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# Research on Online Target Tracking and Control of UAVs Xiuxia Yang<sup>a\*</sup>, Yi Zhang<sup>a</sup>, Hao Yu<sup>a</sup>

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#### ABSTRACT

In order to realize the target tracking and control of multi-rotor unmanned aerial vehicles and ensure that the convergence trajectory is smooth and easy to be realized in engineering, this paper based on Lyapunov vector field method, established the navigation vector field as the online tracking control guidance law of multi-rotor, and gave the flying speed of the UAV at any point in space. The multi-rotor UAV can converge to the tracking ring with the target as the center of the circle. Considering the uncertainty of sensors detection, extended Kalman filter is used to reduce the detection error and predict the trend of target motion. The simulation results show that the guidance law has a good tracking effect on the target in 2D and 3D dimensions.

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# 1. Introduction

Multi-rotor unmanned aerial vehicle (UAVS) has more advantages than fixed-wing UAV in information detection because of its long time hovering in the air and low requirements for take-off site (e.g., Y. Yang et al., 2020; M. Alhafnawi et al., 2023; S. Bhagat et al., 2020). In order to track and detect specific targets and obtain target information, this paper studies the tracking and control of multi-rotor unmanned aerial vehicle.

Currently, the commonly used tracking methods mainly fall into two categories: Persistent tracking and Standoff tracking. Persistent tracking enables the tracked targets to be within the sensor detection range of UAVs as much as possible. However, this method requires multiple UAVs to cooperate in tracking to maintain the robustness of tracking in complex environments and due to factors such as terrain limitation and obstacle avoidance. The latter can also be called range tracking, in this tracking mode, the UAV always maintains a certain standoff distance with the tracked target, ensuring the maximum observation effect of the UAV while reducing the risk of exposure. The Standoff tracking method studied in this paper is the second one, Standoff tracking.

At present, many scholars have conducted researches on Standoff tracking, for example, Good helmsman, track construction, Nonlinear MPC (NMPC), Reference Point Guidance (RPG) and vector field guidance (e.g., LAWRENCE D et al., 2003; M. SINGHA A et al., 2018; OH H et al., 2014). Among them, vector field guidance is the

most simple and convenient, and the tracking path is smooth and easy to implement. For Standoff tracking of the target on a 2-D plane, the Lyapunov vector field method is proposed (e.g., Frew E W et al., 2003; Chen et al. 2018). Under the guidance of the navigation vector field, the required flying velocity of the UAV is given according to the distance between the UAV and the target, so that the UAV converge to the tracking limit loop set. At the same time, in order to solve the problem of the shortest path of the UAV converging to the tracking circle, the tangent vector field guidance method is proposed, but this method also has the drawback that it is only applicable to the UAV located outside the tracking circle (e.g., Chen H et al., 2009, Luo et al., 2018).

The above guidance law is generally aimed at 2-D space and takes little consideration of dynamic targets. The unmanned aerial vehicle flies in 3-D space, so it needs 3-D tracking guidance law. The function of the navigation vector field is to provide the UAV with a desired velocity vector as a reference according to the relative position between the UAV and the target. When the UAV flies according to the desired speed, it can converge to the desired flight path. In this paper, an online target tracking control guidance law based on Lyapunov vector field method is designed, and the expected tracking speed correction control law is designed, which can realize the real-time tracking of static and dynamic targets in 3-D space. Besides, the extended Kalman filter is designed to reduce the detection error and predict the trend of target motion. Finally, The Flight-GEAR (Flight simulation system) shows the flight status of the multi-rotor, and displays the actual flight attitude of the multirotor more intuitively.

### 2. Construction of navigation vector field

Suppose the form of the navigation vector field is as follows:

$$\dot{\boldsymbol{r}}_d = h(\boldsymbol{r}) \tag{1}$$

where r is the position vector of the UAV relative to the target.

Since the UAVs are ultimately expected to converge to the face-off circle centered on the target, it is assumed that the stable state determined by the above navigation vector field is a ring, and the ring is globally attractive, that is, the multi-rotor UAVs starting from any point in space can eventually converge to the limit cycle.



Fig. 1. The limit cycle under the Earth system.

