International Journal of Applied Mathematics in Control Engineering 7 (2024) 10-17

Contents lists available at YXpublications

International Journal of Applied Mathematics in Control Engineering

Journal homepage: http://www.ijamce.com

Review on Cooperative Control of Multi-Agent Systems

Wenguang Li[a,*], Fengming Shi[a], Wei Li[a], Sen Yang[a], Zhili Wang[a], Jing Li[a]

[a] *Shijiazhuang Campus, Army Engineering University of PLA, Shijiazhuang, 050003, China*

ARTICLE INFO

Article history:
Received 24 October 2023
Accepted 8 January 2024
Available online 10 January 2024

Keywords:
Multi-agent systems
Consistency theory
Cooperative control
Formation tracking

ABSTRACT

Multi-agent systems have been widely used in military and civilian fields because of the advantages of good flexibility, strong robustness and high cooperation efficiency. In order to track and study the current research directions and status quo of multi-agent systems, the concepts and basic theories of multi-agent systems are first described, followed by the analyzation of key technologies such as consistency theory, cooperative control strategy and formation tracking control, with emphasis on the current technical status and research hot spots. Finally, with consideration of the real application practice, the problems to be solved are put forward to provide suggestions for further research on the control field of multi-agent systems.

1. Introduction

Behavioral activities of biological groups in nature, such as ant colony cooperative foraging, flying geese formation, fish gathering and swimming, as shown in Fig 1, have been paid attention to and studied by scholars for a long time. In particular, these biological groups can complete group cooperation behavior through mutual cooperation and coordination between individuals, breaking the limited limit of a single individual's ability.

(a) Cooperative foraging behavior of ant colonies

(b) Group behavior of flying geese in formation

(c) Fish gathering and swimming behavior

Fig. 1. (a), (b) and (c) show the group behavior of organisms

Here, agents, which were first proposed in the field of computer science by Professor Minsky of the Massachusetts Institute of Technology, are defined as autonomous or semi-autonomous individuals with certain perception, communication, computation and reaction capabilities, which can be software modules or physical entities, such as drones, robots, etc. These individuals can perceive environmental information, and invoke their own functions, and then have an impact on the surrounding environment through effectors.

A multi-agent systems can also be understood as a "system of systems", which is equipped with a large number of agents that can communicate and interact with each other to complete complex tasks that are difficult for a single agent to complete. At present, multi-agent systems have been widely used and developed in military and civilian fields such as UAV formation, robot collaboration, satellite networking, etc.

In addition to reducing the complexity of system modeling, the application of multi-agent systems has the following obvious

* Corresponding author.
E-mail addresses: oec_lwg@163.com (W. Li)
Digital Object Identifiers: https://doi.org/10.62953/IJAMCE.805830

advantages (Ji L H et al., 2019):

(1) Good flexibility: each agent in the multi-agent systems has a certain autonomy, and completes the appropriate cooperation or competition on the basis of its own performance, which can be used to achieve a stronger complex intelligence than a single agent.

(2) Strong robustness: all agents cooperatively or independently complete a certain goal. If a few agents failed, the corresponding task can be replaced by other agents, without causing the system to be paralyzed or task to be interrupted. So the multi-agent systems has stronger fault tolerance and robustness in the execution of tasks.

(3) High collaborative efficiency: compared with the traditional single complex agent with centralized control strategies, the multi-agent systems can carry out distributed control and coordinate actions through information sharing and rule organization, and distributedly complete the global goal, with high operational efficiency.

Based on the above mentioned advantages of multi-agent systems, key technologies such as cooperative control and formation control of multi-agent systems have been widely concerned by scholars at domestic and abroad. The following will focus on three key technologies in the field of multi-agent systems cooperative control, namely, multi-agent systems consistency theory, multi-agent systems cooperative control strategy, and multi-agent systems formation tracking control strategy.

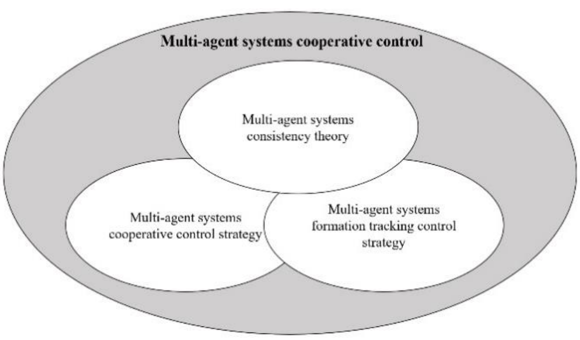

Fig. 2. Relationship between parts

2. Multi-Agent systems consistency theory

2.1 Base content

The core task of cooperative control of multi-agent systems is to design a suitable controller for each agent to complete the overall task goal in total. Distributed control strategy, which is a mainstream control strategy to deal with multi-agent systems, only requires the information of agents and neighboring agents to complete the cooperative control, and can give full play to the advantages of multi-agent systems. Consistency theory is the basis of distributed control strategy.

Consistency refers to that one or several states in a multi-agent system tend to be consistent over time based on information exchange and control protocols. Researchers, both at home and abroad, have carried out extensive research on the consistency of multi-agent systems. With the development of the consistency theory and its application to the formation control problem, a few significant results have been reached by scholars.

The basic idea of the formation control method based on consistency theory is to transform the formation control problem into a consistency problem through an appropriate coordinate transformation, and then carry on the subsequent design and analysis using the consistency theory tools.

