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Adaptive Neural Network Control with Predefined-Time Convergence for Pure-Feedback Nonlinear Systems

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ABSTRACT

Finite/fixed-time control provides a valuable approach for optimizing a system's settling time; however, it lacks the flexibility to independently define both the settling time and the convergence domain. Unlike traditional approaches that address semi-global bounded tracking for pure feedback systems, this paper achieves not only convergence of tracking errors to zero but also ensures that the convergence time can be predefined according to user requirements. To develop the desired predefined-time controller, a mild semi-bounded assumption for non-affine functions is first introduced, which addresses the design challenges posed by pure feedback structures. Then, by leveraging the properties of Radial Basis Function (RBF) neural networks and Young's inequality, an upper bound for unknown nonlinear functions and external disturbances is derived. Finally, a predefined-time virtual control input is provided, and its derivative is estimated using a finite-time differentiator. It is rigorously proven that the proposed novel predefined-time controller guarantees global convergence of tracking errors to zero within the specified time. The effectiveness and practicality of this predefined-time control method are validated through examples.

1. Introduction

Linearization of nonlinear systems is an important method for early research on the control of nonlinear systems, but it has great limitations [1]. As for the research on the control theory of nonlinear systems, the research methods include phase plane method and descriptive function method. However, these two methods are mainly used for the design and stability analysis of simple nonlinear system controllers. In the 80s of the last century, Italy scholar Isidori and others introduced mathematical tools such as differential geometry and differential algebra into the control theory of nonlinear systems, giving sufficient necessary conditions for making the state space observability and controllability of nonlinear systems, which greatly promoted the development of nonlinear system theory [2]. On this basis, many scholars have applied adaptive control, fuzzy control, and other control methods to the control analysis of nonlinear systems [3].

In the field of modern control, convergence is an important indicator of system stability. In order to achieve a faster convergence speed, finite/fixed-time stability has attracted the attention of many scholars [4], [5], [6], [7]which enables the state of the closed-loop system to converge to the equilibrium point in a finite/fixed time.

In recent years, significant advancements have been made in adaptive intelligent finite/fixed-time control, leveraging the strong approximation capabilities of neural networks (NNs) [8], [9], [10]. For instance, Zhang et al. [11] proposed an adaptive neural finite-time control method for single-input single-output (SISO) nonlinear systems with full-state constraints and actuator failures. For multiple-input multiple-output (MIMO) nonlinear systems, a solution for nonsingular fixed-time output feedback control was addressed in [12]. In stochastic nonlinear systems, Sui et al. [13] introduced an adaptive fuzzy finite-time control approach, while Wu et al. [14] developed a fixed-time fuzzy consensus control strategy for multi-agent nonlinear systems. However, a common limitation in these studies is that the settling times cannot be arbitrarily predefined [11], [12], [13], [14]. This becomes a challenge in many engineering applications, such as autonomous vehicle rendezvous and missile guidance, where ensuring system performance within a specified timeframe is crucial. This need drives the research into predefined-time control methods.

Predefined-time stability, a specialized form of fixed-time stability, allows the convergence time to be selected in advance, as introduced

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by Jiménez-Rodríguez E [15]. A sufficient condition for achieving predefined-time stability was later proposed in [11]. Due to the advantageous characteristics of predefined-time stability, it has been applied in areas such as robotics and rigid spacecraft systems [16], [17], [18]. However, in these works, the system nonlinearities are either known or required to meet linear growth conditions. As systems become more complex, the nonlinearities in practical control systems are often unknown, prompting growing interest in adaptive intelligent predefined-time control using FLSs/NNs. Specifically, for unknown strict-feedback nonlinear systems (SFNSs), an adaptive fuzzy predefined-time control approach was explored in [19]. Building on [19], Y Jiang et al. addressed input saturation and output hysteresis by developing a predefined-time adaptive fuzzy controller, [20]. It is acknowledged that pure-feedback nonlinear systems represent a broader class without the affine appearance of state variables. However, previous works [19], [20] did not account for signal quantization, crucial in networked systems where limited communication capacity necessitates quantizing signals before transmission. With the increasing use of networked systems, quantization has garnered significant attention.

The neural network backstepping control method has emerged in recent years as an intelligent control technique for nonlinear systems. This method leverages the powerful function approximation capabilities of neural networks to estimate system uncertainties, and combines this with the backstepping approach to design feedback controllers, thereby significantly mitigating the impact of uncertainties on system stability. Currently, this method has been applied to various nonlinear systems, such as strict-feedback systems [21], pure-feedback systems [22], and partially non-strict-feedback systems [23]. However, due to the structural characteristics of nonlinear systems, the application of the neural network backstepping control method to certain systems faces challenges. Given its immense potential, if this method can be widely applied to underactuated systems, it will greatly advance the development of intelligent control techniques for nonlinear systems. Thus, researching the neural network backstepping control technology for nonlinear systems holds significant theoretical and practical importance.