Suppose  $V_F(\mathbf{r})$  is the Lyapunov function of the navigation vector field, which is required to be positive definite and a real continuous function in the domain,  $V_F(\mathbf{r}) = 0$  when  $\mathbf{r} \in C$ , and  $V_F(\mathbf{r}) > 0$  when  $\mathbf{r} \notin C$ .

It is also required that 
$$V_F(\mathbf{r})$$
 be radially unbounded, i.e.  

$$\lim_{r \to \infty} V_F(\mathbf{r}) \to \infty$$
(2)

for  $\forall M, M \in \mathbb{R}, \exists r_0$ , when  $\left\|\frac{\partial V_F}{\partial r}\right\| \leq M$ .

Suppose the navigation vector field is equal to:

$$\dot{\boldsymbol{r}}_{d} = -\frac{\partial V_{F}}{\partial \boldsymbol{r}} \Gamma(\boldsymbol{r}) + S(\boldsymbol{r})$$
(3)

The navigation vector field is composed of two parts, where the first part  $-\frac{\partial V_F}{\partial r}\Gamma(\mathbf{r})$  is the attractive term, where  $\frac{\partial V_F}{\partial \mathbf{r}}$  is the gradient of Lyapunov function  $V_F(\mathbf{r})$  constructed above,  $\Gamma(\mathbf{r})$  is a symmetric positive definite matrix. The direction of the attraction term is opposite to the direction of the gradient of the Lyapunov function  $V_F(\mathbf{r})$ , and the effect is to reduce  $V_F(\mathbf{r})$ .

The second part  $S(\mathbf{r})$  is the rotation term, and the direction is always perpendicular to the direction of  $\frac{\partial V_F}{\partial \mathbf{r}}$ . The function of the rotation term is to make the UAV fly around the limit cycle when it converges to the limit cycle, without reducing  $V_F(\mathbf{r})$ .

Assume that the position vector of the UAV is  $\mathbf{r} = (x, y, z)^T$ , the limit ring is located in the plane perpendicular to the z axis, the radius of the limit ring is R, and the center of the circle is located at  $(0,0,0)^T$ , as shown in the following figure:



The component of the position vector of the UAV in the limit cycle is  $r_t$ , and the component of the position vector of the UAV in the direction perpendicular to the limit cycle is  $r_n$ .

Select the Lyapunov function as:

$$V_F(\mathbf{r}) = \frac{1}{2}z^2 + \frac{1}{2}\left(\sqrt{x^2 + y^2} - R\right)^2 \tag{4}$$

The gradient of Lyapunov function can be obtained from the above equation:

$$\frac{\partial V_F}{\partial \boldsymbol{r}} = \boldsymbol{r}_n \hat{\boldsymbol{n}}^T + (\boldsymbol{r}_t - R) \hat{\boldsymbol{r}}_t^T$$
(5)

where  $\hat{r}_t$  indicates the unit vector in the direction of  $r_t$ , and  $\hat{r}_t = \frac{r_t}{\|r_t\|}$ .

Let

$$\Gamma(\mathbf{r}) = \frac{1}{\alpha(\mathbf{r})}I$$
(6)

where I is the identity matrix and  $\alpha(\mathbf{r})$  is the normalization factor, the purpose is to keep the navigation vector field  $\|\dot{\mathbf{r}}_d\|$  equal to the velocity of the UAV.

Since the direction of  $S(\mathbf{r})$  is always required to be perpendicular to the direction of  $\frac{\partial V_F}{\partial \mathbf{r}}$ , let

$$S(\mathbf{r}) = \gamma \frac{\widehat{\mathbf{n}} \times \mathbf{r}_t}{\alpha(\mathbf{r})} = \frac{\gamma}{\alpha(\mathbf{r})} (-y, x, 0)^T$$
(7)

The magnitude of  $\gamma$  determines the relative strength of the attraction term and the rotation term, and the symbol of  $\gamma$  determines the direction of the UAV to fly around the target.

