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Reliable H_∞ Filter Design for Time-Delay Systems with Sensor Saturation

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Abstract

This study addresses the control of a class of continuous-time systems subject to both sensor saturation and failure and additive L_2 -bounded disturbances. Such systems have time delay in their state equations and saturation in their output equations. Attention is focused on the analysis and the synthesis problems of a reliable H_∞ filter such that the filtering error is asymptotically stable. New results for H_∞ filter are proposed by means of linear matrix inequalities (LMIs)-based optimisation approach. Based on the Lyapunov-Krasorskii stability theorem and the linear matrix inequalities (LMI) techniques, solutions can be found. Dynamical filter architecture is proposed, and the linear filter designed by the method can guarantee the performance. A numerical example is given to illustrate the effectiveness of the proposed results.

KEYWORDS

Sensor saturation , linear matrix inequality, H_∞ filter , time-delay systems, sensor fault model

1 | INTRODUCTION

In practical control systems it is well-known that every practical system is subjected to actuator and sensor saturation. Because of their theoretical significance and practical importance, the problem of controller design for systems with actuator saturation has been extensively investigated [1],[2],[3],[4],[5].

The control of linear systems subject to amplitude constraints actuator have been extensively studied [6], and some works concerning stability [7], some works concerning anti-windup synthesis [8] and dynamic output feedback controllers and static anti-windup loops [9].The saturation of the sensor output induces an incorrect action of the controller, since the actual state or output of the plant is no longer precisely measured. Comparing with the vast literatures with respect to control problem for systems with actuator saturation, the filtering results for sensor saturation have been relatively few probably because of the technical difficulty. Few results concern the case of sensor saturation, for instance, the effects of sensor saturation on plant observability are studied [10], the design of an output H_∞ filtering for linear continuous- time systems subject to sensor nonlinearities based on the use of a classical sector condition[10], the stabilisation of a linear system is carried out [11] and also, addresses the control of linear systems subject to both sensor and actuator saturations and additive L_2 -bounded disturbances [11]. Time delay is one of the inevitable problems in actual engineering applications. Considering time delay in the controller design process will be very

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important to the system stability and performance. The presence of time delay may result in instability, chaotic mode, and poor performance of the systems. Thus, the control of dynamic systems with time delay has been a research subject of great practical and theoretical significance, which has received considerable attention in the past decades [12]-[13]. Another one of the most critical issues is contingent failures, which are possible for all sensors in real-world systems. A large degree of filter performance may degrade. Relatively few attentions have been focus on the design of reliable filter systems in the literature [14]. In some safety critical systems, such as flight control and navigation systems, filter schemes should ensure performances in spite of the presence sensor [15],[16].

Filtering problems play an important role in signal processing. Various filtering schemes have been presented recently, such as robust filtering, Kalman filtering, set-membership filtering and FIR filter. The filtering problem is one of both theoretical and practical importance for control design and signal processing. It is concerned with the disturbance to estimation errors is within a prescribed level. In contrast with the well-known Kalman filter, the filtering makes no assumptions on the statistics of the process or the measurement noise except for the only bounded energy one. Moreover, filters have been demonstrated more useful in industrial applications. Up to now, various approaches to the filter design have been proposed, such as the algebraic Riccati-equation method, the interpolation method, the linear matrix inequality method, etc. Practical filtering systems are inherently subject to many nonlinear effects, such as adder overflows and amplitude saturations [17],[18]. Especially, the H_∞ filter design problem for the systems with delay-dependent and delay-independent designs has received much attention. One of the practical issues which should be considered in the controller design process is the sensor saturation problem because sensors and actuators cannot provide unlimited signal due primarily to the physical, safety or technological constraints [19].

However, to our best knowledge, in contrast to the existing results on the analysis and synthesis of H_∞ filters for systems either with time delay or with saturation, literature [20] investigated the H_∞ filtering problem for a class of linear continuous-time systems with both time delay and saturation. A reliable H_∞ filter design problem is proposed for a class of continuous-time system with sensor saturation and failures[21]. There are fewer results for the systems with both time delay and sensor failures and saturation. It is therefore, the objective of this paper to fill the gap.

In this paper, we aim to investigate the H_∞ filtering problem for systems with both the sensor failures and saturation. Note that the systems model addressed is quite comprehensive to cover sensor failures and saturations, hence reflecting the reality closely. The main contribution in this paper is that our method can deal with the sensor subject to both the failures and saturations while the existing results cannot be implemented directly.

