i(Z_i) = f_i + \dot{\omega}_i + d_i + k_{i,2}\xi_i^{2q-1} - \xi_{i-1}$ .Similarly, there exists an NNs, exist $\forall\varepsilon_i > 0$, $h_i(Z_i) = \phi_i^{*T} R_i(x) + \delta_i(x)$ with $|\delta_i(Z_i)| \le \varepsilon_i$.

Combining the Young's inequality, we have

$$\xi_i h_i(Z_i) \le \frac{1}{2a_i^2}\xi_i^2\theta_i R_i^T R_i + \frac{a_i^2}{2} + \frac{1}{2}\xi_i^2 + \frac{1}{2}\varepsilon_i^2 \quad (29)$$

where $\theta_i = \|\phi_i^*\|^2, a_i > 0$.

Next, the virtual controller is introduced as α_i ,construct the compensating signal r_i ,and adaptive law $\dot{\hat{\theta}}_i$ as follows:

$$\alpha_i = -k_{i,1}z_i - \frac{1}{2}\xi_i - \frac{1}{2a_i^2}\xi_i\hat{\theta}_i R_i^T R_i \quad (30)$$

$$\dot{r}_i = -k_{i,1}r_i + r_{i+1} + \omega_{i+1} - \alpha_i \quad (31)$$

$$\dot{\hat{\theta}}_i = \frac{\lambda_i}{2a_i^2}\xi_i^2 R_i^T R_i - \eta_i\hat{\theta}_i \quad (32)$$

where $k_{i,1}$ indicates a normal number to be constructed. With the help of (27)-(30), we obtain

$$\dot{V}_i \le -\sum_{j=1}^{i} k_{j,1}\xi_1^2 - \sum_{j=1}^{i} k_{j,2}\xi_1^{2q} + \sum_{j=1}^{i}\frac{\eta_j}{\lambda_j}\tilde{\theta}_j\hat{\theta}_j + \frac{1}{2}\sum_{j=1}^{i}(a_j^2 + \varepsilon_j^2) + \xi_i\xi_{i+1} \quad (33)$$

Step n: With the aid of (1), (12) and (13), one can get the derivative of ξ_n as

$$\begin{aligned}\dot{\xi}_n &= u(t) + f_n - \dot{\omega}_n - \dot{r}_n + d_n \\ &= s(v)v + t(v) + f_n + d_n - \dot{\omega}_n - \dot{r}_n\end{aligned} \quad (34)$$

Select a Lyapunov function V_n as

$$V_n = V_{n-1} + \frac{1}{2}\xi_n^2 + \frac{1}{2\lambda_n}\tilde{\theta}_n^2 \quad (35)$$

Then, we obtain the $\dot{V}_n$ by (32) and (33) as:

$$\begin{aligned}\dot{V}_i \le & -\sum_{j=1}^{n-1} k_{j,1}\xi_1^2 - \sum_{j=1}^{n-1} k_{j,2}\xi_1^{2q} + \sum_{j=1}^{n-1}\frac{\eta_j}{\lambda_j}\tilde{\theta}_j\hat{\theta}_j \\ & + \xi_n(\xi_{n-1} - k_{n,2}\xi_n^{2q-1} + m(v)v + \pi(v) + h_n(Z_n) \\ & - \dot{\omega}_n - \dot{r}_n) - \frac{1}{\lambda_n}\tilde{\theta}_n\dot{\hat{\theta}}_n - \frac{1}{2}\xi_n^2 + \frac{1}{2}\sum_{j=1}^{n-1}(a_j^2 + \varepsilon_j^2)\end{aligned} \quad (36)$$

where $h_n(Z_n) = f_n + \dot{\omega}_n + d_n + k_{n,2}\xi_n^{2q-1} - \xi_{n-1}$ .Similar to (17) and (27), we can get

$$\xi_n h_n(Z_n) \le \frac{1}{2a_n^2}\xi_n^2\theta_n R_n^T R_n + \frac{a_n^2}{2} + \frac{1}{2}\xi_n^2 + \frac{1}{2}\varepsilon_n^2 \quad (37)$$

$$\xi_n\pi(v) \le \frac{1}{2}\xi_n^2 + \frac{1}{2}D^2 \quad (38)$$

where $|\pi(v)| < D$.