The minimum eigenvalue of Laplacian matrix, which is used in most of the design of formation control protocols based on consistency theory, is related to the communication topology of multi-agent systems, hence can be seen as the global information, Formation control can not realised only relying on the state information of the agent itself and the neighboring agents, so it remains a significant research work to use adaptive control technology to avoid the calculation of the eigenvalues of the Laplacian matrix when designing the control protocols.

2.2 Research status

The research of consistency theory can be roughly divided into three stages. The first stage is mainly to model the group behavior in nature. In 1987, Reynolds used computers to simulate the flock behavior of birds and proposed the famous Boids model. This model provides the core criteria that agents should meet in swarm behavior: ① avoiding collision between agents; ② matching the speed of each agent; ③ clustering towards the center (Borkar V et al., 1982). In 1995, literature (Vicsek T et al., 1995) proposed Vicsek model on the basis of Boids model, in which each individual can be described as:

$$\alpha_i(t+1) = \arctan\frac{\sum_{j\in N_i(t)}\sin(\alpha_j(t))}{\sum_{j\in N_i(t)}\cos(\alpha_j(t))} + \Delta\alpha_i, i = 1, 2, ..., N$$

Where, $N_i(t)$ is the neighbor set of independent individual i at time t, and $\alpha_i(t)$ is the movement direction of independent individual i at time t. Researchers usually regard Vicsek model as a special form of Boids model.

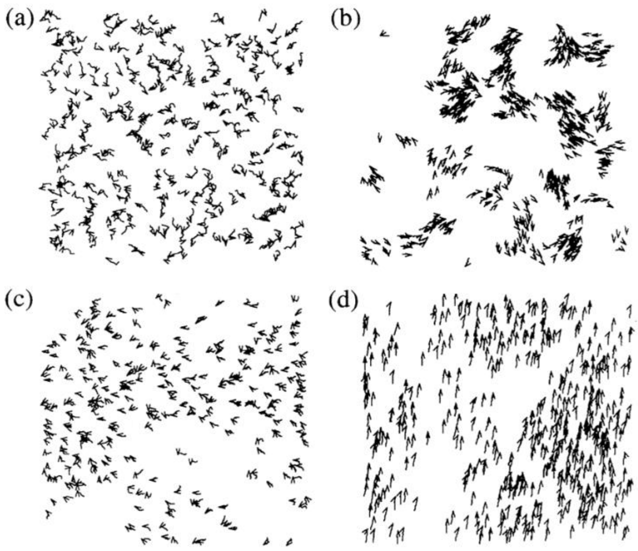

Fig. 3. Vicsek model diagram

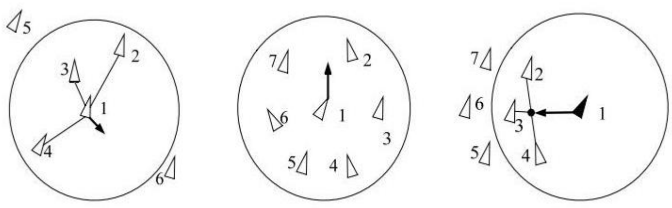

Fig. 4. Three rules of the Boids model

The second stage is the theory research and exploration stage. In 2003, analyzing Vicsek model using matrix theory and graph theory tool theory discovered the influence of connectivity of communication topology graph on system consistency (Jadbabaie A et al., 2002). The results also showed that the motion of particles can achieve consistency if the system topology is unidirectionally connected, as shown in the following formula:

$$\alpha_i\left(t+1\right)=\frac{1}{1+\left|N_i\left(t\right)\right|}\left(\alpha_i+\sum_{j\in N_i(t)}\sin\left(\alpha_j\left(t\right)\right)\right), i=2,3,...,N$$

In 2004, for multi-agent systems with fixed topology and switching topology, Murry and Olfat-Saber studied the connection between Fiedler eigenvalues of communication topological networks and the consistency convergence rate, proposed the basic theoretical framework for studying the consistency problem, and analyzed three cases: ① directed networks with switching topology and no delay; ② directed networks with fixed topology and no time delay; ③ undirected networks with fixed topology and time delay (Olfati-saber R et al., 2004). In 2005, Ren and Beard analyzed the consistency problem of second-order multi-agent systems using Laplacian matrix, and showed the important relationship between consistency and directed spanning tree in communication topology. The study of consistency theory has entered the third stage since the application of graph theory tools (Wei R et al., 2005).

The third stage focuses on the analysis of the consistency model and the design of the consistency protocol. Many scholars carried out research on consistency theory from different directions, including directed/undirected topological network, switched/fixed topological network, event-triggered consistency, packet consistency, etc. The research of consistency theory is becoming more and more completed and gradually in-depth.

The existing consistency algorithms can be roughly divided into two categories (Li Z et al., 2012; Li W et al., 2020): "leaderless" consistency and "leader-follower" consistency. In the former, the states of each agent tend to a certain value, but it is impossible to make a prior judgment (Li G., 2020) on the convergence value. "Leader-follower" consistency is also known as consistency tracking, that is, the status of the follower tracks the status of the leader through information interaction (Dong X., 2021; Luo Y P et al., 2021; Zhou T et al., 2022). In the latter, the leader may have control inputs that are required to achieve specific goals, which may or may not be known. Thus, the "leader-follower" consistency problem is more complex than the "leader-less" consistency problem.