2 | PROBLEM STATEMENT

Consider the following continuous-time linear system with sensor saturation:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-\tau) + B_1 w(t) \\ y(t) = sat(Cx(t)) + B_2 w(t) \\ z(t) = Lx(t) \\ x(\theta) = \tilde{\varphi}(\theta) \end{cases} \tag{1}$$

where $x(t) \in R^n$, $y(t) \in R^p$ and $z(t) \in R^r$ are the state of the system, the measurement output and the unmeasurable signal to be estimated, respectively, $w(t) \in R^m$ is the disturbance input which belongs to $L_2[0,\infty)$, and $A \in R^{n\times n}$, $A_d \in R^{n\times n}$, $B_1 \in R^{n\times m}$, $B_2 \in R^{p\times m}$, $C \in R^{p\times n}$ and $L \in R^{r\times n}$ are known constant matrices. $\tilde{\varphi}(\theta)$ is a given continuous function on $[-\tau, 0]$, the time delay $\tau > 0$ is a constant number.

Here, a general sensor fault model [22]-[23] is adopted in this paper for$i = i, \cdots, m, j = i, \cdots, R$.

$$u_{ij}^{\mathrm{F}}(t) = \left(1 - \rho_i^j\right) y_j(t), 0 \leq \rho_{i(\min)}^j \leq \rho_i^j \leq \rho_{i(\max)}^j$$

where ρ_i^j is an unknown constant. The index j represents the jth failure mode and j stands for the total failure modes. $u_{ij}^{\mathrm{F}}(t)$ denotes the signal from the ith sensor that has failed in the jth fault mode. For every fault mode, $\rho_{i(\min)}^j$ and $\rho_{i(\max)}^j$ denotes the lower and upper bounds of ρ_i^j , respectively. Note that, when $\rho_{i(\min)}^j = \rho_{i(\max)}^j = 0$, there is no fault for the ith sensor $y_i(t)$ is outage in the jth fault mode. When $0 < \rho_{i(\min)}^j \le \rho_{i(\max)}^j < 1$, it corresponds to the partial degradation of $y_i(t)$ in the jth fault mode.

Denote

$$y_j^{\mathrm{F}}(t) = diag\left\{u_{1j}^{\mathrm{F}}, u_{2j}^{\mathrm{F}}, \cdots, u_{mj}^{\mathrm{F}}\right\} = \left(I - \rho^j\right) y(t)$$

where $\rho^j = diag\left\{\rho_1^j, \rho_2^j, \cdots, \rho_m^j\right\}$ with $j = 1, 2, \cdots, R$.

Consider the lower and upper bounds $\left(\rho_{i(\min)}^j, \rho_{i(\max)}^j\right)$, the following set is defined:

$$V_{\rho^j} = \left\{\rho^j \,\middle|\, \rho^j = diag\left\{\rho_1^j, \rho_2^j, \cdots, \rho_m^j\right\}, \rho_i^j = \rho_{i(\min)}^j or \rho_i^j = \rho_{i(\mathrm{m}ax)}^j\right\}$$

Clearly, there are at most 2^m elements in the set V_{ρ^j}. For convenience, we will use a uniform sensor fault model

$$y^{\mathrm{F}}(t) = (I - \rho) y(t), \rho \in \left\{\rho^1, \rho^2, \cdots, \rho^R\right\} \tag{2}$$

with $\rho = diag\{\rho_1, \rho_2, \cdots, \rho_m\}$. The system (1) with sensor failure (2) is described by

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-\tau) + B_1 w(t) \\ y^F(t) = (I - \rho)\left[sat(Cx(t)) + B_2 w(t)\right] \\ z(t) = Lx(t) \\ x(\theta) = \tilde{\varphi}(\theta) \end{cases} \tag{3}$$

The saturation function $sat(\cdot) : R^p \to [-1, 1]^p$ is defined as follows:

$$sat(u) = \left[sat(u_1), sat(u_2), \cdots, sat(u_p)\right]^{\mathrm{T}}$$

where $sat(u_i) = sign(u_i)\min(1, |u_i|), i = 1, 2, \cdots, p$. Here, the notation $sat(\cdot)$ is slightly abused to denote both the scalar valued and the vector valued saturation functions. Without loss of generality, it is assumed that the saturation is a unit level because the level of saturation can always be scaled to unity by scaling the matrix C.

