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Topological Transformation and Neural Network Backstepping Control for Unknown Non-linear Systems

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Abstract

Aiming at second-order electromechanical systems with state coupling and nonlinear disturbances, this paper proposes a predefined-time backstepping control method that combines topological transformation and Radial Basis Function(RBF) neural networks. First, by constructing an invertible topological mapping, the original second-order system is transformed into an n th-order nonlinear state equation, achieving a normalized equivalent transformation of the system structure. Subsequently, a predefined-time control framework is introduced. Based on the backstepping design philosophy, error variables are defined through coordinate transformations, and RBF neural networks are employed to approximate the system's unknown nonlinear functions. At each step of the backstepping design, a Lyapunov function that includes both the error terms and the estimation errors of the neural network weights is constructed. Virtual control laws and adaptive laws are then designed, leading to the derivation of the actual control input. Stability analysis demonstrates that the proposed method guarantees the system states converge to a bounded region within a predefined time, and the convergence time is independent of the system's initial states. This method effectively addresses the predefined-time stabilization problem for unknown nonlinear systems, providing a high-precision and robust control solution for electromechanical systems with coupling and disturbances.

KEYWORDS

Predefined-time; Invertible topological mapping; Neural networks; Backstepping control; Nonlinear systems

1 | INTRODUCTION

System response speed is a core indicator for evaluating the tracking control performance of a system, and related research has become a focus of academic attention. Based on Lyapunov stability theory, Dagne et al. [1] conducted research on the asymptotic stability of nonlinear systems. However, this method has limitations: it can only ensure that the system states converge to the equilibrium point as time approaches infinity. To address this issue and improve system response speed, finite-time control

theory was proposed by scholars [2]. A prominent advantage of this theory is that it enables system states to converge within a finite time.

Subsequent studies on finite-time control theory were further deepened and expanded in [3][4]. It is worth noting that the convergence time of finite-time stability is still closely related to the initial states of the system. To overcome this drawback, Polyakov [5] innovatively proposed the concept of fixed-time control, which effectively eliminates the dependence of convergence time on the system's initial states. On this basis, [6] combined fixed-time control theory to construct multiple practically valuable fixed-time stability criteria. Nevertheless, although fixed-time control decouples the convergence time from the initial states, its convergence time is determined by multiple controller design parameters, which also poses significant challenges to the direct solution of the convergence time. Obviously, exploring how to achieve stable convergence of the system within a predefined time frame is a research topic with both profound theoretical significance and practical application value. Its findings can not only enrich the theoretical system of nonlinear system control, but also provide feasible technical guidance for engineering practice. To further improve the system response speed, [7] took the lead in proposing the innovative concept of "predefined-time control", which can ensure the stable operation of the system within the time range specified by the user. On the basis of the theoretical framework of predefined-time stability, [8][9] respectively carried out systematic and in-depth research on the predefined-time control problems of first-order and second-order systems. [10] innovatively designed a predefined-time leader-follower consensus protocol, which effectively realized the fast convergence of tracking errors within the predefined time. In addition, the research teams of [11][12] successfully applied predefined-time control technology to robotic manipulator systems, providing a new practical idea for the engineering implementation of this technology.

In addition, the application of predefined-time control methods has emerged as a cutting-edge trend in the field of neural network control design, as this method allows for the pre-defined specification of key performance indicators such as tracking errors and convergence rates [13]. [14] further extended the application scope of predefined-time control methods to the output feedback control problem of a class of switched neural networks, forming a more generalizable control scheme. Subsequently, some scholars proposed an adaptive dynamic surface control method based on predefined-time control, which successfully solved the control challenge of state-constrained neural network systems [15]. This paper proposes a predefined-time practical stability criterion for nonlinear systems with uncertainties. Based on this criterion, a complete predefined-time adaptive control scheme is designed for the n -th order feedback parameterized systems subject to external disturbances. A comparison with the traditional predefined-time control methods proposed in [7][8][12] reveals that these methods are not only limited to the control scenarios of second-order nonlinear systems, but also fail to incorporate the influencing factors of external disturbances and parameter uncertainties, thus exhibiting obvious limitations in terms of anti-disturbance performance and parameter robustness. The core characteristics of the proposed method mainly include two aspects: first, the convergence time of the closed-loop system can be predefined by the user, and this time is independent of neither the initial states of the system nor the controller design parameters; second, this method can strictly ensure that the system satisfies the pre-specified performance index requirements and state constraint conditions during its operation. Compared with the research schemes proposed in [16][17], the control method constructed in this paper not only effectively avoids the cumbersomeness in the calculation process and stability analysis, but also significantly reduces the conservative assumption requirements for the target tracking signal.