Literature (Li Z et al., 2012; Lv Y et al., 2017; Li Z et al., 2015) studied the distributed adaptive consistency tracking problem of linear multi-agent systems, namely the "leader-follower" distributed adaptive consistency problem. In literature (Li Z et al., 2012), the leader has unknown bounded input, and the communication between followers is still undirected. Because the Laplacian matrix of the directed graph is asymmetrical, it is more difficult to design a completely distributed consistency protocol for a multi-agent systems with a directed communication topology. Literature (Lv Y et al., 2017) introduced a monotone increasing function to improve the flexibility of design, and proposed a distributed adaptive consistency protocol, for which there is no need for two-way communication between followers, and the communication topology contains a spanning tree with the leader as the root node to meet the communication requirements, while the leader has no control input. Literature (Li Z et al., 2015) further proposed another distributed adaptive consistency protocol.

Considering that most practical multi-agent systems are second-order or higher-order models, and the consistency algorithm of first-order systems is difficult to extend to second-order or higher-order systems, the application range of the consistency algorithm studied in literature (Zhang Y et al., 2015; Wang Z et al., 2017; Shang Y et al., 2019; Li Z et al., 2021; Miao G et al., 2015) is very limited. Literature (Ren W et al., 2007) provides sufficient conditions for second-order multi-agent systems to achieve consistency under the directed communication topology. Reference (Yu W et al., 2010) studies the consistency problem of second-order multi-agent systems with nonlinear dynamics and directed communication topology. In literature (Lin P et al., 2009), a second-order discrete-time consistency algorithm is proposed while considering switching topology and time delay. For heterogeneous multi-agent systems composed of first-order and second-order integrators, literature (Zheng Y et al., 2011) applies graph theory and Lyapunov stability theorem to give sufficient conditions for the system to achieve consistency in the case of undirected communication topology. Literature (Ren W et al., 2007) proposes a consistency algorithm for higher-order multi-agent systems. At present, the research of consistency theory is mostly carried out in the first order, second order or higher order consistency problems.

3. Cooperative control strategy for multi-agent systems

3.1 Base content

In recent years, multi-agent systems study has developed into a new complex system science, and has gradually penetrated into many fields of social life, and its cooperative control problem has derived many branches. Including Formation, Flocking, Coverage, Consensus, Rendezvous, Flocking and other control problems, there are some common missions(objectives) in various research branches.

(1) Formation control. The goal of formation control is to keep the relative distance/position between the agents at the expected value, and the final state of all agents in formation control is more diverse than that of consistent control where the states of all agents tend to be single. Formation control is widely used in satellite formation, cooperative transport and sensor network. Formation control can be roughly divided into formation generation and formation tracking. Formation generation refers to the algorithm design of a group of agents to achieve the desired formation without reference to the baseline state. Formation tracking refers to a group of agents performing a task while following the baseline state, and formation tracking is often more challenging than formation generation (Cao Y et al., 2013; Yan D H et al., 2022).

(2) Swarm control. Swarm control originates from the natural group behavior, such as bird swarm, fish swarm and bacteria swarm, and swarm control is more decentralized than formation control (Peng J S et al., 2023).

(3) Consistency control. Some states of each agent tend to be the same gradually with time through information interaction.

It is mathematically described as follows: Suppose a multi-agent systems consists of N agents, and the state of agent i is determined by $x_i\left(t\right)\left(i=1,2,...,N\right)$ said, when $t\rightarrow\infty$, $\left\|x_i\left(t\right)-x_j\left(t\right)\right\|\rightarrow 0,\forall i\neq j$ set up, means that the multi-agent systems to achieve consistency control. The control problems of multi-agent systems such as formation control and swarm control are essentially a special case of consistency control (Sun F L et al., 2012).

According to the different ways of multi-agent information interaction, the cooperative control strategies can be generally divided into three types (Li G., 2020; Zhao C L et al., 2022; Ye P., 2019): centralized control, decentralized control and distributed control.

(1) Centralized control. The control strategy requires each node to communicate with the central controller (central node or control station) and share information when the network is fully connected. Obviously, a system with this control strategy requires large amount of information transmission and extremely high computing power of the central controller. In addition, it is not able to arbitrarily increase or decrease the number of nodes, and even may crash due to the loss of a node or a controller failure. Therefore, the fault-tolerance, robustness, flexibility and reliability of the centralized control strategy are not satisfactory. Its advantages include simpleness of implementation and theory maturity. It should be pointed out that the communication network of most multi-agent systems is not fully connected. Furthermore, the setting of a central controller can not make full use of the advantages of multi-agent systems, instead, may add limitations because of its shortcomings.

(2) Decentralized control. The decentralized control strategy, which no longer uses the centralized controller, instead, uses the decentralized controller to achieve the control of the multi-agent subsystems, between which achieved via the communication can be connected, but not necessarily. Hence, the decentralized control is also known as the semi-decentralized control strategy. It is obvious that distributed control strategy have merits of small amount of information transmission and computation, and simple structure; in the other hand, the disadvantages include, strong limitations on the scalability and robustness, and disability to ensure the formation and the avoidance of collision accidents.