Now, the actual control signal v ,compensation signal $\dot{r}_n$,and adaptive law $\dot{\hat{\theta}}_n$ can be constructed as follows:

$$v = \frac{1}{\gamma}\left(-k_{n,1}z_n - \xi_n - \frac{1}{2a_n^2}\xi_n\hat{\theta}_n R_n^T R_n\right) \quad (39)$$

$$\dot{r}_n = -k_{n,1}r_n \quad (40)$$

$$\dot{\hat{\theta}}_n = \frac{\lambda_n}{2a_n^2}\xi_n^2 R_n^T R_n - \eta_n\hat{\theta}_n \quad (41)$$

where $k_{n,1}$ indicates a normal number to be constructed. With the help of (35)-(39), we obtain

$$\dot{V}_n \le -\sum_{j=1}^{n} k_{j,1}\xi_1^2 - \sum_{j=1}^{n} k_{j,2}\xi_1^{2q} + \sum_{j=1}^{n} \frac{\eta_j}{\lambda_j}\tilde{\theta}_j\hat{\theta}_j + \frac{1}{2}\sum_{j=1}^{n}(a_j^2+\varepsilon_j^2) + \frac{1}{2}D^2 \tag{42}$$

3.2 Stability Analysis

Now, after the above n-step controller design, the controller construction has been completed. This section will be concluded by the following theorem.

Theorem 1: For the uncertain nonlinear system (1) that meets the conditions of Assumptions 1-3, under adopting the controller (19),(28),(37) and the adaptive law (30), then, the controlled system are practically finite-time stable and the signals of the system are bounded almost surely.

Proof: Recalling the definition $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$,for any $\rho_j > 1/2$,We can obtain the expression of inequality as follows

$$\tilde{\theta}_j\hat{\theta}_j \le \frac{\rho_i}{2}\theta_j^2 - \frac{1}{\lambda_j}\tilde{\theta}_j^2 \tag{43}$$

With the aid of (41), we can get

$$\begin{aligned}\dot{V}_n \le & -a\sum_{j=1}^{n}\frac{1}{2}\xi_j^2 - a\Big(\sum_{j=1}^{n}\frac{1}{2}\xi_j^2\Big)^q - a\sum_{j=1}^{n}\frac{\tilde{\theta}_j^2}{2\lambda_j} \\ & -a\Big(\sum_{j=1}^{n}\frac{\tilde{\theta}_j^2}{2\lambda_j}\Big)^q + \sum_{j=1}^{n}\eta_j\Big(\frac{\tilde{\theta}_j^2}{2\lambda_j}\Big)^q - \sum_{j=1}^{n}\frac{\eta_j}{2\lambda_j}\tilde{\theta}_j^2 \\ & +\frac{1}{2}\sum_{j=1}^{n}(a_j^2+\varepsilon_j^2+\rho_j\eta_j\theta_j^2) + \frac{1}{2}D^2\end{aligned} \tag{44}$$

where $a = \min\left\{2k_{j,1}, 2^q k_{j,2}, \eta_j, j = 1,\ldots,n\right\}$.

By applying Lemma 3, one can get

$$\Big(\sum_{j=1}^{n}\frac{1}{2\lambda_j}\tilde{\theta}_j^2\Big)^q \le \sum_{j=1}^{n}\frac{1}{2\lambda_j}\tilde{\theta}_j^2 + (1-q)q^{\frac{q}{1-q}} \tag{45}$$

where $x = 1, y = \sum_{j=1}^{n}\frac{1}{2\lambda_j}\tilde{\theta}_j^2, b_1 = \frac{1}{4}, b_2 = \frac{3}{4}$.

With the aid of (41)-(43), we can get

$$\dot{V}_n \le -\alpha V_n - \beta V_n^q + \delta_1 \tag{46}$$

in which $\alpha = \beta = a$,

$$\delta_1 = \frac{1}{2}\sum_{j=1}^{n}(a_j^2+\varepsilon_j^2+\rho_j\eta_j\theta_j^2) + \frac{1}{2}D^2 + (1-q)q^{\frac{q}{1-q}} \tag{47}$$