1.1 | System Model

Considering a class of original second-order electromechanical systems with state coupling and nonlinear disturbances

$$\begin{aligned}\dot{z}_1 &= z_2 + z_1 z_2 + d_1(t) \\ \dot{z}_2 &= u + z_1^2 + d_2(t) \\ y &= z_1\end{aligned}\tag{1}$$

$\dot{z}_1$ and $\dot{z}_2$ denote the derivatives of the system's two state variables, $z_1 z_2$ reflects the state coupling characteristic, z_1^2 is a nonlinear term, $d_1(t)$ and $d_2(t)$ stand for external nonlinear disturbances, u is the control input, and $y = z_1$ is the system output. To achieve normalized transformation of the system structure for simplifying control design, an invertible topological mapping is

constructed to perform an equivalent transformation on the original system.

$$\begin{aligned} x_1 &= z_1, \\ x_2 &= z_2 + z_1 z_2. \end{aligned} \quad (2)$$

By differentiating Equation (2) and substituting System (1) into the result, we obtain an expression that can be transformed into an n th-order nonlinear state equation this form is more conducive to subsequent control analysis.

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(x) + d_i(t), & 1 \leq i \leq n-1, \\ \dot{x}_n = u(t) + f_n(x) + d_n(t), \\ y = x_1 \end{cases} \quad (3)$$

in which $x_i = [x_1(t), \dots, x_i(t)] \in \mathbb{R}^n, i = 1, \dots, n$ expresses the system state variables, and $y \in \mathbb{R}$ indicates the system output; $f_i(\bar{x}), i = 1, \dots, n$ denotes the unknown nonlinear function of the system, which describes the internal nonlinear dynamic characteristics of the system; $d_i(t)$ represents unknown bounded external interference with $x_{i+1} + f_i(x) \neq 0$. $u(t)$ is the control input, which is used to adjust the operating state of the system.

1.2 | Mathematical Preparation

Definition 1. Consider the following nonlinear system $\dot{x}(t) = f(x(t)), x(0) = x_0$, the origin of system is said to be predefined-time stable, if for any given predefined constant $T_d > 0$, and any initial condition $x(0) \in \Omega$, the solution of the system satisfies $\|x(t, x_0)\| \in \Omega$, for all $t \geq 0$, and the solution converges to Ω exactly within the predefined time T_d , where T_d is a user-assignable constant. If there exists some design constants $0 < \eta < 1, T_d > 0, 0 \leq \Delta < \infty$, π represents the circumference-diameter ratio of a circle.

$$\dot{V} \leq -\frac{3\pi}{2\eta T_d} \left(V^{1+\frac{\eta}{2}} + V^{1-\frac{\eta}{2}} \right) + \Delta \quad (4)$$

Subsequently, under the predefined time T_d , the origin is a practical predefined-time stable, and for all $t \geq T_d$, the solution $x(x_0, t)$ will remain within the bounded set $\Omega = \left\{ x : V(x) \leq \frac{\eta T_d \Delta}{\pi} \right\}$. Lemma 1:[20] During the design of the system controller, RBF will be utilized to address the uncertainty terms in the uncertain nonlinear system. Specifically as follows:

Supposing $h(Z)$ is a continuous function defined on $\Omega_Z \subset \mathbb{R}^n$, for $\forall \varepsilon > 0, |\delta(Z)| \leq \varepsilon$, being the neural networks node number, there exists a RBF neural networks satisfying:

$$h(Z) = W^* \Phi(Z) + \delta(Z) \quad (5)$$

Here, $W = [\omega_1, \omega_2, \dots, \omega_l] \in \mathbb{R}^l$ refers to the ideal constant weight vector; $\delta(Z)$ represents the approximation error, which satisfies $|\delta(Z)| \leq \varepsilon, l > 0$ denotes the number of nodes in the RBF neural network. $\Phi(Z) = [\Phi_1(Z), \Phi_2(Z), \dots, \Phi_l(Z)]^T$ is the basis function vector, where $\Phi_i(Z) = \exp\left(\frac{-\|Z - \mu_i\|^2}{\eta^2}\right) - \eta$ and μ_i correspond to the width of the Gaussian functions and the center of the receptive field, respectively. Next, we introduce the following lemma. Let $\Phi(\cdot) = [\Phi_1(\cdot), \dots, \Phi_l(\cdot)]^T$ serve as the basis function vector of an RBF neural network. Let $Z = [z_1, \dots, z_n]^T$ and $Z_m = [z_1, \dots, z_m]^T$ denote two input vectors, respectively. Then, for any positive integer $m \leq n$, the following inequality holds: $\|\Phi(Z)\|^2 \leq \|\Phi(Z_m)\|^2$.