In this paper, we design a filter in the following form for system (1)

$$\begin{cases} \dot{\hat{x}}(t) = A_f \hat{x}(t) + B_f y^{\mathrm{F}}(t), \hat{x}(0) = 0 \\ \hat{z}(t) = C_f \hat{x}(t) \end{cases} \tag{4}$$

To handle the saturation term in system (1), we introduce a new matrix $D(\alpha) = diag\{\alpha_i(x)\}$, where the diagonal elements$\alpha_i(x), i = 1, 2, \cdots, p$, satisfying

$$\alpha_i(x) = \begin{cases} -\dfrac{1}{C_i x}, C_i x < -1 \\ 1, -1 \le C_i x \le 1 \\ \dfrac{1}{C_i x}, C_i x > 1 \end{cases} \tag{5}$$

which implies that $0 < \alpha_i(x) \le 1\,(i = 1, 2, \cdots, p)$. Here C_i is the i-th row of C.

With the above notation, systems (1) and (3) can be rewritten as

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-\tau) + B_1 w(t) \\ y^F(t) = (I - \rho)\left[D(\alpha) C x(t) + B_2 w(t)\right] \\ z(t) = Lx(t) \\ x(\theta) = \tilde{\varphi}(\theta) \end{cases} \tag{6}$$

and

$$\begin{cases} \dot{\hat{x}}(t) = A_f \hat{x}(t) + B_f (I-\rho)[D(\alpha) C x(t) + B_2 w(t)] \\ \hat{z}(t) = C_f \hat{x}(t) \end{cases} \tag{7}$$

then,we have $e(t) = z(t) - \hat{z}(t)$, Combining (6) and (7), we obtain a filtering

$$\dot{\xi} = \tilde{A}\xi + \tilde{A}_d H \xi (t-\tau) + \tilde{B} w(t) \tag{8}$$

where
$\xi = \begin{bmatrix} x^{\mathrm{T}}(t) & \hat{x}^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}, e(t) = \begin{bmatrix} L & -C_f \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}, \tilde{A} = \begin{bmatrix} A & 0 \\ B_f (1-\rho) D(\alpha) C & A_f \end{bmatrix}, \tilde{A}_d = \begin{bmatrix} A_d \\ 0 \end{bmatrix},$
$H = \begin{bmatrix} I_n & 0 \end{bmatrix}, \tilde{B} = \begin{bmatrix} B_1 \\ B_f (I-\rho) B_2 \end{bmatrix}, \tilde{C} = \begin{bmatrix} L & -C_f \end{bmatrix}$

The aim of reliable H_∞ filtering problem addressed in this paper is as follows:

1) system (8) with $w(t) = 0$ is asymptotically stable,

2) the L_2 gain of the estimation error vector $e(t)$ is bounded by γ, under the zero initial condition $e(t) = 0\,(t \in [-\tau, 0])$, $\|e(t)\|_2 \leq \gamma \|w\|_2$ is satisfied for any nonzero $w(t) \in L_2[0, \infty)$.

Moreover, if there exists a minimal positive scalar γ satisfying 1) and 2), system (3) is an optimal H_∞ filter of $z(t)$.

3 | MAIN RESULTS

Consider the stability of system (8) in the absence of noise $w(t)$ as

$$\dot{\xi} = \tilde{A}\xi + \tilde{A}_d H \xi (t-\tau) \tag{9}$$

where $e(t) = \begin{bmatrix} L & -C_f \end{bmatrix} \xi$.

Theorem 1 *If there exist symmetric positive definite matrices $P \in R^{2n \times 2n}$ and $Q \in R^{n \times n}$ such as*

$$S = \begin{bmatrix} \tilde{A}^T P + P\tilde{A} + H^T Q H & P\tilde{A}_d \\ * & -Q \end{bmatrix} < 0$$

then system (9) is asymptotically stable.

Proof 1 *For system (9), construct the following Lyapunov-Krasovskii function*

$$V(\xi(t)) = \xi^T(t) P \xi(t) + \int_{t-\tau}^{t} \xi^T(s) H^T Q H \xi(s)\, ds \tag{10}$$

where $\xi_t(\theta) = \xi(t+\theta), \forall \theta \in [-\tau, 0]$. Computing the derivative of $V(\xi(t))$ along the trajectory of system (9), we have

$$\begin{aligned} \dot{V}(\xi(t)) &= 2\xi^T(t) P \dot{\xi}(t) + \xi^T(t) H^T Q H \xi(t) - \xi^T(t-\tau) H^T Q H \xi(t-\tau) \\ &= 2\xi^T(t) P \left(\tilde{A}\xi + \tilde{A}_d H \xi(t-\tau)\right) + \xi^T(t) H^T Q H \xi(t) - \xi^T(t-\tau) H^T Q H \xi(t-\tau) \\ &= \begin{bmatrix} \xi(t) \\ H\xi(t-\tau) \end{bmatrix}^T S \begin{bmatrix} \xi(t) \\ H\xi(t-\tau) \end{bmatrix} \end{aligned}$$

where $S = \begin{bmatrix} \tilde{A}^T P + P\tilde{A} + H^T Q H & P\tilde{A}_d \\ * & -Q \end{bmatrix}$.