The characteristics of adaptive backstepping methods allow for achieving asymptotic stability in nonlinear systems and ensuring signal boundedness under parameter uncertainties, which has led to numerous significant results [24], [25]. By combining backstepping methods with fuzzy or neural adaptive technologies, effective control tools for uncertain nonlinear systems have been developed [26]. In [27], adaptive tracking control for nonlinear systems with unknown input constraints and unpredictable variables was investigated. In [28], a control design strategy based on variable separation was developed for non-strict-feedback nonlinear systems.

Inspired by the above discussion, we have applied a novel control scheme based on adaptive neural networks to pure-feedback nonlinear systems. This approach ensures that the tracking error reaches zero within a predetermined time frame. In summary, the main contributions of our work are as follows:

1) Unlike traditional finite-time and fixed-time control methods, the proposed controller addresses a significant issue by ensuring that the tracking error converges from any initial condition within a user-specified fixed time. Many practical control systems require a rapid transition from transient to steady-state response.
2) The derivatives of virtual control laws are estimated using finite-time differentiators instead of being directly applied in the recursive design. This approach effectively prevents the issue of "explosion of complexity." This innovation allows for more efficient and manageable control system designs, streamlining the development process and enhancing system robustness.
3) By applying neural networks, their strong adaptability and learning capability enable continuous adjustment of control strategies in response to system dynamics, improving the system's accuracy and response speed. This approach also reduces the reliance on precise system models, making control system design more simplified and efficient.

2. System preview

The system is:

$$\begin{cases} \dot{x}_i = f_i(x) + h_i(x_{i+1}) \\ \dot{x}_n = f_n(x) + h_n(u) \\ y = x_1 \end{cases} \tag{1}$$

where n stands for the order of the system and x_i are the system states. $u \in R$, $y \in R$ represent the control input and the output of the system, respectively. Nonlinear continuous nonaffine functions $f_i(x)$ and $h_i(x)$ are unknown.

Assumption 1 ([29]): Define $P_i(x_{i+1}) = h_i(x_{i+1}) - h_i(0)$, where $i = 1,\ldots,n$, We assume that

$$\begin{cases} \underline{q}_i x_{i+1} + o_{1i} \le P_i(x_{i+1}) \le \overline{q}_i x_{i+1} + o_{2i}, & x_{i+i} \ge 0 \\ \underline{q}_i' x_{i+1} + o_{3i} \le P_i(x_{i+1}) \le \overline{q}_i' x_{i+1} + o_{4i}, & x_{i+1} < 0 \end{cases} \tag{2}$$

where $\overline{q}_i, \underline{q}_i, \underline{q}_i'$, and $\overline{q}_i'$ stand for unknown positive constants and $\overline{o}_{1i}, o_{2i}, o_{3i}, o_{4i}$ represent unknown constants, $i = 1,\ldots,n$.

From(2), we can know that there exist functions $\mho_{i1}(\overline{x}_{i+1}) \in [0,1]$ and $\mho_{i2}(\overline{x}_{i+1}) \in [0,1]$ satisfying

$$\begin{cases} P_i(x_{i+1}) = (1 - \mho_{i1}(\overline{x}_{i+1}))(\underline{q}_i x_{i+1} + o_{1i}) + \mho_{i1}(\overline{x}_{i+1})(\overline{q}_i x_{i+1} + o_{2i}), & x_{i+1} > 0 \\ P_i(x_{i+1}) = (1 - \mho_{i2}(\overline{x}_{i+1}))(\underline{q}_i' x_{i+1} + o_{3i}) + \mho_{i2}(\overline{x}_{i+1})(\overline{q}_i' x_{i+1} + o_{4i}), & x_{i+1} < 0. \end{cases} \tag{3}$$

To simplify the design of the control system, define the functions clearly $E_i(\overline{x}_{i+1})$ and $\Delta(\overline{x}_{i+1})$ as follows:

$$\begin{cases} \Delta_i(x_{i+1}) = (1 - \mho_{i1}(\overline{x}_{i+1}))\underline{q}_i + \mho_{i1}(\overline{x}_{i+1})\overline{q}_i, & x_{i+1} > 0 \\ \Delta_i(\overline{x}_{i+1}) = (1 - \mho_{i2}(\overline{x}_{i+1}))\underline{q}_i' + \mho_{i2}(\overline{x}_{i+1})\overline{q}_i', & x_{i+1} < 0 \\ E_i(\overline{x}_{i+1}) = (1 - \mho_{i1}(\overline{x}_{i+1}))o_{1i}n + \mho_{i1}(\overline{x}_{i+1})o_{2i}, x_{i+1} > 0 \\ E_i(\overline{x}_{i+1}) = (1 - \mho_{i2}(\overline{x}_{i+1}))o_{3i}n + \mho_{i2}(\overline{x}_{i+1})o_{41}, x_{i+1} < 0. \end{cases} \tag{4}$$