2 | CONTROLLER DESIGN AND STABILITY ANALYSIS

2.1 | predefined time Controller Design

Following the backstepping approach, a coordinate transformation is first introduced to support the subsequent design process.

$$\begin{cases} z_1 = x_1 - y_d, \\ z_i = x_i - \alpha_{i-1}, \end{cases} \quad (6)$$

in which y_d expresses the desired reference trajectory signal, α_{i-1} is the intermediate control law to be designed. In what follows, we will elaborate on the specific steps of the design process. Meanwhile, neural networks (NNs) will be utilized as an approximator to handle the unknown terms present in the system.

Step 1: We start with the first equation of System (3). By applying Equations (4), we can derive the following result:

$$\begin{aligned}\dot{z}_1 &= x_2 + f_1 + d_1 - \dot{y}_d \\ &= z_2 + \alpha_1 + f_1 + d_1 - \dot{y}_d\end{aligned}\quad (7)$$

Select a Lyapunov function V_1 as

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2\lambda_1}\tilde{\theta}_1^2 \quad (8)$$

Taking the time derivative of V_1 , one can get

$$\begin{aligned}\dot{V}_1 &= z_1\dot{z}_1 - \frac{1}{\lambda_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \\ &= z_1(z_2 + \alpha_1 + f_1 + d_1 - \dot{y}_d) - \frac{1}{\lambda_1}\tilde{\theta}_1\dot{\hat{\theta}}_1\end{aligned}\quad (9)$$

where $h_1(Z_1) = f_1 - \dot{y}_d + d_1$. It should be noted that $h_1(Z_1)$ cannot be used for feedback, as it contains the unknown nonlinear function $f_1(x)$. To address this issue, neural networks (NNs) can serve as an approximator for this unknown component: they can approximate $h_1(Z_1)$ to any desired precision $\varepsilon_1 > 0$. As indicated by Equation (5), we know that

$$h_1(Z_1) = W_1^*\Phi_1(Z_1) + \delta_1(Z_1), \quad |\delta_1(Z_1)| \leq \varepsilon_1 \quad (10)$$

where $Z_1 = [x_1, y_d, \dot{y}_d]^T$, and $\delta_1(Z_1)$ is the estimate error, with arbitrary constants $\varepsilon_1 > 0$. Applying Young's inequality, it produces

$$\begin{aligned}z_1 h_1 &= z_1 W_1^* \Phi_1(Z_1) + z_1 \varepsilon_1 \\ &\leq \frac{1}{2a_1^2} z_1^2 \theta_1 \Phi_1^T(Z_1) \Phi_1(Z_1) + \frac{a_1^2}{2} + \frac{1}{2} z_1^2 + \frac{1}{2} \varepsilon_1^2\end{aligned}\quad (11)$$

where $\theta_1 = \|W_1^*\|^2$ is an unknown constant to be estimated, $a_1 > 0$ is a known constant. By substituting Equation (11) into Equation (9), we obtain

$$\dot{V}_1 \leq z_1(z_2 + \alpha_1) + \frac{1}{2a_1^2} z_1^2 \theta_1 \Phi_1^T \Phi_1 + \frac{a_1^2}{2} + \frac{1}{2} z_1^2 + \frac{1}{2} \varepsilon_1^2 - \frac{1}{\lambda_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1 \quad (12)$$