(3) Distributed control. The distributed control strategy does not require the setting of a central controller or full network communication, and only requires the nodes to know the information of themselves and neighboring agents to achieve collaborative control Hence, the distributed control strategy is also known as the decentralized control strategy. Compared with the centralized control strategy, the control accuracy of the distributed control strategy is slightly lower, but it greatly reduces the communication requirements and calculation amount, enhances the scalability and stability of the system, and is more in line with the needs of practical projects. Distributed control strategy has the advantages of both distributed control strategy and centralized control strategy, and its cost performance is relatively high. Therefore, the distributed control strategy of multi-agent systems has been widely recognized and favored by researchers at home and abroad, and has become a mainstream control strategy.

3.2 Research status

In literature (Zuo Z., 2015; Yu S et al., 2015; Liu Z et al., 2021), the consistency protocol only has the state information between adjacent agents, but its design strictly relies on some eigenvalue information of the Laplace matrix related to the communication topology. When the communication topology is undirected, the minimum non-zero eigenvalue of the Laplace matrix needs to be known; when the communication topology is directed, the minimum non-zero eigenvalue of the Laplace matrix needs to be known. The minimum real part of the non-zero eigenvalue of the Laplacian matrix needs to be known in advance. However, the non-zero eigenvalue of the Laplace matrix belongs to global information, that is, the entire communication topology is required to be known to each agent. Therefore, the consistency protocol in literature (Zuo Z., 2015; Yu S et al., 2015; Liu Z et al., 2021) can be implemented in a distributed way, but the design process involves global information. In other words, the consistency protocol in literature (Zuo Z., 2015; Yu S et al., 2015; Liu Z et al., 2021) is not completely distributed.

To overcome this limitation, Li Zhongkui and other scholars proposed a distributed adaptive consistency protocol (Li Z et al., 2013; Li Z et al., 2013; Li Z et al., 2013; Li Z et al., 2014), which aims to ensure that the state of multi-agent systems is consistent in a completely distributed way without using any global information. Based on the state information of adjacent agents and the adaptive theory, literature (Li Z et al., 2012) designed edge adaptive consistency protocols and node adaptive consistency protocols for linear multi-agent systems with undirected communication topology. The former adjusts the coupling weights of each edge in the communication topology, and the latter adjusts the coupling weights of each node in the communication topology, thus eliminating the requirement of eigenvalue information. Reference (Li Z et al., 2013) proposes an edge-adaptive consistency protocol for linear and nonlinear multi-agent systems with undirected communication topologies.

4. Multi-Agent systems formation tracking control strategy

4.1 Base content

Formation control of multi-agent systems is one of the current research hotspots. The goal of formation control is to keep the relative distance/position between agents at the expected value. The final states of all agents in formation control are more diversified than those in uniform control where the states of all agents tend to be single.

Fig. 5. UAV formation flight

At present, common formation control methods mainly include Leader-follower approach (Zhao C L et al., 2022; Peng J S et al., 2023), Behavior-based Control approach, Virtual Structual Approack, etc (Balch T et al., 1998; Lawton J R et al., 2003; Lewis M A et al., 1997; Beard R W et al., 2001).

(1) Pilot-Following approach. In this method, the leader and the follower are designated. The leader's movement is pre-set, and the follower keeps the position and speed deviation from the leader by some way. There are two ways for the leader to maintain a relative relationship with the follower: the "distance-direction" $(l-\phi)$ way and the "distance-distance" way $(l-l)$ (Hu Z W et al., 2012).

The advantage of the pilot-following method is that the formation configuration can be determined by one or several leaders, which is convenient for mathematical description and realization and requires less calculation. The disadvantages are: (1) when the leader moves too fast, it is easy to result to the follower's tracking failure, because the leader can not get the explicit state feedback of the follower; (2) if the leader fails, the formation control will fail immediately; (3) as the number of followers gradually increases, the error of the following followers will gradually increase. considering the shortcomings of the pilot-follow method, domestic and foreign scholars combined backstepping technique, artificial neural network and model predictive control, MPC (MPC) and other control technologies to improve the method performance.

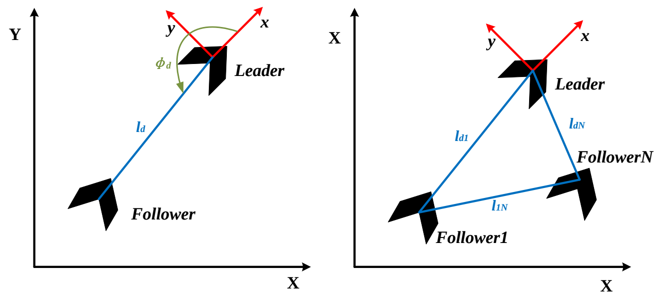

Fig. 6. Two ways of pilot-following method

(2) Based on behavioral law. This method needs to initially design the desired behavior of each agent, such as obstacle avoidance, collision avoidance, target search and formation keeping, etc, and then obtain its control input by weighting the desired behavior of the agent, which comes from itself or neighboring agents. Most of the information interactions in this method are local interactions, which are easy to be realized by decentralized or distributed control strategies, leading to good robustness and flexibility. The limitation of this method is the difficulty to explain its behavior through mathematical models and scarceness of stability analysis tools, so there are little research on formation control based on behavioral method.