Because  $\|\dot{\boldsymbol{r}}_d\| = v$ , we can get:

$$\alpha(\mathbf{r}) = \frac{1}{v} (\mathbf{r}_n^2 + (\mathbf{r}_t - R)^2 + R^2 \gamma^2)^{1/2}$$
$$= \frac{1}{v} (z^2 + (\sqrt{x^2 + y^2} - R) + R^2 \gamma^2)$$
(8)

Synthesize the above formula can be obtained

$$\dot{\boldsymbol{r}}_{d} = -\frac{\partial v_{F}}{\partial \boldsymbol{r}} \Gamma(\boldsymbol{r}) + S(\boldsymbol{r})$$

$$= \frac{v}{\sqrt{x^{2} + y^{2}} \sqrt{z^{2} (\sqrt{x^{2} + y^{2}} - R)^{2} + R^{2} \gamma^{2}}} \begin{bmatrix} -(\sqrt{x^{2} + y^{2}} - R)x - \gamma Ry \\ -(\sqrt{x^{2} + y^{2}} - R)y - \gamma Rx \\ -2\sqrt{x^{2} + y^{2}} \end{bmatrix}$$
(9)

# 3. Numerical approach Target motion observation filtering

In order to solve the problem of inaccurate target information, extended Kalman filter is designed to reduce the detection error and predict the trend of target motion. Denote the target's state as  $\boldsymbol{X}_{t,k} = \begin{bmatrix} x_{t,k}, y_{t,k}, v_{t,x}, v_{t,y} \end{bmatrix}^T$ , and the corresponding observation value as  $\boldsymbol{Z}_{t,k} = \begin{bmatrix} x_{t,k}, y_{t,k} \end{bmatrix}^T$ , so the state equation and observation equation of EKF are as follows:

$$\begin{cases} \boldsymbol{X}_{t,k+1} = \boldsymbol{f}\left(\boldsymbol{X}_{t,k}\right) + \boldsymbol{w}_{t,k} \\ \boldsymbol{Z}_{t,k+1} = \boldsymbol{h}\left(\boldsymbol{X}_{t,k}\right) + \boldsymbol{v}_{t,k} \end{cases}$$
(10)

Fig. 2. Decomposition of the position vector r under the limit cycle.

where  $f \cdot h$  represent the state equation and observation equation

of dynamic target.  $\mathbf{w}_{t,k}$ ,  $\mathbf{v}_{t,k}$  represent zero mean Gaussian white noise and observed noise, which satisfy  $p(\mathbf{w}) \sim N(0, \mathbf{Q})$  and  $p(\mathbf{v}) \sim N(0, \mathbf{R})$ .  $\mathbf{Q} \sim \mathbf{R}$  are the covariance matrix. The EKF process is shown in Fig. 3. The state matrix  $F_k$  and the observation matrix  $H_k$  are Jacobian matrices calculated by taking the partial derivatives of  $f \sim h$  respectively.

$$\boldsymbol{F}_{k} = \begin{bmatrix} \frac{\partial f_{1}}{\partial x_{t,k}} & \cdots & \frac{\partial f_{1}}{\partial v_{t,y}} \\ \vdots & \vdots \\ \frac{\partial f_{4}}{\partial x_{t,k}} & \cdots & \frac{\partial f_{4}}{\partial v_{t,y}} \end{bmatrix}, \quad \boldsymbol{H}_{k} = \begin{bmatrix} \frac{\partial h_{1}}{\partial x_{t,k}} & \frac{\partial h_{1}}{\partial y_{t,k}} \\ \frac{\partial h_{2}}{\partial x_{t,k}} & \frac{\partial h_{2}}{\partial y_{t,k}} \end{bmatrix}$$
Time update (prediction)
$$\begin{array}{|c|c|c|c|c|c|c|c|c|c} 1 & 1 & \text{Calculate the Kalman gain} \\ \hat{x}_{k+1} = f\left(\hat{x}_{k}\right) \\ 2 & \text{Error covariance} \\ \text{transfer} \\ P_{k+1} = F_{k}P_{k}F_{k}^{T} + Q_{k} \end{bmatrix}$$

$$\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c} 1 & 1 & \text{Calculate the Kalman gain} \\ \hat{x}_{k+1} = P_{k+1}H_{k}^{T}\left(H_{k}P_{k+1}H_{k}^{T} + R_{k}\right) \\ 2 & 0 & \text{State update} \\ \hat{x}_{k+1} = \hat{x}_{k+1} + K_{k+1}\left(Z_{k} & h\left(\hat{x}_{k+1}\right)\right) \\ 3 & 0 & \text{Update error covariance} \\ P_{k+1} = (I & K_{k+1}H_{k})P_{k+1} \end{bmatrix}$$

Fig. 3. EKF process.