Then, by applying (4), we can model the nonaffine terms $P_i(x_{i+1})$ as

$$P_i(x_{i+1}) = \Delta_i(\overline{x}_{i+1})x_{i+1} + E_i(\overline{x}_{i+1}). \tag{5}$$

From (5), it follows that

$$0 < \underline{\Delta}_i \le \Delta_i(\overline{x}_{i+1}) \le \overline{\Delta}_i$$
$$0 \le \left|\overline{E}_i(\overline{x}_{i+1})\right| \le E_{iM}$$

where $\underline{\Delta}_i = \min_{i=1,\ldots,n}\{\overline{q}_i, \underline{q}_i, \underline{q}_i', \overline{q}_i'\}$, $\overline{\Delta}_i = \max_{i=1,\ldots,n}\{\overline{q}_i, \underline{q}_i, \underline{q}_i', \overline{q}_i'\}$ and $E_{iM} = \max\{|o_{1i}| + |o_{2i}| + |o_{3i}| + |o_{4i}|\}$, and one can find a positive constant g_{mi} such that $g_{mi} < i$. With the help of (5) and the definition of $P_i(x_{i+1})$ can be rewrote as

$$\begin{cases} \dot{x}_i = f_i(x_i) + h_i(0) + \Delta_i(\bar{x}_{i+1})x_{i+1} + E_i(\bar{x}_{i+1}), \quad i = 1,\dots,n-1 \\ \dot{x}_n = f_n(x_n) + h_n(0) + \Delta_n(\bar{x}_{n+1})u + E_n(\bar{x}_{n+1}) \\ y = x_1 \end{cases} \tag{6}$$

3. Controller design

Step 1:

Definition $z_1 = y - y_r$.

Let the Lyapunov function:

$$V_1 = \frac{1}{2}z_1^2 \tag{7}$$

Seek guidance for it

$$\begin{aligned} \dot{V}_1 &= z_1 \times \dot{z}_1 \\ &= z_1[f_1(x_1) + h_1(0) + \Delta(\bar{x}_2)x_2 + E_1(\bar{x}_2) - \dot{y}_r] \\ &= z_1 f_1(x_1) + z_1 h_1(0) + z_1\Delta(\bar{x}_2)z_2 + z_1\Delta(\bar{x}_2)\alpha_1 + z_1 E_1(\bar{x}_2) - z_1\dot{y}_r \\ &= z_1\Delta(\bar{x}_2)z_2 + z_1\Delta(\bar{x}_2)\alpha_1 + z_1[f_1(x_1) + h_1(0) + E_1(\bar{x}_2) - \dot{y}_r] \end{aligned} \tag{8}$$

Perform $f_1(x_1) + h_1(0)$ an approximation,

$$f_1(x_1) + h_1(0) = W_1^T\Phi_1 + \delta_1(x_1)$$

Cause $\bar{W}_1 = [\hat{W}_1, \varepsilon_{1M}, E_{1M}, A_2]^T$, $\bar{\Phi}_1 = [\Phi_1, 1, 1, 1]$,thereinto $|y_r| \le A_1$, $|\dot{y}_r| \le A_2$, $\delta_1(x_1) \le \varepsilon_{1M}$.

We can get

$$\begin{aligned} & f_1(x_1) + h_1(0) + E_1(\bar{x}_2) - \dot{y}_r \\ &= W_1^T\Phi_1 + \delta_1(x_1) + E_1(\bar{x}_2) - \dot{y}_r \\ &\le W_1^T\Phi_1 + \varepsilon_{1M} + E_{1M} + A_2 \\ &\le \|\bar{W}_1\|\|\bar{\Phi}_1\| \\ &\le \|\bar{W}_1\|\sqrt{q+3} \end{aligned}$$

Assumption 2: The desired reference signal y_r is a continuous and bounded function whose bound is unknown. Generally speaking, its time derivative is also a bounded function with unknown bound. We can find unknown constants A_1, A_2 such that $|y_r| \le A_1$, $|\dot{y}_r| \le A_2$.

There is also Young's inequality

$$\|\bar{W}_1\|\sqrt{q+3} \le \frac{1}{2}\left(q + 3 + \|\bar{W}_1\|^2\right) = \frac{1}{2}(q + 3 + \theta_1) \tag{9}$$

thereinto $\theta_1 = \|\bar{W}_1\|^2$.