To achieve the stabilization of this subsystem within a predefined time, we proceed to design the predefined-time virtual controller α_1 as follows:

$$\alpha_1 = -\frac{1}{2a_1^2} z_1 \hat{\theta}_1 \Phi_1^T \Phi_1 - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_1^{1+\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_1^{1-\eta} - \frac{1}{2} z_1 \quad (13)$$

and the adaptation law as

$$\dot{\hat{\theta}}_1 = \frac{\lambda_1}{2a_1^2} z_1^2 \Phi_1^T \Phi_1 - \lambda_1 \sigma_1 \hat{\theta}_1 - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} \tilde{\theta}_1^{1+\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} \tilde{\theta}_1^{1-\eta} \quad (14)$$

substituting (13)-(14) into (12), one can get

$$\begin{aligned}\dot{V}_1 &\leq -\frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_1^{2+\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_1^{2-\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2\lambda_1} \tilde{\theta}_1^{2+\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2\lambda_1} \tilde{\theta}_1^{2-\eta} \\ &\quad + \frac{a_1^2}{2} + \frac{1}{2} \varepsilon_1^2 + \sigma_1 \tilde{\theta}_1 \hat{\theta}_1 + z_1 z_2\end{aligned}\quad (15)$$

Step i: With the aid of (6) and (3), similar to step 1 as

$$\begin{aligned}\dot{z}_i &= x_{i+1} + f_i + d_i - \dot{\alpha}_{i-1} \\ &= z_{i+1} + \alpha_i + f_i + d_i - \dot{\alpha}_{i-1}\end{aligned}\quad (16)$$

Select a Lyapunov function V_i as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2\lambda_i}\tilde{\theta}_i^2 \quad (17)$$

Then, similarly to step 1, we get

$$\begin{aligned} \dot{V}_i &= \dot{V}_{i-1} + z_i \dot{z}_i - \frac{1}{\lambda_i} \tilde{\theta}_i \dot{\hat{\theta}}_i \\ &= - \sum_{j=1}^{i-1} \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_j^{2+\eta} - \sum_{j=1}^{i-1} \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_j^{2-\eta} - \sum_{j=1}^{i-1} \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2\lambda_j} \tilde{\theta}_j^{2+\eta} - \sum_{j=1}^{i-1} \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2\lambda_j} \tilde{\theta}_j^{2-\eta} \\ &\quad + \sum_{j=1}^{i-1} \left(\frac{a_j^2}{2} + \frac{1}{2} \varepsilon_j^2 \right) + \sum_{j=1}^{i-1} \sigma_j \tilde{\theta}_j \hat{\theta}_j + z_i (z_{i+1} + \alpha_i + f_i + d_i - \dot{\alpha}_{i-1}) - \frac{1}{\lambda_i} \tilde{\theta}_i \dot{\hat{\theta}}_i \end{aligned} \quad (18)$$

in which $h_i(Z_i) = \alpha_{i-1} + f_i + d_i$. Similarly, there exists an NNs, for $\forall \varepsilon_i > 0$, $h_i(Z_i) = W_i^* \Phi_i(Z_i) + \delta_i(Z_i)$ with $|\delta_i(Z_i)| \leq \varepsilon_i$. Combining the Young's inequality, we have

$$\begin{aligned} z_i h_i &= z_i W_i^* \Phi_i(Z_i) + z_i \varepsilon_i \\ &\leq \frac{1}{2a_i^2} z_i^2 \theta_i^T \Phi_i^T \Phi_i + \frac{a_i^2}{2} + \frac{1}{2} z_i^2 + \frac{1}{2} \varepsilon_i^2 \end{aligned} \quad (19)$$

where $\theta_i = \|W_i^*\|^2$, $a_i > 0$. Similarly to step 1, for the purpose of stabilizing this subsystem within a pre-set time frame, we design the corresponding predefined-time virtual controller in the following manner:

$$\alpha_i = -\frac{1}{2a_i^2} z_i \hat{\theta}_i \Phi_i^T \Phi_i - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_i^{1+\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_i^{1-\eta} - \frac{1}{2} z_i - z_{i-1} \quad (20)$$

and the adaptation law as

$$\dot{\hat{\theta}}_i = \frac{\lambda_i}{2a_i^2} z_i^2 \Phi_i^T \Phi_i - \lambda_i \sigma_i \hat{\theta}_i - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} \tilde{\theta}_i^{1+\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} \tilde{\theta}_i^{1-\eta} \quad (21)$$

With the help of (19)-(21), we obtain

$$\begin{aligned} \dot{V}_i &\leq - \sum_{j=1}^i \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_j^{2+\eta} - \sum_{j=1}^i \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_j^{2-\eta} - \sum_{j=1}^i \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2\lambda_j} \tilde{\theta}_j^{2+\eta} \\ &\quad - \sum_{j=1}^i \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2\lambda_j} \tilde{\theta}_j^{2-\eta} + \sum_{j=1}^i \left(\frac{a_j^2}{2} + \frac{1}{2} \varepsilon_j^2 \right) + \sum_{j=1}^i \sigma_j \tilde{\theta}_j \hat{\theta}_j + z_i z_{i+1} \end{aligned} \quad (22)$$