# 4. Tracking 2-D targets based on Lyapunov navigation vector field method

### 4.1 Tracking of ground targets

**Г с с** 

When a UAV performs a flight tracking task, the position and speed of the UAV itself are obtained by the onboard sensor, and the position and speed of the target are obtained by the onboard camera of the UAV. In the process of designing the guidance law, the location and speed of the target are assumed to be known for convenience. At the same time, because the UAV has a flying height, the target can be tracked and detected from a high altitude, that is, the height of the UAV is constant, and the target can be tracked on a 2-D plane.

In the Earth coordinate system, the absolute speed of the UAV is assumed to be  $v = [\dot{x}, \dot{y}]^T$ , and

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} w_x \\ w_y \end{bmatrix} + \begin{bmatrix} v_{ax} \\ v_{ay} \end{bmatrix}$$
(11)

and

$$\begin{bmatrix} v_{ax} \\ v_{ay} \end{bmatrix} = v_a \begin{bmatrix} \cos \varphi(t) \\ \sin \varphi(t) \end{bmatrix}$$
(12)

where  $\begin{bmatrix} w_x \\ w_y \end{bmatrix}$  Represents the speed of the wind in the Earth coordinate system,  $v_a$  represents the magnitude of the airspeed, which is the speed of the UAV relative to the wind,  $\varphi(t)$  is the yaw angle of the UAV.

In the Earth coordinate system, the absolute speed of the target  $v_t = \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \end{bmatrix}$ , where  $\begin{bmatrix} x_t \\ y_t \end{bmatrix}$  is the position of the UAV. Therefore, it can be obtained that the relative velocity of the UAV relative to the target  $v_r$  is:

$$\boldsymbol{v}_{r} = \boldsymbol{v} - \boldsymbol{v}_{t} = \begin{bmatrix} v_{ax} \\ v_{ay} \end{bmatrix} - \begin{bmatrix} \dot{x}_{t} - w_{x} \\ \dot{y}_{t} - w_{y} \end{bmatrix}$$
(13)

The control methods for unmanned aerial vehicles are as follows:

$$\begin{cases} v_{ax} = u_1(t)\cos\varphi(t) \\ v_{ay} = u_1(t)\sin\varphi(t) \\ \dot{\phi} = u_2(t) \end{cases}$$
(14)

Among them,  $u_1(t)$  and  $u_2(t)$  are control variables,  $u_1(t)$  is the airspeed of the UAV, and  $u_2(t)$  is the turning rate of the UAV. Considering the actual flight environment of UAVs, there are limits to their airspeed and turning rate, namely:

$$\begin{aligned} 0 < v_{min} \le u_1(t) \le v_{max} & (15) \\ \|u_2(t)\| \le w_{max} & (16) \end{aligned}$$

This article considers a maximum turning rate of 0.1 rad/s for unmanned aerial vehicles. In order to facilitate the design of the tracking control law, it is assumed that the influence of wind speed can be ignored during flight, that is, the wind speed is set to zero.

The UAV tracks a stationary target. Assuming that during the tracking process, the position of the target is a known quantity and the UAV's flight speed is constant at  $v_0$ , the chosen displacement Lyapunov function is:

and

g

$$V = (r^2 - R^2)^2$$
(17)

$$r = \sqrt{x_r^2 + y_r^2} = \sqrt{(x - x_t)^2 + (y - y_t)^2}$$
(18)

Among them, r is the relative distance between the UAV and the target, and R is the set standoff distance.