Will (9) be brought in (8)

$$\dot{V} \le z_1\Delta(\bar{x}_2)z_2 + z_1\Delta_1(\bar{x}_2)\alpha_1 + \frac{1}{2}z_1(q + 3 + \theta_1) \tag{10}$$

Design the virtual controller α_1 as

$$\begin{aligned} \alpha_1 = -\frac{1}{g_m}[&\frac{n}{lT_c}sig(z_1)^{1-l}\exp(z^l) + \frac{1}{2}(q + 3 + \hat{\theta}_1)sign(z_1) \\ &+ k_1 sign(z_1)] \end{aligned} \tag{11}$$

Bring the (11) virtual controller α_1 in (10)

$$\dot{V}_1 \le -\frac{n}{lT_c}|z_1|^{2-l}\exp(z^l) + \frac{1}{2}|z_1|\tilde{\theta}_1 - k_1|z_1| + z_1\Delta_1(\bar{x}_2)z_2$$

Order $W_1 = V_1 + \frac{1}{2}\tilde{\theta}_1^2$, among them $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$.

rule

$$\begin{aligned} \dot{W}_1 &= \dot{V}_1 - \dot{\hat{\theta}}_1\tilde{\theta}_1 \\ &\le -\frac{n}{lT_c}|z_1|^{2-l}\exp(z^l) + \frac{1}{2}|z_1|\tilde{\theta}_1 - k_1|z_1| + z_1\Delta(\bar{x}_2)z_2 - \dot{\hat{\theta}}_1\tilde{\theta}_1 \end{aligned}$$

The adaptive law is

$$\dot{\hat{\theta}}_1 = \frac{1}{2}|z_1|, \ \hat{\theta}_1(0) > 0.$$

Step i:

The finite-time differential function used to approximate α_{i-1} the first derivative is constructed as follows:

$$\begin{aligned} \dot{\omega}_{(i-1)1} &= \omega_{(i-1)2} - \kappa_1 sig(\omega_{(i-1)1} - \alpha_{i-1})^{1/2} \\ \dot{\omega}_{(i-1)2} &= -\kappa_2 sig(\omega_{(i-1)1} - \alpha_{i-1}) \end{aligned}$$

Here, $\omega_{(i-1)1}$、$\omega_{(i-1)2}$、κ_1、κ_2 the state and parameters of the differentiator are described, respectively. According to Levant and Chen and Ge, if the initial deviation $\omega_{(i-1)1}(0) - \alpha_{i-1}(0)$ and are $\omega_{(i-1)2}(0) - \dot{\alpha}_{i-1}(0)$ bounded, then a finite-time differentiator can be provided with arbitrary precision α_{i-1}. Thus, we get $\dot{\alpha}_{i-1} = \omega_{(i-1)2}(t) - \sigma_{i-1}$ the bounded estimation error σ_{i-1}, we can find σ_{i-1} such that: $|\sigma_{i-1}| \le \sigma_{(i-1)M}$.

At this time $z_i = x_i - \alpha_{i-1}$.

$$\begin{aligned} \dot{z}_i &= \dot{x}_i - \dot{\alpha}_{i-1} \\ &= f_i(x_i) + h_i(0) + \Delta_i(\bar{x}_{i+1})x_{i+1} + E_i(\bar{x}_{i+1}) - \dot{\alpha}_{i-1} \\ &= f_i(x_i) + h_i(0) + \Delta_i(\bar{x}_{i+1})(z_{i+1} + \alpha) + E_i(\bar{x}_{i+1}) - \dot{\alpha}_{i-1} \end{aligned}$$

The Lyapunov function is constructed as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2$$

Seek guidance for it

$$\begin{aligned} \dot{V}_i &= \dot{V}_{i-1} + z_i\dot{z}_i \\ &= \dot{V}_{i-1} + z_i f_i(x_i) + z_i h_i(0) + z_i\Delta(\bar{x}_{i+1})z_{i+1} \\ &\quad + z_i E_i(\bar{x}_{i+1}) + z_i\Delta(\bar{x}_{i+1})\alpha_i - \dot{\alpha}_{i-1}z_i \end{aligned} \tag{12}$$

To $f_i(x_i) + h_i(0)$ carry out the approximation, order $f_i(x_i) + h_i(0) = W_i^T\Phi_i + \delta_i(x_i)$, $\bar{W}_i = [\hat{W}_i, \varepsilon_{iM}, E_{iM}, \sigma_{(i-1)M}]^T$, $\bar{\Phi}_i = [\Phi_i, 1, 1, 1]$, rule

$$\begin{aligned} & f_i(x_i) + h_i(0) + E_i(\bar{x}_{i+1}) + \omega_{(i-1)2} - \dot{\alpha}_{i-1} \\ &= W_i^T\Phi_i + \delta_i(x_i) + E_i(\bar{x}_{i+1}) + \omega_{(i-1)2} - \dot{\alpha}_{i-1} \\ &\le W_i^T\Phi_i + \varepsilon_{iM} + E_{iM} + \sigma_{(i-1)M} \\ &\le \|\bar{W}_i\|\|\bar{\Phi}_i\| \\ &\le \|\bar{W}_i\|\sqrt{q+3} \end{aligned}$$

There is also Young's inequality

$$\|\bar{W}_i\|\sqrt{q+3} \le \frac{1}{2}\left(q + 3 + \|\bar{W}_i\|^2\right) = \frac{1}{2}(q + 3 + \theta_i) \tag{13}$$