Therefore, if $S < 0$, then we can get $\dot{V}(\xi(t)) < 0$.

According to the Lyapunov-Krasovskii stability theorem, system (9) is asymptotically stable. ■

In order to design the optimal H_∞ filter, we present the following performance result.

Theorem 2 *If there exists symmetric positive definite matrices $P \in R^{2n\times 2n}$ and $Q \in R^{n\times n}$, and a positive scalar γ, such that*

$$S_3 = \begin{bmatrix} \tilde{A}^T P + P\tilde{A} + H^T QH & P\tilde{B} & \tilde{C}^T & P\tilde{A}_d \\ * & -\gamma^2 I & 0 & 0 \\ * & 0 & -I & 0 \\ * & 0 & 0 & -Q \end{bmatrix} < 0$$

then there exists an optimal H_∞ filter such that system (8) is asymptotically stable, and the upper bound of the L_2 gain for the estimation error vector $e(t)$ is finite. Moreover, the upper bound γ^* is obtained by

$$\min \quad \gamma^2 s.t. \quad S_3 < 0, P > 0, Q > 0 \tag{11}$$

Proof 2 *Considering system (8), and using the Lyapunov-Krasovskii functional in (10), it is easy to see that $\|e(t)\|_2 \le \gamma \|w\|_2$ is equivalent to*

$$J(w(t)) = \int_0^\infty \left[e^T(t)\, e(t) - \gamma^2 w^T(t)\, w(t) \right] dt$$

Since $V(\xi(t))|_{t=0} = 0$ under zero initial condition and $V(\xi(t))|_{t=\infty} \ge 0$, we have

$$\begin{aligned} J(w(t)) &\le \int_0^\infty \left[e^T(t)\, e(t) - \gamma^2 w^T(t)\, w(t) + \dot{V}(\xi(t)) \right] dt - V(\xi(t))|_{t=0} + V(\xi(t))|_{t=\infty} \\ &\le \int_0^\infty \left[e^T(t)\, e(t) - \gamma^2 w^T(t)\, w(t) + \dot{V}(\xi(t)) \right] dt \end{aligned}$$

then

$$\dot{V}(\xi(t)) = \begin{bmatrix} \xi(t) \\ w(t) \\ H\xi(t-\tau) \end{bmatrix}^T S_1 \begin{bmatrix} \xi(t) \\ w(t) \\ H\xi(t-\tau) \end{bmatrix}$$

$$S_1 = \begin{bmatrix} \tilde{A}^T P + P\tilde{A} + H^T QH & P\tilde{B} & P\tilde{A}_d \\ * & 0 & 0 \\ * & * & -Q \end{bmatrix}$$

we have

$$\begin{aligned} J(w(t)) &\le \int_0^\infty \left[e^T(t)\, e(t) - \gamma^2 w^T(t)\, w(t) + \dot{V}(\xi(t)) \right] dt \\ &= \int_0^\infty \left[e^T(t)\, e(t) - \gamma^2 w^T(t)\, w(t) + \dot{V}(\xi(t)) \right] dt \\ &= \int_0^\infty \begin{bmatrix} \xi(t) \\ w(t) \\ H\xi(t-\tau) \end{bmatrix}^T S_2 \begin{bmatrix} \xi(t) \\ w(t) \\ H\xi(t-\tau) \end{bmatrix} \end{aligned}$$

where

$$S_2 = \begin{bmatrix} \tilde{A}^T P + P\tilde{A} + H^T QH + \tilde{C}^T\tilde{C} & P\tilde{B} & P\tilde{A}_d \\ * & -\gamma^2 I & 0 \\ * & * & -Q \end{bmatrix}$$

Therefore, if $S_2 < 0$, then we can get $J(w(t)) < 0$. According to the matrix Schur complement,

$$S_3 = \begin{bmatrix} \tilde{A}^T P + P\tilde{A} + H^T QH & P\tilde{B} & \tilde{C}^T & P\tilde{A}_d \\ * & -\gamma^2 I & 0 & 0 \\ * & 0 & -I & 0 \\ * & 0 & 0 & -Q \end{bmatrix} < 0$$

Then, $S_2 < 0$, $J(w(t)) < 0$. Hence, the L_2 gain of the estimation error vector $e(t)$ does not exceed γ, the gain is finite. Moreover, it is easy to see that such γ can be minimized by the optimization problem (11).