The following Lyapunov navigation vector field can be obtained from (18):

$$(x, y) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \left(\frac{v_0}{r \cdot (r^2 + R^2)}\right) \begin{bmatrix} -x_r \cdot (r^2 - R^2) - y_r (2 \cdot r \cdot R) \\ -y_r \cdot (r^2 - R^2) + x_r (2 \cdot r \cdot R) \end{bmatrix} (19)$$

Since the target is stationary, i.e.  $(\dot{x}_t, \dot{y}_t) = 0$ , the expected flight speed of the UAV is determined by the Lyapunov navigation vector field as:

$$\boldsymbol{v}_d = \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix} = \begin{bmatrix} g_x \\ g_y \end{bmatrix}$$
(20)

The expected flight direction can be obtained from the expected flight speed as follows:

$$\varphi_d = \tan^{-1} \left( \frac{\dot{y}_d}{\dot{x}_d} \right) \tag{21}$$

Considering the neglect of the influence of wind speed, the constant motion speed of the UAV, and the target being in a stationary state, it can be considered that the expected flying speed of the UAV is equal to the absolute speed of the UAV at this time, that is:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} g_x \\ g_y \end{bmatrix}$$
(22)

Taking the full derivative of equation (17) with respect to time t yields:

$$\frac{dV}{dt} = \left[\frac{\partial V}{\partial x_r}, \frac{\partial V}{\partial y_r}\right] \cdot \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \end{bmatrix} = \left[\frac{\partial V}{\partial r} \cdot \frac{\partial r}{\partial x_r}, \frac{\partial V}{\partial r} \cdot \frac{\partial r}{\partial y_r}\right] \begin{bmatrix} \dot{x} - \dot{x}_t \\ \dot{y} - \dot{y}_t \end{bmatrix}$$
$$= \frac{-4rv_0 \cdot (r^2 - R_0^2)^2}{r^2 + R_0^2} \le 0$$
(23)

From this, it can be concluded that the derivative of the displacement Lyapunov function over time is always negative, satisfying the asymptotic stability in the equilibrium state. According to the LaSalle invariance principle, the distance r between the UAV and the target can converge to the set standoff distance and track the target.

In order to ensure that the UAV's heading can converge to the desired tracking loop through the Lyapunov navigation vector field during flight, which requires the UAV's absolute velocity direction to converge to the desired flight velocity direction, the following PID control loop is introduced:

$$u_2(t) = \dot{\varphi}_d - k_1(\varphi - \varphi_d)$$
(24)  
where  $k_1$  is the proportional coefficient.

From the above equation, it can be concluded that

$$\dot{\varphi}_d = 4v_0 \frac{R^3}{(r^2 + R^2)^2} \tag{25}$$

## 4.2 Tracking of ground moving target in 2-D plane

In order to enable the UAV to track moving targets, it is necessary to modify the UAV's flying speed based on the previous content.

Assuming that during the tracking process, the expected flight speed of the corrected UAV is

$$\boldsymbol{\nu}_{d} = \begin{bmatrix} \dot{x}_{d} \\ \dot{y}_{d} \end{bmatrix} = \alpha \begin{bmatrix} g_{x} \\ g_{y} \end{bmatrix} + \begin{bmatrix} \dot{x}_{t} \\ \dot{y}_{t} \end{bmatrix}$$
(26)

where  $\alpha$  is the correction factor for the expected flight speed.

So the relative speed of the UAV relative to the target is

$$\boldsymbol{v}_{r} = \begin{bmatrix} \dot{x}_{r} \\ \dot{y}_{r} \end{bmatrix} = \begin{bmatrix} \dot{x}_{d} \\ \dot{y}_{d} \end{bmatrix} - \begin{bmatrix} \dot{x}_{t} \\ \dot{y}_{t} \end{bmatrix} = \alpha \begin{bmatrix} g_{x} \\ g_{y} \end{bmatrix}$$
(27)

When the UAV tracks the moving target at the corrected expected speed, it can be concluded that

$$\frac{dV}{dt} = \frac{-4\alpha r v_0 \cdot (r^2 - R_0^2)^2}{r^2 + R_0^2}$$
(28)

In order to ensure that the UAV can successfully track the upper target during its flight, the expected flight speed of the modified UAV is equal to the airspeed of the UAV, that is  $\|\boldsymbol{v}_d\| = v_0$ , the following can be obtained:

 $\alpha^{2}(g_{x}^{2} + g_{y}^{2}) + 2\alpha(g_{x}\dot{x}_{t} + g_{y}\dot{y}_{t}) + \dot{x}_{t}^{2} + \dot{y}_{t}^{2} - v_{0}^{2} = 0$ (29)

The above equation applies when the airspeed of the UAV is greater than the absolute speed of the target,  $\alpha$  must be a positive solution, from which equation (29) is always negative. Therefore, when the UAV tracks a moving target, it can still converge to the tracking loop centered on the target.