Will (13) be brought in (12)

$$\begin{aligned} \dot{V}_i \le \dot{V}_{i-1} &+ z_i\Delta(\bar{x}_{i+1})z_{i+1} + \frac{1}{2}|z_i|(q + 3 + \theta_i) + z_i\Delta(\bar{x}_{i+1})\alpha_i \\ &- |z_i|\omega_{(n-1)2} \end{aligned} \tag{14}$$

Construct a virtual controller α_i as

$$\begin{aligned}\alpha_i = -\frac{1}{g_{mi}}[&\frac{n}{lT_c} sig(z_i)^{1-l}\exp(z_i^{\ l}) + \frac{1}{2}(q+3+\hat{\theta}_i) sign(z_i) \\ &+k_i sign(z_i) - \omega_{(i-2)2} + z_{i-1}\Delta_{i-1}(\bar{x}_i)]\end{aligned} \tag{15}$$

Will (15) be brought in (14)

$$\begin{aligned}\dot{V}_i \le \dot{V}_{i-1} - \frac{n}{lT_c}|z_i|^{2-l}\exp(z_i^{\ l}) + \frac{1}{2}|z_i|\tilde{\theta}_i - k_i|z_i| \\ +(z_i\Delta_i(\bar{x}_{i+1})z_{i+1} - z_{i-1}\Delta_{i-1}(\bar{x}_i)z_i)\end{aligned} \tag{16}$$

Definitely (16)

$$\begin{aligned}\dot{V}_i \le &-\sum_{j=1}^{i}\frac{n}{lT_c}|z_j|^{2-l}\exp(z_j^{\ l}) + \sum_{j=1}^{i}\frac{1}{2}|z_j|\tilde{\theta}_j - \sum_{j=1}^{i}k_j|z_j| \\ &+z_i\Delta_i(\bar{x}_{i+1})z_{i+1}\end{aligned} \tag{17}$$

Order $W_i = V_i + \sum_{j=1}^{i}\frac{1}{2}\tilde{\theta}_j^2$, among them $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$.

rule

$$\begin{aligned}\dot{W}_i &= \dot{V}_i - \sum_{j=1}^{i}\dot{\hat{\theta}}_j\tilde{\theta}_j \\ &\le \dot{V}_i - \sum_{j=1}^{i}\frac{n}{lT_c}|z_j|^{2-l}\exp(z_j^{\ l}) + \sum_{j=1}^{i}\frac{1}{2}|z_j|\tilde{\theta}_j - \sum_{j=1}^{i}k_j|z_j| \\ &+z_i\Delta_i(\bar{x}_{i+1})z_{i+1} + z_iz_{i+1} - \sum_{j=1}^{i}\dot{\hat{\theta}}_j\tilde{\theta}_j\end{aligned}$$

The adaptive law is

$$\dot{\hat{\theta}}_i = \frac{1}{2}|z_i|,\ \hat{\theta}_i(0) > 0 .$$

Step n:

Similar to step I, approximately $\dot{\alpha}_{n-1} = \omega_{(n-1)2}(t) + \sigma_{n-1}$, among them $|\sigma_{n-1}| \le \sigma_{(n-1)M}$.

At this time $z_n = x_n - \alpha_{n-1}$.

$$\begin{aligned}\dot{z}_n &= \dot{x}_n - \dot{\alpha}_{n-1} \\ &= f_i(x_i) + h_i(0) + \Delta_n(\bar{x}_{n+1})u + E_i(\bar{x}_{n+1}) - \dot{\alpha}_{n-1}\end{aligned}$$

The Lyapunov function is constructed as

$$V_i = V_{n-1} + \frac{1}{2}z_n^2$$

Seek guidance for it

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + z_n\dot{z}_n \\ &= \dot{V}_{n-1} + z_nf_n(x_n) + z_nh(0) + z_nG(\bar{x}_{n+1})u + z_nE_n(\bar{x}_{n+1}) \\ &+\alpha_i(\Delta(\bar{x}_{i+1})+1) - \dot{\alpha}_{n-1}z_n \\ &= \dot{V}_{n-1} + z_n[f_n(x_n) + h(0) + E_n(\bar{x}_{n+1}) - \dot{\alpha}_{n-1} + \Delta(\bar{x}_{n+1})u]\end{aligned} \tag{18}$$

To $f_n(x_n) + h(0)$ carry out the approximation, order $f_n(x_n) + h(0) = W_n^T\Phi_n + \delta_n(x_n)$, $\bar{W}_n = [\bar{W}_n, \varepsilon_{nM}, E_{nM}, \sigma_{(n-1)M}]^T$, $\bar{\Phi}_n = [\Phi_n, 1,1,1]$, rule

$$\begin{aligned}&f_n(x_n) + h(0) + E_n(\bar{x}_{n+1}) + \omega_{(n-1)2} - \dot{\alpha}_{n-1} \\ &= W_n^T\Phi_n + \delta_n(x_n) + E_n(\bar{x}_{n+1}) + \omega_{(n-1)2} - \dot{\alpha}_{n-1} \\ &\le W_n^T\Phi_n + \varepsilon_{nM} + E_{nM} + \sigma_{(n-1)M} \\ &\le \|\bar{W}_n\|\|\bar{\Phi}_n\| \\ &\le \|\bar{W}_n\|\sqrt{q+3}\end{aligned}$$