Since $S_3 < 0$ is a nonlinear matrix inequality, one can hardly obtain the parameters of the filter. Next section we need to change $S_3 < 0$ into a linear one by using the LMI technique. ■

Let $S_0 \subset R^n$ be any compact set and note that (5) holds, For any $x(t) \in S_0$, one can define a lower bound of $\alpha_i(x)$ as $\alpha_{\min(i)} = \min\{\alpha_i(x), x \in S_0\}$, which implies that scalars $\alpha_i(x), i = 1,2,\cdots,p$, satisfy $\alpha_{\min(i)} \le \alpha_i(x) \le 1$ and $D\left(\alpha_{\min(i)}\right) \le D(\alpha_i(x)) \le 1, \forall x(t) \in S_0$.

lemma[21] Let $D(\gamma_j) = diag\left\{\gamma_{j(i)}\right\}$, where $\gamma_{j(i)} = 1 \; or \; \alpha_{\min(i)}, i = 1,2,\cdots,p$. Then $D(\alpha)$ can be written as $\sum_{j=1}^{2^p} \lambda_{j,t} D(\gamma_j)$ with $\sum_{j=1}^{2^p} \lambda_{j,t} = 1, \lambda_{j,t} \ge 0$.

Theorem 3 *Assume that $D(\alpha_{\min})$ is given, if there exists a $\gamma > 0$, three matrices $M \in R^{n\times n}$, $N \in R^{r\times n}$ and $Z \in R^{n\times p}$, and three symmetric positive definite matrices $Q \in R^{n\times n}, R \in R^{n\times n}$ and $X \in R^{n\times n}$ satisfying the following LMIs*

$$\begin{bmatrix} R & R \\ R & X \end{bmatrix} > 0 \tag{12}$$

$$\begin{bmatrix} -I & \Psi_j^T \\ * & \Psi_j + \Psi_j^T \end{bmatrix} \le 0, j = 1,\cdots,2^p \tag{13}$$

and the following optimization problem has a solution γ^*:

$$\min \quad \gamma^2$$

$$s.t. \begin{bmatrix} RA + AR^T + Q + I & \Xi & RB_1 & L^T - N^T & RA_d \\ * & \Xi_1 & XB_1 + ZB_2 & L^T & XA_d \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & 0 & -I & 0 \\ * & * & 0 & 0 & -Q \end{bmatrix} < 0 \tag{14}$$

for all $\rho \in \left\{\rho^1,\cdots,\rho^R\right\}$ with $\rho^j \in V_{\rho^j}, j = 1,2,\cdots,R$. where

$$\Psi_j = Z(I-\rho)D(\gamma_j)C - Z(I-\rho)D(\alpha_{\min})C$$

$$\Xi = RA + A^T X + (Z(I-\rho)D(\alpha_{\min})C)^T + M^T + Q$$

$$\Xi_1 = XA + A^T X + (Z(I-\rho)D(\alpha_{\min})C)^T + (Z(I-\rho)D(\alpha_{\min})C) + Q$$

Then there exists an optimal H_∞ filter in the form (4) such that the system (8) is asymptotically stable, and the upper bound of the L_2 gain given in the form $\|e(t)\|_2 \le \gamma\|w\|_2$ is γ^*. Furthermore, a suitable filter is given by

$$\begin{cases} A_f = S^{-1}MR^{-1}\left(W^T\right)^{-1} \\ B_f = S^{-1}Z \\ C_f = NR^{-1}\left(W^T\right)^{-1} \end{cases}$$