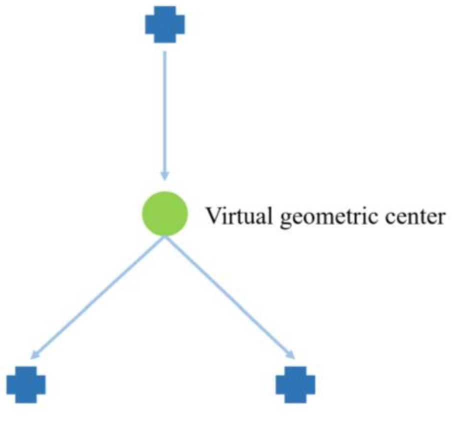

Fig. 7. Virtual structure method diagram

(3) Virtual structure method. In this method, an agent is regarded as a point on the virtual rigid structure, which represents the formation configuration. The agent can track the fixed point corresponding to the rigid structure over time to achieve the formation. In the process of the implementation of the virtual structure method, the dynamic description of the rigid structure needs to be determined first, and then the dynamics of the rigid structure is transformed into the expected motion of each agent. Finally, the control method of the tracking trajectory of the agent needs to be obtained. Compared with the pilot-follow method, this method does not set the display leader. With the ability to describe the formation behavior as a whole, this method can make use of the formation error feedback in the design of the control law to achieve a higher control precision. However, the description of rigid structure also limits the number of agents, which means it can only be used to describe the motion of small-scale multi-agent systems formation.

(4) Consistency method. Due to the large number of unified multi-agent systems and complex control, communication and topological relationships, the consistency method mainly uses graph theory related tools to complete the modeling of multi-agent systems. The nodes in the graph represent the agents, and the edges in the graph represent the information flow between agents. After the modeling completion, theoretical analysis can be carried out based on graph theory and matrix theory. The representation, transformation and properties of the graph can be described by the knowledge of matrix theory, and the stability analysis of the formation control law can be conducted conveniently by combining the Lyapunov stability theory and the properties of Laplacian matrix. At present, the consistency method is a mainstream method of formation direction control.

4.2 Research status

Aiming at the formation control problem of second-order multi-agent systems, Ren constructed a distributed formation control protocol by using the state information of the agents themselves and neighboring agents (Ren W., 2006).

Oh and Ahn built a control protocol based on adjacent displacement for first-order multi-agent systems to realize formation in time-varying topology (Oh K K et al., 2014). Based on the properties of Laplacian matrix, reference (Lin Z Y et al., 2014) studied the sufficient and necessary conditions for a first-order multi-agent system to realize formation in undirected communication topology. Literature (Wang C et al., 2013; El-hawwary M I., 2015) discusses the control of circular formation of first-order multi-agent systems in one and three dimensional space respectively. Tayebi et al. designed formation control protocols for multi-agent systems with undirected communication topology and realized time-invariant formation with communication delay (Abdessameud A et al., 2011). Literature (Seo J et al., 2012; Fei W B et al., 2021) applied consistency theory and output feedback linearization to deal with formation control problems of multi-agent systems, where formation can be partially time-varying. Turpin et al. applied the consistency method to study the formation control problem of multi-agent systems (Turpin M et al., 2012).

Literature (Dong X W et al., 2016; Dong X W et al., 2015) gives the necessary and sufficient conditions for realizing time-varying formation of second-order multi-agent systems with undirected topology and directed topology respectively. For the time-varying formation control problem of high order multi-agent systems with communication constraints, literature (Wang L et al., 2018) gives sufficient conditions for formation realization based on Linear Matrix Inequality. A time-varying formation control protocol based on event triggering is proposed in literature (Li X et al., 2018). The problem of time-varying formation control for heterogeneous multi-agent systems is studied in literature (Rahimi R et al., 2014). For higher-order multi-agent systems with directed communication topology, reference (Chai X et al., 2020) studies the time-varying formation control problem under nonlinear uncertainty, and a robust compensator is designed to suppress the nonlinear uncertainty. For general linear multi-agent systems with switching topology, the time-varying formation control problem with communication delay is studied in literature (Xiao W et al., 2018). A time-varying formation control protocol is constructed based on the consistency theory, and sufficient conditions for the system to realize time-varying formation are given by using Lyapunov stability theory.

It is not enough to only consider formation and stability (Dong X W et al., 2016; Dong X W et al., 2015; Wang L et al., 2018; Li X et al., 2018; Rahimi R er al., 2014; Chai X et al., 2020; Xiao W et al., 2018) in the study of formation control of multi-agent systems. In some practical applications, the formation of a multi-agent systems is only the first step. In order to complete tasks such as tracking the leader's trajectory or surrounding a moving target, the multi-agent systems is required to realize a time-varying formation tracking. In literature (Wu Z P et al., 2007), formation and trajectory tracking of first-order multi-agent systems based on consistency theory were studied. Sorensen et al. studied the formation tracking problem of first-order multi-agent systems, where one leader provides a desired trajectory for a group of followers (Ren W et al., 2008). For first-order multi-agent systems with switching topology, literature (Guo J et al., 2010) proposed a distributed control approach to solve the target alignment problem, which can be seen as a case where followers track the leader. Reference (Mylvaganam T et al., 2015; Yan Z T et al., 2021) solved the problem of collision avoidance for multi-agent systems in formation tracking.

5. Problems to be solved

Although domestic and foreign scholars have carried out extensive research on multi-agent systems, limited by the practical application of multi-agent systems, there are still problems that need to be further studied and explored as follows:

(1) The design of the time-varying formation tracking control protocol for multi-agent systems still relies on the global information of the eigenvalue of the Laplacian matrix, so formation tracking cannot be implemented in a completely distributed way;

(2) When the communication topology of multi-agent systems is a directed graph, the corresponding Laplacian matrix is asymmetrical; so it is difficult to design the time-varying formation tracking control protocol for such multi-agent systems;

(3) The existing time-varying formation tracking control protocols are designed based on the assumption that the leader's control input is zero or known, which is different from the case when the leader has unknown control input.