Step n: With the aid of (3) and (6), one can get the derivative of z_n as

$$\dot{z}_n = \dot{x}_n - \dot{\alpha}_{n-1} = u + f_n + d_n - \dot{\alpha}_{n-1} \quad (23)$$

Select a Lyapunov function V_n as

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2\lambda_n}\tilde{\theta}_n^2 \quad (24)$$

Then, we obtain the following by (23) and (24):

$$\begin{aligned} \dot{V}_n &= \dot{V}_{n-1} + z_n \dot{z}_n - \frac{1}{\lambda_n} \tilde{\theta}_n \dot{\hat{\theta}}_n \\ &= - \sum_{j=1}^{n-1} \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_j^{2+\eta} - \sum_{j=1}^{n-1} \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_j^{2-\eta} - \sum_{j=1}^{n-1} \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2\lambda_j} \tilde{\theta}_j^{2+\eta} - \sum_{j=1}^{n-1} \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2\lambda_j} \tilde{\theta}_j^{2-\eta} \\ &\quad + \sum_{j=1}^{n-1} \left(\frac{a_j^2}{2} + \frac{1}{2} \varepsilon_j^2 \right) + \sum_{j=1}^{n-1} \sigma_j \tilde{\theta}_j \hat{\theta}_j + z_n (u + f_n + d_n - \dot{\alpha}_{n-1}) - \frac{1}{\lambda_n} \tilde{\theta}_n \dot{\hat{\theta}}_n \end{aligned} \quad (25)$$

where $h_n(Z_n) = f_n + d_n - \dot{\alpha}_{n-1}$. Similar to (19), we can get

$$\begin{aligned} z_n h_n &= z_n W_n^* \Phi_n(Z_n) + z_n \varepsilon_n \\ &\leq \frac{1}{2a_n^2} z_n^2 \theta_n \Phi_n^T \Phi_n + \frac{a_n^2}{2} + \frac{1}{2} z_n^2 + \frac{1}{2} \varepsilon_n^2 \end{aligned} \quad (26)$$

where $\theta_n = \|W_n^*\|^2$, $a_n > 0$. Now, we define the actual control law as

$$u = -\frac{1}{2a_n^2} z_n \hat{\theta}_n \Phi_n^T \Phi_n - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_n^{1+\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_n^{1-\eta} - \frac{1}{2} z_n - z_{n-1} \quad (27)$$

and the adaptation law as

$$\dot{\hat{\theta}}_n = \frac{\lambda_n}{2a_n^2} z_n^2 \Phi_n^T \Phi_n - \lambda_n \sigma_n \hat{\theta}_n - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} \tilde{\theta}_n^{1+\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} \tilde{\theta}_n^{1-\eta} \quad (28)$$

With the help of (26)-(28), we obtain

$$\begin{aligned} \dot{V}_n &\leq -\sum_{j=1}^n \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_j^{2+\eta} - \sum_{j=1}^n \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_j^{2-\eta} - \sum_{j=1}^n \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2\lambda_j} \tilde{\theta}_j^{2+\eta} \\ &\quad - \sum_{j=1}^n \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2\lambda_j} \tilde{\theta}_j^{2-\eta} + \sum_{j=1}^n \left(\frac{a_j^2}{2} + \frac{1}{2} \varepsilon_j^2 \right) + \sum_{j=1}^n \sigma_j \tilde{\theta}_j \hat{\theta}_j \end{aligned} \quad (29)$$

2.2 | Stability Analysis

Theorem 1: The closed-loop system, which is composed of the non-strict feedback system (3), virtual control laws (13) and (20), the actual controller (27), and adaptive law (14), (21) and (28), is taken into account herein. Subsequently, the closed-loop stability is ensured, and the tracking error performance is additionally established in a predefined-time interval.