## 5. Tracking of moving targets in 3D plane

The above only considers tracking targets in 2D plane. Due to the fact that in a real environment, the height of the target will change accordingly as it moves. Therefore, when the UAV continues to fly at a constant altitude, it may cause the altitude difference to exceed the optimal detection range of the UAV sensor. Therefore, altitude information needs to be taken into consideration to ensure that the UAV can maintain an optimal altitude difference with the target.

In 3-D space, construct a displacement Lyapunov function as follows

$$\frac{V = (r^2 - R^2)^2 + (z_r^2 - H^2)^2}{\sqrt{(x - x_t)^2 + (y - y_t)^2}}$$
(30)  
where  $r = \sqrt{x_r^2 + y_r^2} = \sqrt{(x - x_t)^2 + (y - y_t)^2}$  represents  
the relative distance between the UAV and the target on the horizontal  
plane,  $z_r$  represents the relative height between the UAV and the  
target,  $R$  is the set standoff distance, and  $H$  represents the expected  
standoff height between the UAV and the target.

According to the above equation, the navigation vector field in 3-D space is

$$g(x, y, z) = \left(\frac{v_0}{r \cdot (r^2 + R^2)}\right) \begin{bmatrix} -x_r \cdot (r^2 - R^2) - y_r (2 \cdot r \cdot R) \\ -y_r \cdot (r^2 - R^2) + x_r (2 \cdot r \cdot R) \\ \kappa \cdot r \cdot (z_r^2 - H^2) \end{bmatrix} (31)$$

where  $\kappa$  Determine the convergence rate of the navigation vector field in the vertical direction to the desired confrontation height.

Taking the derivative of (31) yields

$$\frac{dV}{dt} = \frac{-4rv_0 \cdot (r^2 - R_0^2)^2 - 4\kappa v_0 \cdot (z_r^2 - H^2)^2}{r^2 + R_0^2}$$
(32)

From the above equation, it can be seen that  $\frac{dv}{dt}$  is always less than 0. Therefore, in 3-D space, when the UAV tracks the target, it can still converge to the tracking loop centered on the target, and the height can also converge to the desired confrontation height.

If a moving target is tracked in 3-D space, in order for the UAV to successfully track, it is necessary to modify the UAV's flying speed based on equation (31)

The revised UAV's expected flying speed is

$$\boldsymbol{\nu}_{d} = \begin{bmatrix} x_{d} \\ \dot{y}_{d} \\ \dot{z}_{d} \end{bmatrix} = \alpha \begin{bmatrix} g_{x} \\ g_{y} \\ g_{z} \end{bmatrix} + \begin{bmatrix} x_{t} \\ \dot{y}_{t} \\ \dot{z}_{t} \end{bmatrix}$$
(33)

where  $\alpha$  is the correction coefficient for the expected flight speed, and the speed of the moving target is  $v_t = (\dot{x}_t, \dot{y}_t, \dot{z}_t)^T$ .

During the flight of a UAV, if the expected flight speed of the corrected UAV is equal to the airspeed of the UAV, i.e.,  $\|\boldsymbol{v}_d\| = v_0$ Substituting the corrected speed into (23) yields

$$\frac{dV_d}{dt} = \frac{-4\alpha r v_0 \cdot (r^2 - R_0^2)^2 - 4\kappa v_0 \cdot (z_r^2 - H^2)^2}{r^2 + R_0^2}$$
(34)

From equation (32), it can be seen that when the airspeed of the UAV is greater than the absolute speed of the target,  $\alpha$ must be a positive solution, so  $\frac{dV_d}{dt}$  is always less than 0. Therefore, when the UAV tracks a moving target in 3-D space, it can still converge to the tracking loop centered on the target, and the height can also converge to the desired standoff height.

# 6. Simulation

In order to be more realistic and meet the requirements of the actual Flight of the unmanned aerial vehicle, the tracking control guidance law derived above is now further verified by software inloop simulation, and the flight status of the multi-rotor is displayed through flight-GEAR (simulated flight simulation system), and the actual flight attitude of the multi-rotor is displayed more intuitively.