(4) There are limitations in the connection capacity and communication bandwidth of individual agents, which means it is risky when exchanging large amounts of data. There are also difficulties in determining who is sending and who is receiving information between agents.

(5) The limited computing power of the individual agents means that the system may breakdown if the data is not distributed properly.

6. Summary

To sum up, the control of multi-agent systems, as one of the research hotspots in the field of control, attracts scholars at home and abroad to carry out continuous research and exploration. This paper first describes the concept and basic theory of multi-agent systems, then analyzes the current technical status including the consistency theory, cooperative control strategy, formation tracking control and other key technologies, and finally analyzes the future direction to be further studied considering the application of multi-agent systems, which would have an important impact on breaking through the application limits of multi-agent systems in the future and better applying multi-agent systems to complex problem modeling and solving.

References

Ji L H, Yang S S, Pu X C, et al. 2019. Theory and technology of multi-agent systems consistency co-evolutionary control [M]. Science Press.

Borkar V, Varaiya P. 1982. Asymptotic agreement in distributed estimation [J]. IEEE Transactions on Automatic Control, 27(3): 650-5.

Vicsek T, Czirok A, Benjacob E, et al. 1995. Novel type of phase-transition in a system of self-driven particles [J]. Physical Review Letters, 75(6): 1226-9.

Jadbabaie A, Lin J, Morse A S, et al. 2002. Coordination of groups of mobile autonomous agents using nearest neighbor rules; proceedings of the 41st IEEE Conference on Decision and Control, Las Vegas, Nv, F 2002 Dec 10-13.

Olfati-saber R, Murray R M. 2004. Consensus problems in networks of agents with switching topology and time-delays [J]. IEEE Transactions on Automatic Control, 49(9): 1520-33.

Wei R, Beard R W. 2005. Consensus seeking in multiagent systems under dynamically changing interaction topologies [J]. IEEE Transactions on Automatic Control, 50(5): 655-61.

Li Z, Liu X. 2012. Distributed tracking control for linear multi-agent systems with a leader of bounded input using output information; proceedings of the Proceedings of the 10th World Congress on Intelligent Control and Automation, F 6-8 July 2012.

Li W, Huo X Y. 2020. Path planning algorithm for quadrotor unmanned aerial vehicles based on improved artificial potential field[J]. International Journal of Applied Mathematics in Control Engineering, 3(2): 164-172.

Li G. 2020. Research on consensus control for leaderless multi-agent systems [D]; Nanjing University of Information Science and Technology.

Dong X. 2021. Leader-following consensus of multi agent systems under switching topology [D]; Harbin University of Science and Technology.

Luo Y P, Cai C, Xiao X, et al. 2021. Event-triggered consensus of second-order leader-follower multi-agent system [J]. Journal of Hunan Institute of Engineering, 31(01): 1-7.

Zhou T, Liu Q L Wang D, et al. 2022. Leader-following consensus for linear multi-agent systems based on integral-type event-triggered strategy [J]. Control and Decision, 37(5): 1258-1266.

Li Z, Liu X, Ren W. 2012. Distributed containment control of linear multi-agent systems with multiple leaders of bounded unknown inputs; proceedings of the Proceedings of the 31st Chinese Control Conference, F 25-27 July 2012.

Lv Y, Li Z, Duan Z, et al. 2017. Novel distributed robust adaptive consensus protocols for linear multi-agent systems with directed graphs and external disturbances [J]. International Journal of Control, 90(2): 153-63.

Li Z, Wen G, Duan Z, et al. 2015. Designing fully distributed consensus protocols for linear multi-agent systems with directed graphs [J]. IEEE Transactions on Automatic Control, 60(4): 1152-7.

Zhang Y, Gao J, Ren J. 2015. A novel consensus protocol for two-dimension first-order multiagent systems [J]. Mathematical Problems in Engineering.

Wang Z, Song X, Zhang H, et al. 2017. Consensus of first-order agents with neighbors' delayed information; proceedings of the Chinese Automation Congress (CAC), Jinan, China, F 2017 Oct 20-22, 2017 [C].

Shang Y, Liu C L, Liu F, et al. 2019. An event-triggered consensus protocol for first-order nonlinear multi-agent Systems; proceedings of the Chinese Automation Congress (CAC), Hangzhou, China, F 2019 Nov 22-24, 2019 [C].

Li Z, Wang Z, Feng Y. 2021. Fixed-time consensus of first-order multi-agent systems over signed directed graphs [J]. Transactions of the Institute of Measurement and Control, 43(10): 2392-401.

Miao G, Ma Q. 2015. Group consensus of the first-order multi-agent systems with nonlinear input constraints [J]. Neurocomputing, 161: 113-119.

Ren W, Atkins E. 2007. Distributed multi-vehicle coordinated control via local information exchange [J]. International Journal of Robust and Nonlinear Control, 17(10-11): 1002-1033.

Yu W, Chen G, Cao M, et al. 2010. Second-order consensus for multiagent systems with directed topologies and nonlinear dynamics [J]. IEEE Transactions on Systems Man and Cybernetics Part B-Cybernetics, 40(3): 881-91.