By using the formulation of Theorem 4, it can be easily obtained that the system (1) considered in this paper is practically finite-time stable and converges to the following compact set

$$x \in \min\left\{V(x) \le \frac{\sigma}{(1-\varphi)\alpha}, \Big(\frac{\sigma}{(1-\varphi)\beta}\Big)^{\frac{1}{q}}\right\} \tag{48}$$

where $\varphi \in (0,1)$. Then, the upper limit of the settling time can be expressed as

$$\begin{aligned}T \le T_r = \max & \left\{t_0 + \frac{1}{\varphi\alpha(1-q)}\ln\frac{\varphi\alpha V^{1-q}(t_0)+\beta}{\beta},\right. \\ & \left. t_0 + \frac{1}{\alpha(1-q)}\ln\frac{\alpha V^{1-q}(t_0)+\varphi\beta}{\varphi\beta}\right\}\end{aligned} \tag{49}$$

By the above description, it is proved to be completed. To get a clearer picture of the controller designed in this paper and to facilitate the design of the simulation in the next section, the adaptive NNs command filtering control algorithm scheme is shown in Figure 1.

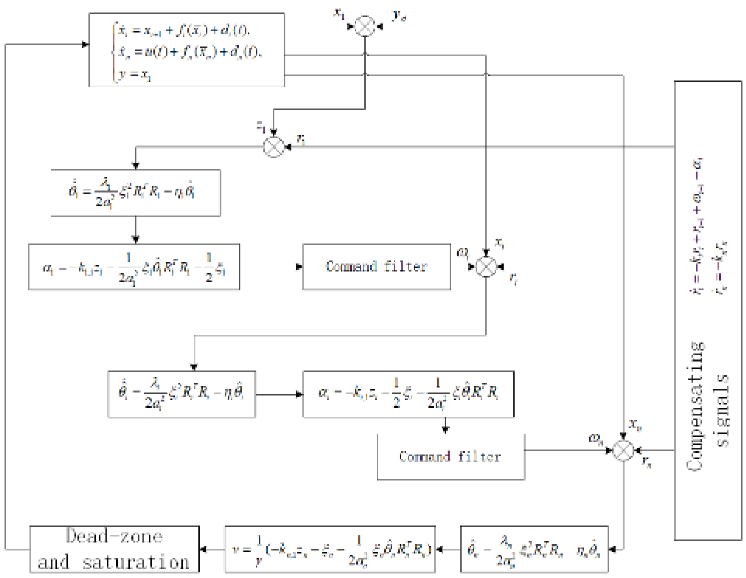

Fig. 1. The block diagram of control scheme.

4. Simulation

In the above formulation of the paper, the research work of this paper has been completed. In this section, the simulation verification of the designed finite-time controller will be done.

Example 1: The following second-order nonlinear system is used as the simulation object:

$$\begin{cases} \dot{x}_1 = (1+0.1\cos(x_1))x_2 + x_1 x_2^2 + d_1(t) \\ \dot{x}_2 = u + x_2^2 \sin(x_1) + d_2(t) \\ y = x_1 \end{cases} \tag{50}$$

where $d_1(t) = 0.1\sin(0.1t), d_2(t) = 0.1\sin(0.5t)$. The desired trajectory tracking signal is $y_d = 0.5\sin(1.5t)$, the control objective is to utilize a controller u designed to enable the input to track the expected target path y_d.

The relationship between the actual input signal u and the actual control signal v of the system is defined as follows

$$u = \begin{cases} 6, & v > 6 \\ 0.6(v-0.6), & 0.6 < v < 6 \\ 0, & -0.6 < v < 0.6 \\ 0.6(v+0.6), & -6 < v < -0.6 \\ -6, & v < -6 \end{cases} \tag{51}$$

The control law, the adaptive law and its related parameters are designed as follows: $k_{1,1} = 10, k_{2,1} = 15$, $a_1 = a_2 = 10$, $\gamma = 1$, $\omega_\tau = 60$, $\varsigma = 0.85, \lambda_1 = 0.5, \lambda_2 = 0.1, \eta_1 = 1, \eta_2 = 0.8$. The initial states and updating laws are selected as $[x_1(0), x_2(0)]^T = [0.3, 0.5]^T$, $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0.02, 0.03]^T$.

Since RBF neural networks have excellent approximation performance, they are often used as an approximate model for unknown nonlinearities. In the present study, the RBF NNs is used to approximate the unknown nonlinear term $h(Z)$. The following Gaussian function is chosen as the basis function of RBF NNs, and its expression as

$$R_i(Z) = \exp(-\frac{(Z-c_i)^T (Z-c_i)}{2}) \tag{52}$$

where $i = 1, 2, \ldots, 8$, The distribution interval of the center c_i of the Gaussian function is $[-1, 1]$.