Proof 3 *Based on Lemma, (13) is equivalent to*

$$\sum_{j=1}^{2^p} \lambda_{j,t} \begin{bmatrix} -I & \Psi_j^T \\ * & \Psi_j + \Psi_j^T \end{bmatrix} \le 0, j = 1,\cdots,2^p$$

i.e.,

$$\begin{bmatrix} -I & \Phi^T \\ * & \Phi^T+\Phi \end{bmatrix} \le 0$$

From which it follows that

$$\begin{bmatrix} -I & \Phi^T & 0 & 0 & 0 \\ * & \Phi^T+\Phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \le 0 \tag{15}$$

where

$$\Phi = Z(I-\rho)D(\alpha)C - Z(I-\rho)D(\alpha_{\min})C.$$

According to (14) and (15), we have

$$\begin{bmatrix} RA+AR^T+Q & \tilde{\Xi} & RB_1 & L^T-N^T & RA_d \\ * & \tilde{\Xi}_1 & XB_1+ZB_2 & L^T & XA_d \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & 0 & -I & 0 \\ * & * & 0 & 0 & -Q \end{bmatrix} < 0 \tag{16}$$

where $\tilde{\Xi} = RA + A^TX + (Z(I-\rho)D(\alpha)C)^T + M^T + Q$ *and* $\tilde{\Xi}_1 = XA + A^TX + (Z(I-\rho)D(\alpha)C)^T + (Z(I-\rho)D(\alpha)C) + Q$.

From (12), it is easy to obtain that $R > 0$ *and* $X - R > 0$*. Therefore,* $I - XR^{-1}$ *is nonsingular, which implies that there exist two nonsingular matrices* $S \in R^{n\times n}$ *and* $W \in R^{n\times n}$ *such that* $XR^{-1} + SW^T = I$.

Letting $J = \begin{bmatrix} R^{-1} & I \\ W^T & 0 \end{bmatrix}$, $\tilde{J} = \begin{bmatrix} I & X \\ 0 & S^T \end{bmatrix}$, *and the matrix P be parameterized as follows:*

$$\tilde{J}J^{-1} = \begin{bmatrix} X & S \\ S^T & W^{-1}R^{-1}(X-R)R^{-1}W^{-T} \end{bmatrix} = P$$

We have

$$X - R > 0 \Leftrightarrow \begin{bmatrix} R^{-1} & I \\ I & X \end{bmatrix} = J^TPJ > 0 \Leftrightarrow P > 0$$

i.e., P is a symmetric positive definite matrix.

Pre-multiplying and post-multiplying (16) by diag $\{R^{-1}, I, I, I, I\}$*, we readily obtain*

$$\begin{bmatrix} \Omega_1 & \Omega_2 & RB_1 & L^T-N^T & RA_d \\ * & \tilde{\Xi}_1 & XB_1+ZB_2 & L^T & XA_d \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & 0 & -I & 0 \\ * & * & 0 & 0 & -Q \end{bmatrix} < 0 \tag{17}$$

where $\Omega_1 = RA + AR^T + R^{-1}QR^{-1}, \Omega_2 = A + R^{-1}A^TX + R^{-1}C^T(Z(I-\rho)D(\alpha))^T + \hat{Z}^T + R^{-1}Q$ *and*

$$\begin{cases} \hat{Z} = MR^{-1} \\ \tilde{Z} = NR^{-1} \end{cases}$$

Let

$$\begin{cases} SB_f = Z \\ SA_f W^T = \hat{Z} \\ C_f W^T = \tilde{Z} \end{cases} \tag{18}$$

Next, introduce the matrix $T_a = diag\left\{J^{-1}, I, I, I\right\}$, performing congruence transformation $T_a^T(\cdot)T_a$ on inequality (17). And using (8) and (18), we have

$$\begin{bmatrix} \tilde{A}^T P + P\tilde{A} + H^T Q H & P\tilde{B} & \tilde{C}^T & P\tilde{A}_d \\ * & -\gamma^2 I & 0 & 0 \\ * & 0 & -I & 0 \\ * & 0 & 0 & -Q \end{bmatrix} < 0$$

Finally, we gave the parameters of the desired filter. The filter parametersA_f, B_f and C_f can be designed as

$$\begin{cases} A_f = S^{-1}\hat{Z}W^{-T} \\ B_f = S^{-1}Z \\ C_f = \tilde{Z}W^{-T} \end{cases}$$

■

4 | ILLUSTRATIVE EXAMPLE

In the section, we give an illustrative example to show how to apply the method proposed in this paper. The corresponding system is follows:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -2 & -1.5 \\ 0.3 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} x(t-1) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} w(t) \\ y(t) = sat\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t)\right) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} w(t) \\ z(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(t) \end{cases}$$

where $sat(\cdot)$ is the saturation functions defined in precious section, and $w(t)$ is the external disturbance.

We carry out some simulation results with the following choices: For this example, by choosing $\alpha_{\min} = 0.1$ and using MATLAB LMI toolbox to solve (13), it is easy to obtain the solutions.