There is also Young's inequality

$$\|\bar{W}_n\|\sqrt{q+3} \le \frac{1}{2}\left(q+3+\|\bar{W}_n\|^2\right) = \frac{1}{2}(q+3+\theta_n) . \tag{19}$$

Will (19) be brought in (18)

$$\dot{V}_n \le \dot{V}_{n-1} + z_n\Delta(\bar{x}_{n+1})u + \frac{1}{2}|z_n|(q+3+\theta_n) . \tag{20}$$

The construction controller u is

$$\begin{aligned}u = -\frac{1}{g_{mn}}[&\frac{n}{lT_c} sig(z_n)^{1-l}\exp(z_n^{\ l}) + \frac{1}{2}(q+3+\hat{\theta}_n) sign(z_n) \\ &+k_n sign(z_n) - \omega_{(n-2)2} + z_{n-1}\Delta_{n-1}(\bar{x}_n)]\end{aligned} \tag{21}$$

Will (21) be brought in (20)

$$\begin{aligned}\dot{V}_n \le \dot{V}_{n-1} - \frac{n}{lT_c}|z_n|^{2-l}\exp(z_n^{\ l}) + \frac{1}{2}|z_n|\tilde{\theta}_n - k_n|z_n| \\ -z_{n-1}\Delta_{n-1}(\bar{x}_n)z_n - z_{n-1}z_n)\end{aligned} \tag{22}$$

Definitely (22)

$$\dot{V}_i \le -\sum_{j=1}^{n}\frac{n}{lT_c}|z_j|^{2-l}\exp(z_j^{\ l}) + \sum_{j=1}^{n}\frac{1}{2}|z_j|\tilde{\theta}_j - \sum_{j=1}^{n}k_j|z_j| \tag{23}$$

Order $W_n = V_n + \sum_{j=1}^{n}\frac{1}{2}\tilde{\theta}_j^2$, among them $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$.

rule

$$\begin{aligned}\dot{W}_n &= \dot{V}_n - \sum_{j=1}^{n}\dot{\hat{\theta}}_j\tilde{\theta}_j \\ &\le -\sum_{j=1}^{n}\frac{n}{lT_c}|z_j|^{2-l}\exp(z_j^{\ l}) + \sum_{j=1}^{n}\frac{1}{2}|z_j|\tilde{\theta}_j - \sum_{j=1}^{n}k_j|z_j| - \sum_{j=1}^{n}\dot{\hat{\theta}}_j\tilde{\theta}_j\end{aligned} \tag{24}$$

The adaptive law is

$$\dot{\hat{\theta}}_n = \frac{1}{2}|z_n|,\ \hat{\theta}_n(0) > 0 .$$

Bring it in (24)

$$\dot{W}_n \le -\sum_{j=1}^{n}\frac{n}{lT_c}|z_j|^{2-l}\exp(z_j^{\ l}) - \sum_{j=1}^{n}k_j|z_j| < 0 .$$

The stability of the nonstrict-feedback nonlinear system (1) is analyzed in this section.

Theorem 1: For the considered pure-feedback system (1) subjected to unknown disturbances, the gains k_i satisfy $(1/2)\bar{\theta}_i \le k_i$, The controller, composed of adaptive laws and virtual control inputs, guarantees that all closed-loop signals remain bounded, and the tracking error converges to zero within a specified time T_c .

Proof: we can know that $V_1, z_i, \bar{\theta}_i$ are bounded. Due to the boundedness property of y_r , we can know that is bounded. Because of the fact that θ_i are constants and $\tilde{\theta}_i$ are bounded, we can obtain the boundedness property of $\hat{\theta}_i$. This combined with the boundedness of z_1 and $\dot{y}_r$ and constants q, g_{m1}, n, T_c , and k_1 contributes to the boundedness of α_1 . Ulteriorly, we have x_2 is bounded. Because $(\partial\alpha_1/\partial z_1)$, $(\partial\alpha_1/\partial y_r), (\partial\alpha_1/\partial\dot{y}_r), (\partial\alpha_1/\partial\theta_1)$ are continuous functions that have bounded arguments, and σ_1 is bounded, we can get that $\dot{\alpha}_1$ and ω_{12} are bounded. Considering that $z_2, M(Z_2), \hat{\theta}_2, \Delta_1(x_2)$ are bound and the constants q, g_{m2}, n, T_c , and k_2 , we have α_2 is bounded. From the boundedness property of α_2 and Z_2 , we get the boundedness property of x_3 . Likewise, we can obtain that x_i, α_i and ω_{i2} are bounded. As a result, we are able to obtain the boundedness property of all the close-loop signals.