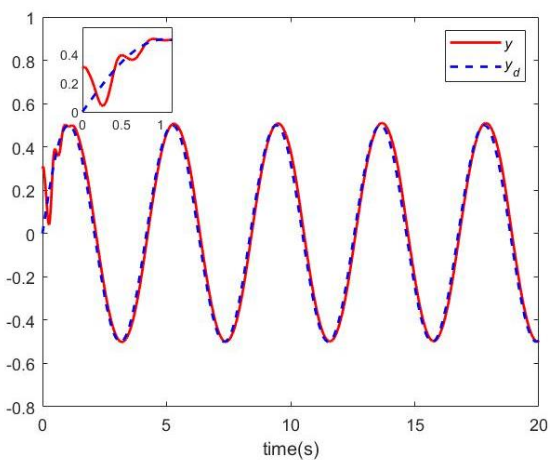

Fig. 2. System output and desired trajectory.

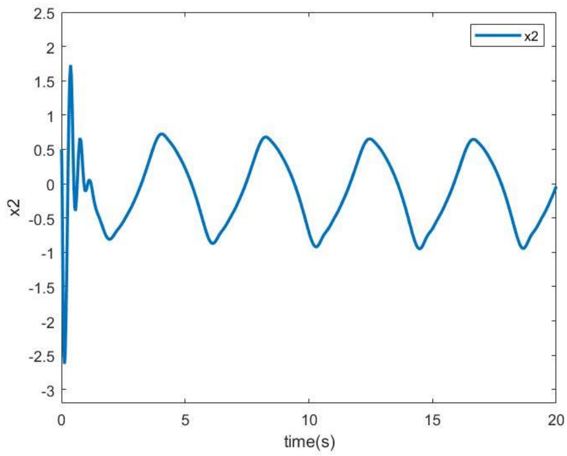

Fig. 3. The trajectory of State variable x_2.

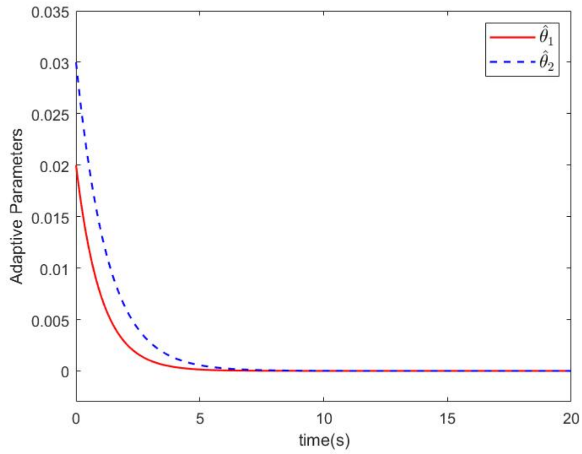

Fig. 4. The trajectory of adaptive parameters.

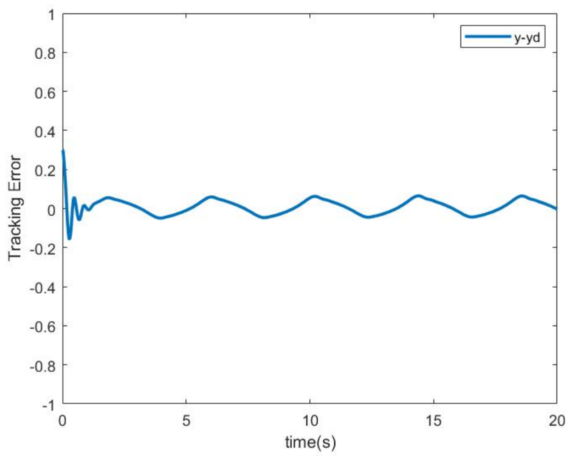

Fig. 5. The trajectory of tracking error.

The simulation results in this example are presented in Fig 2~6. Figure 2 depicts the trajectory of the system output y, the expectation trajectory y_d From Figure 2, it is easy to see that the control designed using this paper has good control performance, and the system output y can track our desired trajectory signal y_d very well after 1.8 seconds. Fig 3 and Fig 4 shows the trajectory of the state x_2 of the system and the trajectory curve of the built adaptive law $\hat{\theta}_1, \hat{\theta}_2$. And Fig 5 shows the tracking error of the system fluctuates in a small range. Fig. 6 illustrates the trajectories of u and

v ,with deadband and saturation constraints, the control input is free of peaks and becomes smoother without affecting the control performance, which meets the realities of actual industrial production.