Lin P, Jia Y. 2009. Consensus of second-order discrete-time multi-agent systems with nonuniform time-delays and dynamically changing topologies [J]. Automatica, 45(9): 2154-8.

Zheng Y, Zhu Y, Wang L. 2011. Consensus of heterogeneous multi-agent systems [J]. IET Contr Theory Appl, 5(16): 1881-8.

Ren W, Moore K L, Chen Y. 2007. High-order and model reference consensus algorithms in cooperative control of multivehicle systems [J]. Journal of Dynamic Systems Measurement and Control-Transactions of the Asme, 129(5): 678-88.

Cao Y, Yu W, Ren W, et al. 2013. An overview of recent progress in the study of distributed multi-agent coordination [J]. IEEE Transactions on Industrial Informatics, 9(1): 427-38.

Yan D H. 2022. Two-layered decentralized prescribed performance formation control for networked UAVs with uncertain communication delay[J]. International Journal of Applied Mathematics in Control Engineering, 5(1): 101-108.

Peng J S, Fu X J. 2023. Formation control of leader-follower UAV based on the behavior of geese swarm[J]. Control Engineering of China, 30(1): 113-118.

Sun F L. 2012. On consensus of multi-agent networks[D]. Huazhong University of Science and Technology.

Zhao C L, Dai S W, Zhao G R, et al. 2022. Formation control of multi-UAV based on distributed model predictive control algorithm [J]. Control and Decision, 37(7): 1763-1771.

Ye P. 2019. Distributed cooperative guidance law of multiple missiles based on multi-agent consensus algorithm [D]; Nanjing University of Science and Technology.

Zuo Z. 2015. Nonsingular fixed-time consensus tracking for second-order multi-agent networks [J]. Automatica, 54: 305-309.

Yu S, Long X. 2015. Finite-time consensus for second-order multi-agent systems with disturbances by integral sliding mode [J]. Automatica, 54: 158-165.

Liu Z, Zhang A, Qiu J, et al. 2021. Event-triggered control of second-order nonlinear multi-agent systems with directed topology & nbsp [J]. Neurocomputing, 452: 820-826.

Li Z, Ren W, Liu X, et al. 2013. Distributed consensus of linear multi-agent systems with adaptive dynamic protocols [J]. Automatica, 49(7): 1986-1995.

Li Z, Ren W, Liu X, et al. 2013. Distributed containment control of multi-agent systems with general linear dynamics in the presence of multiple leaders [J]. International Journal of Robust and Nonlinear Control, 23(5): 534-547.

Li Z, Liu X, Ren W, et al. 2013. Distributed tracking control for linear multi-agent systems with a leader of bounded unknown input [J]. IEEE Transactions on Automatic Control, 58(2): 518-523.

Li Z, Duan Z, Lewis F L. 2014. Distributed robust consensus control of multi-agent systems with heterogeneous matching uncertainties [J]. Automatica, 50(3): 883-889.

Li Z, Liu X, Ren W, et al. 2012. Consensus control of linear multi-agent systems with distributed adaptive protocols; proceedings of the 2012 American Control Conference (ACC), F 27-29 June 2012[C].

Li Z, Ren W, Liu X, et al. 2013. Consensus of multi-agent systems with general linear and lipschitz nonlinear dynamics using distributed adaptive protocols [J]. IEEE Transactions on Automatic Control, 58(7): 1786-1791.

Peng J S, Fu X J. 2023. Formation control of leader-follower UAV based on the behavior of geese swarm [J]. Control Engineering of China, 30(1): 113-118.

Balch T, Arkin R C. 1998. Behavior-based formation control for multirobot teams [J]. IEEE Transactions on Robotics and Automation, 14(6): 926-39.

Lawton J R T, Beard R W, Young B J. 2003. A decentralized approach to formation maneuvers [J]. IEEE Transactions on Robotics and Automation, 19(6): 933-41.

Lewis M A, Tan K H. 1997. High precision formation control of mobile robots using virtual structures [J]. Autonomous Robots, 4(4): 387-403.

Beard R W, Lawton J, Hadaegh F Y. 2001. A coordination architecture for spacecraft formation control [J]. IEEE Transactions on Control Systems Technology, 9(6): 777-90.

Hu Z W, Liang J H, Chen L, et al. 2012. Survey on the formation control of multi-agent system; proceedings of the 31st Chinese Control Conference, Hefei, PEOPLES R CHINA, F 2012 Jul 25-27.

Ren W. 2006. Consensus based formation control strategies for multi-vehicle systems; proceedings of the American Control Conference 2006, Minneapolis, MN, F 2006 Jun 14-16, 2006 [C].

Oh K K, Ahn H S. 2014. Formation control and network localization via orientation alignment [J]. IEEE Transactions on Automatic Control, 59(2): 540-545.

Lin Z Y, Wang L L, Han Z M, et al. 2014. Distributed formation control of multi-agent systems using complex laplacian [J]. IEEE Transactions on Automatic Control, 59(7): 1765-1777.

Wang C, Xie G M, Cao M. 2013. Forming circle formations of anonymous mobile agents with order preservation [J]. IEEE Transactions on Automatic Control, 58(12): 3248-3254.

El-hawwary M I. 2015. Three-dimensional circular formations via set stabilization [J]. Automatica, 54: 374-381.