Flight-gear receives real-time Flight status sent by Simulink through UDP and is used to observe flight status during Simulink simulation, as shown in Fig.4.



Fig. 4. Software in the loop simulation framework.

6.1 Simulation verification of ground stationary target tracking in 2D plane

Assuming that the UAV starts from the position of (400, 400) and tracks a stationary target at the origin at an airspeed of 20m/s, with a set standoff distance of 200 meters, simulation verification is conducted using Matlab as shown in Fig. 5.



Fig. 5. Tracking of stationary ground targets in 2-D plane.

From the Fig. 5, it can be seen that the UAV can successfully converge to the desired tracking loop according to the Lyapunov navigation vector field, achieving tracking of stationary targets. The Flight results of the UAV in the flight-Gear are shown in Fig. 6.



(a)



(b)











(f)



(g)



Fig. 6. Flight-gear display of trajectory of 2-D stationary target.

# 6.2 Simulation verification of ground moving target tracking in 2D plane

Therefore, assuming that the UAV departs from the position of (800, 800) and tracks the target from the origin at an airspeed of 20m/s, with an absolute velocity of 10m/s, flying at a constant speed along the positive x-axis, and a set standoff distance of 300 meters, Matlab simulation is shown in Fig. 7.





Fig. 7. Tracking of ground moving targets in 2-D plane.







(b)





(d)



(e)

Fig. 8. Flight-gear display trajectory of 2-D moving target.

The Flight results of the UAV tracking the moving target in the flight-Gear are shown in Fig. 8. As can be seen, the UAV with realistic physical models are still able to track moving targets.

# 6.3 Simulation verification of moving target tracking in 3-D plane

Assuming that the UAV starts from the position of (800, 800, 800), tracks from the origin at an airspeed of 20m/s, moves at a speed of (10, 0, 2) target, sets a confrontation distance of 300 meters, and a confrontation height of 100 meters, as shown in Fig. 9

through Matlab simulation.





Fig. 10. Motion Trajectory of UAV and target in inertial frame.

Fig. 9-(a) shows the relative distance between the UAV and the target in the Earth frame, Fig. 9- (b) shows the flight trajectory of the UAV in the coordinate system relative to the target, Fig. 9-(c) shows the motion trajectory of the target in the Earth frame, and Fig. 9-(d) shows the flight trajectory of the UAV in the Earth frame. Fig. 10 shows the synthesis of the motion trajectories of the UAV and the target in the Earth system. As shown in the above 5 figures, in 3-D space, when the UAV tracks the moving target, by adjusting the expected flight speed of the UAV, the Lyapunov navigation vector field can still enable the UAV to track the moving target.



(a)





Fig.9. Tracking of targets in 3-D plane.

(c)















Fig.11. Flight-gear display trajectory of a 3D moving object.

The software in the loop simulation verification results are shown in Fig. 11, it can be seen that through the design of tracking control guidance law based on Lyapunov vector field method and the course control, the UAV is gradually converging to the tracking ring with the target as the center and a certain tracking distance as the radius. This method is feasible, and no matter for the stationary target or the moving target, the UAV will be able to converge to the tracking ring with a certain tracking distance as the radius. Its flight trajectories are relatively smooth, which is conducive to the flight of unmanned aerial vehicles in actual conditions. Moreover, by setting the standoff distance, the distance between unmanned aerial vehicles and targets can always be in a suitable position for observation, so as to facilitate the tracking and observation of important high-value targets.

# 7. Summary

A well resolved and highly accurate direct numerical simulation tool has been developed to understand the fundamental difference in the hydrodynamics of flow over two-dimensional and threedimensional ripples in a channel geometry and its implications on sediment transport. As a first step, in this paper we focus on the steady flow over the ripples. This article is based on the Lyapunov navigation vector field method to determine the flight speed of the UAV in space, so that the UAV can converge to the desired trajectory. It is demonstrated that the designed navigation vector field is simulated and verified in the order of 2-D to 3-D, stationary to motion. It can be seen that the tracking effect is better, the tracking curve is smoother, and it is more suitable for the practical flight application of the UAV.

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