Proof: By recalling the definition of $\tilde{\theta}_j$, for any $\rho_j > \frac{1}{2}$, the inequality below holds:

$$\sigma_j \tilde{\theta}_j \hat{\theta}_j = -\sigma_j \tilde{\theta}_j (\tilde{\theta}_j - \theta_j) \leq -\frac{\sigma_j}{\gamma_j} \tilde{\theta}_j^2 + \frac{\rho_j \sigma_j}{2} \theta_j^2 \quad (30)$$

Substituting (30) into (29) yields

$$\dot{V}_n \leq -\frac{3\pi}{2\eta T_d} \left(V_n^{1+\frac{\eta}{2}} + V_n^{1-\frac{\eta}{2}} \right) + \Delta \quad (31)$$

where $\Delta = \sum_{j=1}^n \left(\frac{a_j^2}{2} + \frac{1}{2} \varepsilon_j^2 + \frac{\rho_j \sigma_j}{2} \theta_j^2 \right)$.

Remark 3.1 It is worth noting that predefined-time adaptive neural control is different from the finite-time adaptive fuzzy neural control in [19-21]. The predefined settling time of predefined-time control can be bounded even when the initial system state approaches infinity. Whereas the settling time of finite-time control grows unboundedly when the initial system state tends to infinity.

3 | SIMULATION

In this section, we apply the presented fixed-time adaptive control strategy to an electromechanical system example for validation. Following the setup in [37], the dynamics of the electromechanical system are formulated as:

$$\begin{cases} \dot{\xi}_1 = \xi_2, \\ \dot{\xi}_2 = -\frac{G}{A} \sin(\xi_1) - \frac{C}{A} \xi_2 + \frac{1}{A} \xi_3 + \frac{C}{E} \xi_2^2 \xi_3^3, \\ \dot{\xi}_3 = \frac{K}{E} \xi_2 - \frac{F}{E} \xi_3 + \frac{1}{E} v + \frac{F}{E} \xi_2^2 \sin(\xi_3), \\ \varphi = \xi_1 \end{cases} \quad (32)$$

where $\frac{C}{E} \xi_2^2 \xi_3^3$ and $\frac{F}{E} \xi_2^2 \sin(\xi_3)$ stand for the model uncertainties. The definitions of I, V, A, C, E, F, G and K can be found in

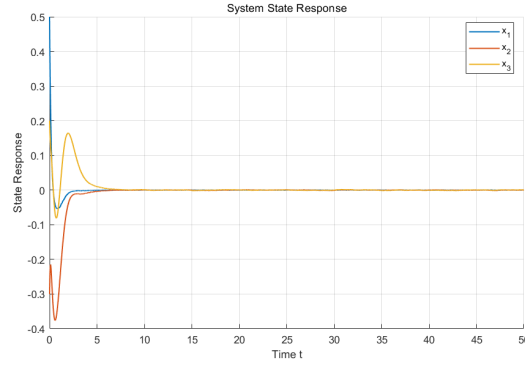


FIGURE 1 System state trajectories x_1, x_2, x_3

the recent work [21]. The system parameters are selected as $A = 1, C = 1, E = 0.05, F = 0.5, G = 10$ and $K = 10$. The control objective is to design a fixed-time controller such that the system output φ tracks the reference signal $y_d = 0.5 \sin(t) + \sin(0.5t)$. To achieve this goal, we design

$$\alpha_1 = -\frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_1^{1+\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_1^{1-\eta} - \frac{1}{2} z_1 + \dot{y}_d \quad (33)$$

$$\alpha_2 = -\frac{1}{2a_2^2} z_2 \hat{\theta}_2 \Phi_2^T \Phi_2 - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_2^{1+\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_2^{1-\eta} - \frac{1}{2} z_2 - z_1 \quad (34)$$

$$u = -\frac{1}{2a_3^2} z_3 \hat{\theta}_3 \Phi_3^T \Phi_3 - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_3^{1+\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} z_3^{1-\eta} - \frac{1}{2} z_3 - z_2 \quad (35)$$

$$\dot{\hat{\theta}}_i = \frac{\lambda_i}{2a_i^2} z_i^2 \Phi_i^T \Phi_i - \lambda_i \sigma_i \hat{\theta}_i - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} \hat{\theta}_i^{1+\eta} - \frac{3\pi}{2\eta T_d} \cdot \frac{1}{2} \hat{\theta}_i^{1-\eta} \quad (36)$$

The design parameter η is chosen as $\eta = 0.07$, which ensures $1 + \frac{\eta}{2} > 1$ and $0 < 1 - \frac{\eta}{2} < 1$ to satisfy the fixed-time convergence condition.