Let initial conditions be $x(0) = [0.4, -0.3]^{\mathrm{T}}$ and $\hat{x}(0) = [0,0]^{\mathrm{T}}$. To test the robustness of the filter, we add a disturbance $w(t) = 5\exp(-0.05t)\sin(0.02t)$. The simulation results of $z(t)$ and its estimator $\hat{z}(t)$.

Case 1: Normal model 1: The two sensors are normal, $\rho_1^1 = \rho_2^1 = 0$.

Q=$\begin{bmatrix} 1.4980 & -0.1718 \\ -0.1718 & 1.4752 \end{bmatrix}, R = \begin{bmatrix} 0.3833 & -0.1346 \\ -0.1346 & 0.2647 \end{bmatrix}, M = \begin{bmatrix} 1.6752 & 0.6947 \\ -0.8950 & 0.3677 \end{bmatrix}$,

$N = \begin{bmatrix} 1 & 1 \end{bmatrix}, Z = \begin{bmatrix} -0.6455 & -0.0581 \\ 0.0594 & -0.5999 \end{bmatrix}, X = \begin{bmatrix} 1.2041 & -0.4524 \\ -0.4524 & 0.9459 \end{bmatrix}$.

Shown in Figs.1-2 are the simulation results of state response and estimation signals of the designed filter under sensor failure model 1. From Figs.1-2, it can be concluded that when two sensors are normal and saturations occur, the reliable H_∞ filter obtained from this paper satisfies the requirements.

Case 2: Fault model 2: The first sensor is normal and the second sensor is loss of effectiveness. $\rho_1^2 = 0, 0 \leq \rho_2^2 = a$,

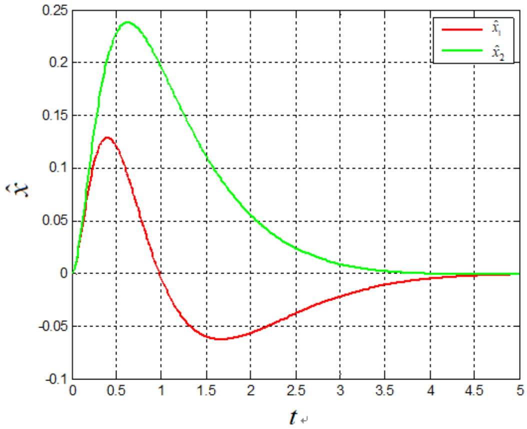

FIGURE 1 State responses $\hat{x}(t)$ of the design filter.

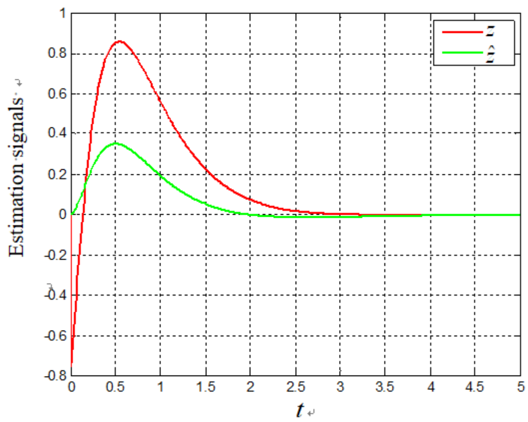

FIGURE 2 Estimation signals

$$a=0.7, Q=\begin{bmatrix}1.5518 & -0.2498\\ -0.2498 & 1.6440\end{bmatrix}, R=\begin{bmatrix}0.7107 & -0.2310\\ -0.2310 & 0.5184\end{bmatrix}, M=\begin{bmatrix}3.0558 & 1.1848\\ -0.80042 & 2.1176\end{bmatrix},$$

$$N=\begin{bmatrix}1.0261 & 1.0168\end{bmatrix}, Z=\begin{bmatrix}-0.7970 & -0.1043\\ 0.5885 & -1.9064\end{bmatrix}, X=\begin{bmatrix}1.6737 & -0.4461\\ -0.4461 & 1.6131\end{bmatrix}.$$

State responses and estimation signals of the designed filter under sensor failure model 2 are given in Figs.3-4. It can be seen that when one of sensors failures and saturations occur, the reliable H_∞ filter obtained from this paper can work well.