Due to the boundedness of all close-loop signals, we can find constants k_i such that $(1/2)\tilde{\theta}_i \le k_i$, and we have

$$\dot{V}_n \le \sum_{i=1}^{n} -\frac{n}{lT_c}|z_i|^{2-l}\exp\left(|z_i|^l\right)$$

$$\sum_{i=1}^{n} -\frac{n}{lT_c}|z_i|^{2-l}\exp\left(|z_i|^l\right) = -\frac{n}{lT_c}\sum_{i=1}^{n}\left(z_i^2\right)^{1-\frac{l}{2}}\exp\left(\left(z_i^2\right)^{\frac{l}{2}}\right).$$

we can see that $\exp\left(x^{(l/2)}\right)$ and $x^{1-(l/2)}, \forall x \geq 0$ are monotonously increasing, we can get the following inequality:

$$\sum_{i=1}^{n}\left(z_i^2\right)^{1-\frac{l}{2}} \exp\left(\left(z_i^2\right)^{\frac{l}{2}}\right) \geq \frac{1}{n}\left(\sum_{i=1}^{n}\left(z_i^2\right)^{1-\frac{l}{2}}\right)\left(\sum_{i=1}^{n}\exp\left(\left(z_i^2\right)^{\frac{l}{2}}\right)\right)$$

$$\frac{1}{n}\sum_{i=1}^{n}\exp\left(\left(z_i^2\right)^{\frac{l}{2}}\right) \geq \left(\prod_{i=1}^{n}\exp\left(\left(z_i^2\right)^{\frac{l}{2}}\right)\right)^{\frac{1}{n}} = \exp\left(\frac{1}{n}\sum_{i=1}^{n}\left(\left(z_i^2\right)^{\frac{1}{2}}\right)\right)$$

Therefore, we have

$$\sum_{i=1}^{n}\left(z_i^2\right)^{1-\frac{l}{2}} \exp\left(\left(z_i^2\right)^{\frac{l}{2}}\right) \geq \left(\sum_{i=1}^{n}\left(z_i^2\right)^{1-\frac{l}{2}}\right)\exp\left(\frac{1}{n}\sum_{i=1}^{n}\left(\left(z_i^2\right)^{\frac{l}{2}}\right)\right).$$

Due to $0 < (l/2) < 1$, one has

$$\begin{aligned}\sum_{i=1}^{n}\left(z_i^2\right)^{1-\frac{l}{2}} \exp\left(\left(z_i^2\right)^{\frac{l}{2}}\right) &\geq \left(\sum_{i=1}^{n}\left(z_i^2\right)^{1-\frac{l}{2}}\right)\exp\left(\frac{1}{n}\sum_{i=1}^{n}\left(\left(z_i^2\right)^{\frac{l}{2}}\right)\right)\\ &\geq \left(\sum_{i=1}^{n}\left(z_i^2\right)^{1-\frac{l}{2}}\right)\exp\left(\frac{1}{n}\left(\sum_{i=1}^{n} z_i^2\right)^{\frac{l}{2}}\right)\end{aligned}$$

The above formula becomes

$$\begin{aligned}\dot{V}_n &\leq -\frac{n}{lT_c}\sum_{i=1}^{n}\left(z_i^2\right)^{1-\frac{l}{2}} \exp\left(\left(z_i^2\right)^{\frac{l}{2}}\right) \leq -\frac{n}{lT_c}\sum_{i=1}^{n}\left(z_i^2\right)^{1-\frac{l}{2}} \exp\left(\frac{1}{n}\left(\sum_{i=1}^{n} z_i^2\right)^{\frac{l}{2}}\right)\\ &= -\frac{n}{lT_c}\sum_{i=1}^{n}\left(2V_n\right)^{1-\frac{l}{2}} \exp\left(\frac{1}{n}\left(2V_n\right)^{\frac{l}{2}}\right).\end{aligned}$$

We define $\mu = 2V_n$, and we can rewrite above formula as

$$\dot{\mu} \leq -\frac{2n}{lT_c}\mu^{1-\frac{l}{2}} \exp\left(\frac{1}{n}\mu^{\frac{l}{2}}\right)$$

At time t, $V_n(t) = 0$ and $\mu_n(t) = 0$, and we estimate the stability time bound as

$$t \leq T_c - T_c \lim_{\mu(0)\to\infty} \exp\left(-\frac{1}{n}\mu(0)^{\frac{1}{2}}\right) = T_c.$$

As a result, the tracking error z_1 will converge to zero within predefined time T_c.

4. Emulation

In the above formulation of the paper, the research work has been completed. In this section, the simulation verification of the designed finite-time controller will be done using matlab.

In the preceding formulation of the paper, the research work has been accomplished.