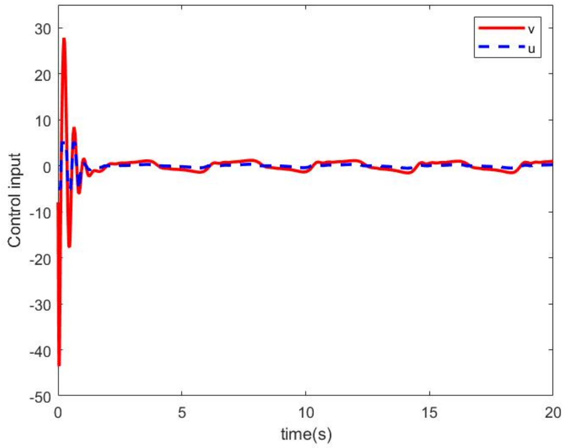

Fig. 6. The trajectory of Control input u and v .

Example 2: A single-link manipulator system containing stochastic perturbations is used as an example to prove the practicality of the designed controller. The single-link manipulator system model is given as:

$$J\ddot{q} = -Mgl_0 \sin(q) - B\dot{q} + u(v) \tag{53}$$

in which $q, \dot{q}$ and $\ddot{q}$ are the coordinate, velocity and acceleration of angles respectively. $u(v)$ is the input torque subject to saturation and deadband. Table I lists all the parameters of the single-link manipulator system.

Tab. 1. Example 2 Parameters of a single link robotic arm system.

Parameter	Description	Value
J	torsion coefficient	$0.5kg \cdot m^2$
M	mess of the link	$1kg$
g	acceleration of gravity	$9.8\,m/s^2$
l_0	length of the connecting rod	$1m$
B	coefficient of friction	$0.5\,N/m^2$

We can rewrite the systems (50) as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 2u(v) - 19.6\sin(x_1) - x_2 - d(t) \\ y = x_1 \end{cases} \tag{54}$$

where $d(t) = 0.2\sin(0.1t)$ .The desired trajectory tracking signal is $y_d = 0.5\sin(t) + 0.5\sin(0.5t)$. The dead zone and saturation nonlinear models is description as follows

$$u = \begin{cases} 10, & v \geq 10 \\ 0.85(v - 0.6), & 0.5 < v < 10 \\ 0, & -0.5 < v < 0.5 \\ 0.85(v + 0.6), & -10 < v < -0.5 \\ -10, & v \leq -10 \end{cases} \tag{55}$$

The control law, the adaptive law and its related parameters are designed as follows: $k_{1,1} = 15, k_{2,1} = 15$, $a_1 = a_2 = 10$, $\gamma = 1$, $\omega_\tau = 60$, $\varsigma = 0.85, \lambda_1 = 0.5, \lambda_2 = 0.2$, $\eta_1 = 1, \eta_2 = 0.8$ .The initial states and updating laws are selected as $[x_1(0), x_2(0)]^T = [0.3, 0.5]^T$, $\left[\hat{\theta}_1(0), \hat{\theta}_2(0)\right]^T = [0.02, 0.03]^T$.

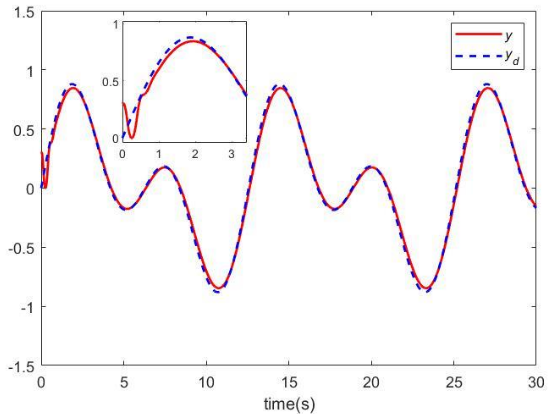

Fig. 7. Example 2: System output and desired trajectory.