Case 3: Fault model 3: The two sensors are loss of effectiveness, $0 \le \rho_1^3 = b_1, 0 \le \rho_2^3 = b_2, b_1 = 0.6, b_2 = 0.8$.

$$Q=\begin{bmatrix}1.6059 & -0.1703\\ -0.1703 & 1.5473\end{bmatrix}, R=\begin{bmatrix}0.4174 & -0.1412\\ -0.1412 & 0.2854\end{bmatrix}, M=\begin{bmatrix}1.8142 & 0.7832\\ -0.9766 & 0.4309\end{bmatrix}, N=\begin{bmatrix}1 & 1\end{bmatrix},$$

$$Z=\begin{bmatrix}-1.2383 & 0.2429\\ 1.4752 & -2.1305\end{bmatrix}, X=\begin{bmatrix}1.3040 & -0.4653\\ -0.4653 & 1.0102\end{bmatrix}.$$

The simulation results of state responses and estimation signals of the designed filter under sensor failure model 3 are given in Figs.5-6. It can be seen that when two sensors failure and saturation occur, the reliable H_∞ filter obtained from this paper satisfies the requirements.

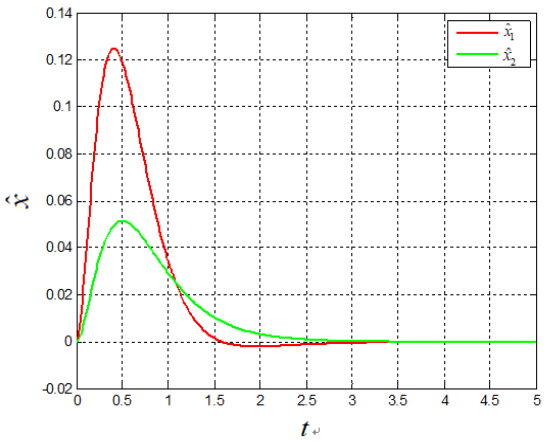

FIGURE 3 State responses $\hat{x}(t)$ of the design filter.

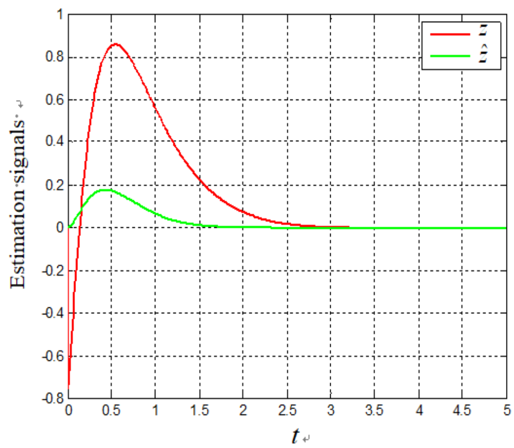

FIGURE 4 Estimation signals

5 | CONCLUSIONS

This paper provides results of the filtering problem for a class of continuous-time systems with sensor saturation and failure. Based on the Lyapunov-Krasorskii stability theorem and the linear matrix inequalities (LMI) techniques, the filter architecture is proposed, and the linear filter designed by the method can guarantee the performance. The effectiveness of the proposed approach is illustrated by numerical examples.

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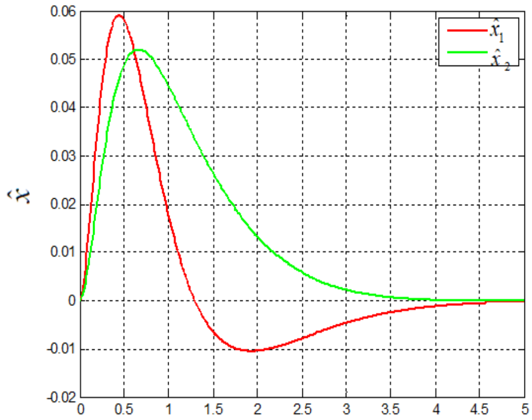

FIGURE 5 State responses $\hat{x}(t)$ of the design filter.

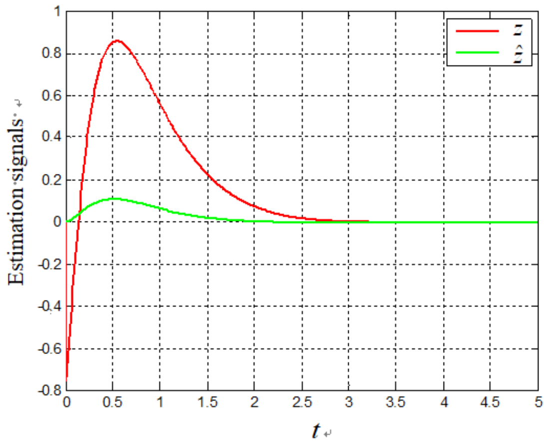

FIGURE 6 Estimation signals

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