To demonstrate the effectiveness of our proposed scheme, we compare the convergence results between the fixed-time control scheme presented in [18] and the typical finite-time-based control scheme corresponding to the case where $q=97/101$ in [21]. Both control schemes are applied to the same electromechanical system modeled by (32), utilizing identical neural networks (NNs) and design parameters to ensure a fair comparison. The design parameters are chosen as $a_1 = a_2 = a_3 = 1, \lambda_1 = \lambda_2 = \lambda_3 = 10, \sigma_1 = \sigma_2 = \sigma_3 = 0.05, T_d = 1$. The initial conditions of system states are $[x_1(0), x_2(0), x_3(0)]^T = [0.15, 0.15, 0.15]$ and $\hat{\theta}(0) = [0.3, 0.3]^T$.

The comparative simulation results are presented in FIGS. 1–4. As can be seen from the results, the proposed predefined-time adaptive control scheme achieves significantly superior tracking performance compared with the finite-time control scheme in [21]. Specifically, FIG. 1 shows the trajectories of the system states x_1, x_2, x_3 ; FIG. 2 presents the tracking error responses z_1, z_2, z_3 ; FIG. 3 illustrates the boundedness of the neural network weight estimates; and FIG. 4 depicts the system output tracking curve.

4 | CONCLUSIONS

In this paper, a predefined-time adaptive control scheme is proposed for a class of non-strict-feedback nonlinear systems with state coupling and nonlinear disturbances. A novel predefined-time stability criterion (SPFTS) is established to guarantee the closed-loop stability of the system. Based on the established SPFTS criterion, it is rigorously proven that both the tracking performance and the closed-loop stability can be preserved within a predefined time via the presented approach, and the convergence

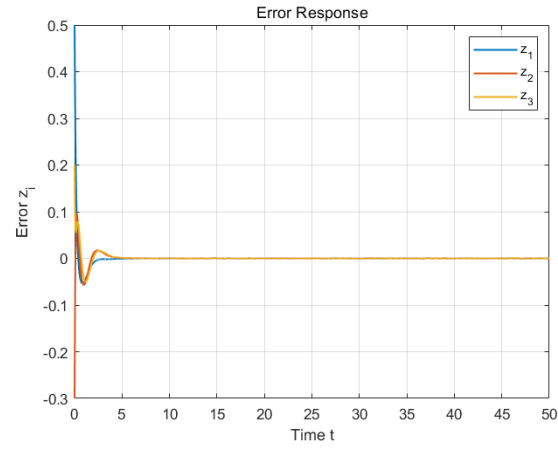


FIGURE 2 Tracking error responses z_1, z_2, z_3 .

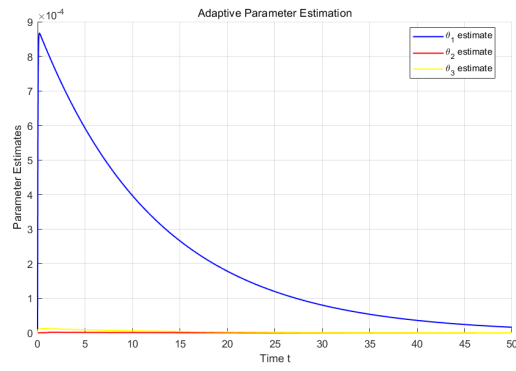


FIGURE 3 Boundedness of neural network weight estimates

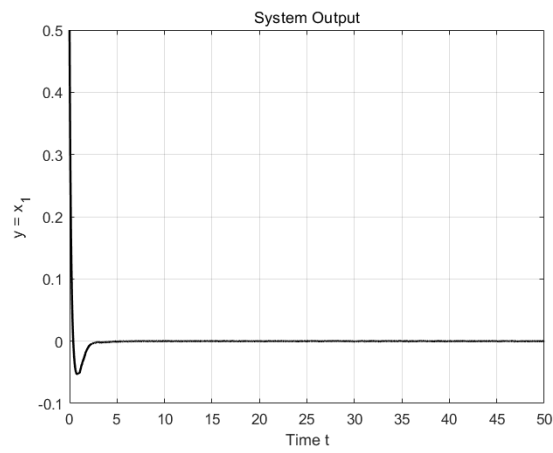


FIGURE 4 system output tracking curve

time is independent of the initial system states. Compared with the conventional finite-time control methods, the convergence time of the proposed predefined-time control scheme can be explicitly predefined by design parameters, which enhances the flexibility and predictability of the system performance. Finally, the effectiveness of the proposed technique is demonstrated by simulation results.

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