Abdessameud A, Tayebi A. 2011. Formation control of VTOL unmanned aerial vehicles with communication delays [J]. Automatica, 47(11): 2383-2394.

Seo J, Kim Y, Kim S, et al. 2012. Consensus-based reconfigurable controller design for unmanned aerial vehicle formation flight [J]. Proceedings of the Institution of Mechanical Engineers Part G-Journal of Aerospace Engineering, 226(G7): 817-829.

Fei W B, Tian J, Wu X L. 2021. Fixed-time adaptive neural network sliding mode control for uncertain nonlinear systems and application to quadrotor UAVs tracking control[J]. International Journal of Applied Mathematics in Control Engineering, 4(1): 62-73.

Turpin M, Michael N, Kumar V, et al. 2012. Decentralized formation control with variable shapes for aerial robots; proceedings of the IEEE International Conference on Robotics and Automation (ICRA), St Paul, MN, F May 14-18, 2012 [C].

Dong X W, Zhou Y, Ren Z, et al. 2016. Time-varying formation control for unmanned aerial vehicles with switching interaction topologies [J]. Control Engineering Practice, 46: 26-36.

Dong X W, Yu B C, Shi Z Y, et al. 2015. Time-varying formation control for unmanned aerial vehicles: theories and applications [J]. IEEE Transactions on Control Systems Technology, 23(1): 340-348.

Wang L, Han X, Chen Q, et al. 2018. Time-varying formation control for swarm systems with communication constraints; proceedings of the 37th Chinese Control Conference (CCC), Wuhan, PEOPLES R CHINA, F 2018 Jul 25-27, 2018 [C].

Li X, Dong X, Xie L, et al. 2018. Periodic event-triggered time-varying formation of multi-agent systems; proceedings of the 15th International Conference on Control, Automation, Robotics and Vision (ICARCV), Singapore, SINGAPORE, F 2018 Nov 18-21, 2018 [C].

Rahimi R, Abdollahi F, Naqshi K. 2014. Time-varying formation control of a collaborative heterogeneous multi agent system [J]. Robotics and Autonomous Systems, 62(12): 1799-1805.

Chai X, Wang Q, Yu Y, et al. 2020. Robust time-varying output formation control for swarm systems with nonlinear uncertainties [J]. Complexity, 2020.

Xiao W, Wang R, Dong X, et al. 2018. Time-varying formation control for general linear multi-agent systems with time-varying delays and switching topologies; proceedings of the 14th IEEE International Conference on Control and Automation (ICCA), Anchorage, AK, F 2018 Jun 12-15, 2018 [C].

Wu Z P, Guan Z H, Wu X Y, et al. 2007. Consensus based formation control and trajectory tracing of multi-agent robot systems [J]. Journal of Intelligent & Robotic Systems, 48(3): 397-410.

Ren W, Sorensen N. 2008. Distributed coordination architecture for multi-robot formation control [J]. Robotics and Autonomous Systems, 56(4): 324-33.

Guo J, Yan G F, Lin Z Y. 2010. Local control strategy for moving-target-enclosing under dynamically changing network topology [J]. Systems & Control Letters, 59(10): 654-61.

Mylvaganam T, Astolfi A. 2015. A differential game approach to formation control for a team of agents with one leader; proceedings of the American Control Conference, Chicago, IL, F Jul 01-03, 2015.

Yan Z T, Li Y, Ye R M, Yang Y L. 2021. Finite-time synchronization of an uncertain complex dynamical network based on sliding mode control technology[J]. International Journal of Applied Mathematics in Control Engineering, 4(1): 113-118.

Wenguang Li is currently working in the Department of UAV Engineering at the Shijiazhuang Campus of the Army Engineering University, Shijiazhuang, China. He obtained M.S. degree from Army Engineering University of PLA, China in 2018. His main research interests are in the areas of intelligent control, UAV cluster control and control applications.

Fengming Shi is currently working in the Department of UAV Engineering at the Shijiazhuang Campus of the Army Engineering University, Shijiazhuang, China. He obtained BS degree from Nation University of Defense Technology, China in 2008 and PHD degree from Hull University, UK in 2013. His main research interests are in the areas of intelligent control, fault tolerate control, generalized systems, and control applications.

Wei Li is currently working in the Department of UAV Engineering at the Shijiazhuang Campus of the Army Engineering University, Shijiazhuang, China. She obtained her BS degree from North University of China in 2006 and her MS degree from Hebei University of Science and Technology, China in 2009. Her main research direction is signal and information processing.

Sen Yang is currently working in the Department of UAV Engineering at the Shijiazhuang Campus of the Army Engineering University, Shijiazhuang, China. He obtained M.S. degree from Ordnance Engineering College, China in 2010 and PHD degree from Ordnance Engineering College, China in 2013. His main research interests are in the areas of intelligent control, fault tolerate control, generalized systems, and control applications.

Zhili Wang is currently working in the Department of UAV Engineering at the Shijiazhuang Campus of the Army Engineering University, Shijiazhuang, China. She obtained M.S. degree from University of Science and Technology Beijing, China in 2007. Her main research interests are in the areas of intelligent control, fault tolerate control, generalized systems, and control applications.

Jing Li is currently working in the Department of UAV Engineering at the Shijiazhuang Campus of the Army Engineering University, Shijiazhuang, China. She obtained M.S. degree from Ordnance Engineering College, China in 2017. Her main research interests are in the areas of intelligent control, fault tolerate control, generalized systems, and control applications.