This section employs simulation examples to validate the efficacy of the proposed control strategy. The subsequent electromechanical systems are examined to illustrate the effectiveness and superiority of the developed control scheme in the physical system:

$$\begin{cases} MQ'' + BQ' + N\sin(Q) = I \\ LI' + RI = V_e - KBQ' \end{cases}$$

where

$$M = \frac{J}{K_\tau} + \frac{mL_0^2}{3K_\tau} + \frac{M_0L_0^2}{K_\tau} + \frac{2M_0R_0^2}{5K_\tau}, N = \frac{mL_0G}{2K_\tau} + \frac{M_0L_0G}{K_\tau}, B = \frac{B_0}{K_\tau},$$

G represents the gravity coefficient, $I(t)$ is the motor armature current, and $q(t)$ represents the angular position of the motor. V_e represents the input control voltage.

By designing the input voltage, the desired motion of the motor driving the load can be realized.

Via a coordinate transformation $x_1 = q$, $x_2 = \dot{q}$, $x_3 = I$, the above kinetic model can be rewritten in the following form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{N}{M}\sin(x_1) - \dfrac{B}{M}x_2 + \dfrac{1}{M}x_3 + d_2(t) \\ \dot{x}_3 = -\dfrac{K_B}{L}x_2 - \dfrac{R}{L}x_3 + \dfrac{1}{L}u + d_3(t) \end{cases}$$

The deception attack signals suffered by the sensor network are chosen as $\lambda_1 = 1 + 0.2\sin(t)$, $\lambda_2 = 1 + 0.1\cos(t)$, $\lambda_3 = 1 + 0.05\sin(t)\cos(t)$.

In this section, we verify and analyze the performance of the control scheme designed.

In this paper, and the controller parameters are designed as g_{m1}=1, g_{m2}=1, g_{m3}= 2, k_1 =12, k_2=15, k_3 = 25, = 0.25, q = 10, r_{11} = 2, r_{12} = 2.7, r_{21} = 3, r_{22} =3.46, r_{31} = 4.48, and r_{41} = 4.49.

The simulation results in this subsection are presented in Fig.1.

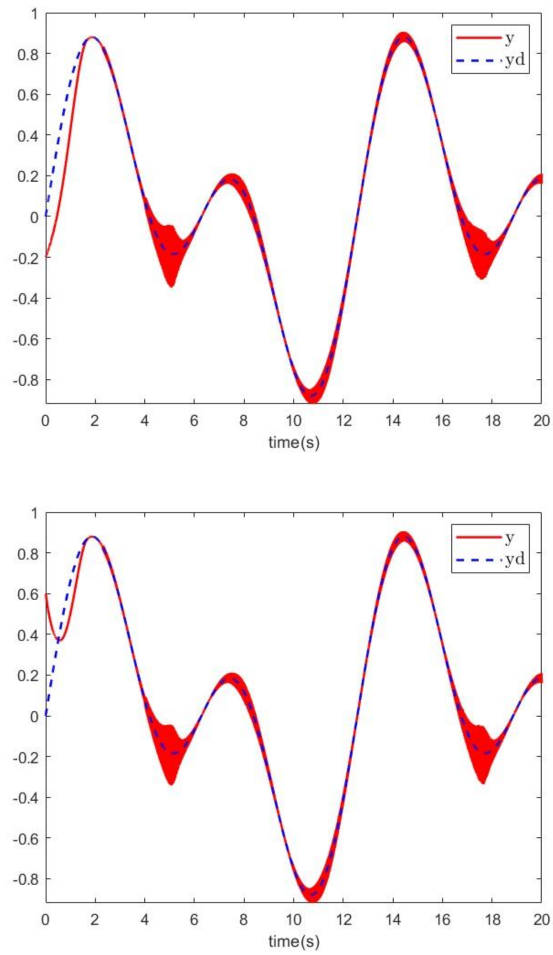

Fig. 1. The trajectories of $y(t)$ and $y_d(t)$

We assign a fixed predefined time Tc= 2, and the outcomes depicted in the figure. demonstrate that the system outputs successfully follow the desired tracking signal within Tc= 2. This confirms that the time required for convergence using our control strategy is independent of the system's initial states.

5. Summary

In this study, we focus on the problem of global predefined-time tracking control for pure-feedback nonlinear systems that are affected by unknown disturbances. We introduced an innovative adaptive neural network (NN) control approach, specifically utilizing radial basis function (RBF) NN control combined with robust control techniques to address unknown nonlinearities. This scheme guarantees that the tracking errors converge to zero globally within a predetermined timeframe. Additionally, we developed virtual controllers whose derivatives are estimated using a finite-time differentiator. It's important to highlight the growing importance of consensus control in multi-agent systems, which is increasingly relevant in various practical engineering applications like traffic flow management, coordination of robot teams, sensor networks, maritime navigation, and cooperative monitoring. Looking ahead, we aim to further explore and extend our predefined-time control strategy to nonlinear multi-agent systems, enhancing its applicability across diverse fields.

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