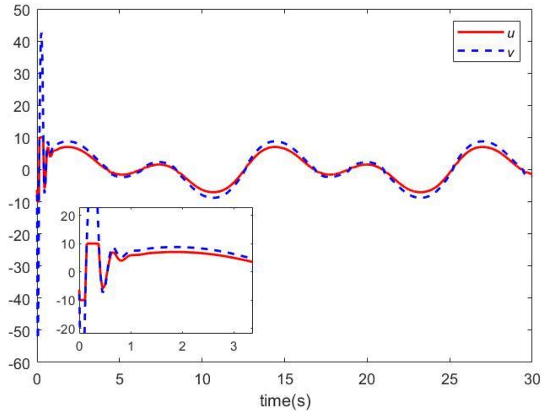

Fig. 8. Example 2: The trajectory of Control input u and v .

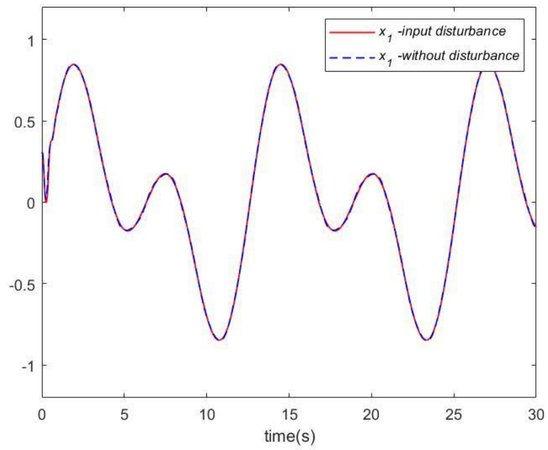

Fig. 9. Example 2: System output with disturbance and without disturbance.

The simulation results in this example are presented in Fig 7~10. Figure 7 depicts the trajectory of the system output and control input. From Figure 1, it is easy to see that the control designed using this paper has good control performance, and the system output y can track our desired trajectory signal y_d very well. Figure 8 also demonstrates the controller trajectory and the trajectory of the control input, from which it is clear that after the deadband and saturation constraints, the control input amplitude is greatly reduced while ensuring good tracking performance. In order to demonstrate the robustness and stability of the adaptive neural network tracking control algorithm proposed in this paper, we add perturbations

$d(t) = 1.5\sin(5t)$ to the control input signal u .Fig 9 and Fig 10 gives a comparison of the trajectories of the control input signal and the system output for additional disturbances and no disturbances. From the figure, it can be seen that the adaptive NN controller designed in this paper has good robustness and stability.

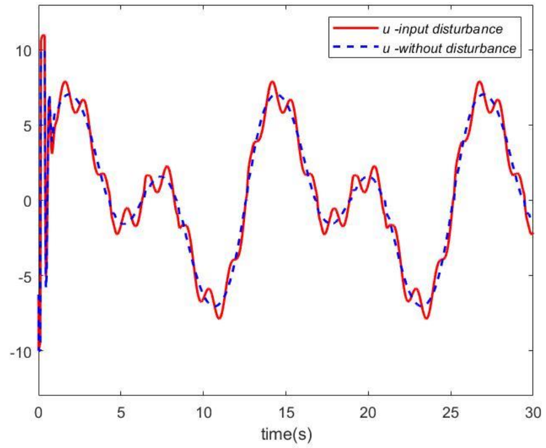

Fig. 10. Example 2: Control input with disturbance and without disturbance.

From the simulation results shown in this section, it is easy to see that applying the finite-time neural network tracking controller designed in this paper and selecting the appropriate parameters, the system can have good tracking performance with all signals bounded and the actual control inputs satisfying the dead zone and saturation constraints, which proves the usability of our designed controller.

5. Conclusions

For the actual physical system, this paper designs a finite-time tracking controller based on the command filter backstepping method for uncertain systems with multiple actuator constraints and nonlinear disturbances. The approximation property of RBF neural network is used to solve the interference of unknown nonlinear functions and simplify the controller design process. The problem of exploding computational complexity caused by the classical backstepping technique is solved by means of command filtering and compensation. The translation of the multi-actuator constraints (deadband and saturation) into a concise mathematical model largely simplifies the design of the control algorithm in this paper. The designed finite-time tracking control algorithm ensures that all variables of the system are bounded and the tracking error fluctuates in a small range. With the corresponding actuator constraints, the actual control inputs can meet the demands of a realistic industrial production environment. Finally, the simulation demonstrates the control effect of the algorithm designed in this paper and verifies the effectiveness of the proposed method.

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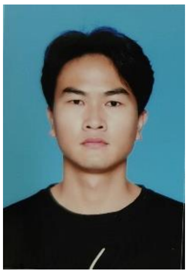

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