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Research on the Application of Adaptive Second Order Sliding Mode Control Based on Lyapunov Method

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Abstract

For nonlinear systems with uncertainty, traditional second-order sliding mode control often faces problems of overestimation of control gain and increased chattering. This article uses a Lyapunov method for adaptive second-order sliding mode control, which only requires bounded uncertainty and does not require the assumption of bounded derivatives. By designing a dynamic gain adjustment mechanism, the control gain can adaptively change with uncertainty, effectively avoiding gain overestimation. Based on the Lyapunov stability theory, the finite time stability of the closed-loop system has been rigorously proven, ensuring that the system state converges to the true sliding mode domain within a finite time. The simulation results have verified the superiority of the proposed control method in reducing chattering and improving tracking accuracy.

KEYWORDS

Adaptive control, Second-order sliding mode control, Lyapunov method

1 | INTRODUCTION

1.1 | Research Background

In the field of modern control engineering, problems such as nonlinear systems, parameter perturbations, and external disturbances are commonly present, which impose strict requirements on the robustness and convergence speed of controllers. From high-precision trajectory tracking of industrial robotic arms, attitude stability control of spacecraft, to energy regulation of new energy equipment and dynamic response of intelligent robots, various complex controlled objects need to achieve high-performance control goals under uncertain working conditions. Although traditional linear control methods have a simple structure and are easy to implement, they are difficult to balance control accuracy and robustness in strong nonlinearity, time-varying parameters, and unknown disturbance scenarios; Classic sliding mode control has gained widespread attention in uncertain system control due to its variable structure characteristics, invariance to matching disturbances, and simple algorithm design with fast response. However, its inherent chattering problem and prior knowledge dependence on disturbance boundaries greatly limit its application in engineering scenarios that require high precision and smoothness.

In order to solve the chattering defect of traditional sliding mode control, high-order sliding mode control has emerged. Its core idea is to transfer the discontinuity of the control law to the high-order derivative of the system state, while retaining

the strong robustness of sliding mode control, and achieving the continuity of the control input, thereby effectively reducing chattering. As the most practical branch of high-order sliding mode control, second-order sliding mode control constructs a second-order sliding surface to enable the system state and its first derivative to converge to the sliding surface simultaneously. This simplifies the algorithm complexity of high-order sliding mode and enables finite time convergence through classical algorithms such as hyper helix, becoming an important bridge connecting theoretical research and engineering applications. However, the control performance of second-order sliding mode control still heavily relies on prior information of disturbance boundaries and system parameters. When faced with unknown disturbances, time-varying parameters, and unmatched uncertainties, fixed gain second-order sliding mode controllers are prone to additional chattering problems caused by insufficient robustness or over estimation of gain.

Adaptive control relies on its ability to identify system uncertainties online and self adjust parameters. By designing adaptive laws to update controller parameters in real time, the system can adaptively match changes in operating conditions without the need for precise mathematical models or disturbance boundary information, achieving dynamic optimization of control performance. In 1957, Qian Xuesen first formally proposed the term "adaptive" in his book, laying the conceptual foundation for adaptive control as an independent research direction. In 1993, Levant [2] conducted an in-depth analysis of the sliding order and sliding accuracy of second-order sliding mode control, providing key support for the construction of the theoretical system of second-order sliding mode control and marking an important milestone in the formation of the theory of second-order sliding mode control.

The adaptive second-order sliding mode control formed by combining adaptive mechanisms with second-order sliding mode control inherits the advantages of finite time convergence, strong robustness, and continuous control of second-order sliding mode control, and breaks through the dependence on prior information through dynamic adjustment of adaptive laws, effectively suppressing chattering and improving the system's adaptability to complex uncertainties. Therefore, conducting theoretical research and engineering application exploration on adaptive second-order sliding mode control has important theoretical value and engineering guidance significance for promoting the development of high-performance control technology for complex uncertain systems.

In 1993, Sira Ramirez et al. [3] combined adaptive backstepping with sliding mode control for linearized systems, achieving adaptive sliding mode control. In 1997, Rios Bolívar et al. [4] designed a dynamic adaptive variable structure controller for a class of minimum phase linearized nonlinear systems, achieving robust control of nonlinear systems with uncertainty and unknown external disturbances. In the same year, Wheeler et al. [5] proposed a new parameter adaptive estimation method that ensured the rationality of variable structure control gain. In 2010, Y. Shtessel and J. Moreno et al. [6] presented a conference paper for the first time combining adaptive mechanisms with second-order sliding mode hyper spiral algorithms. They designed adaptive laws using the Lyapunov method and solved the problem of hyper spiral algorithms relying on perturbation boundary prior knowledge, providing a core theoretical framework for subsequent research. In 2012, Y. Shtessel et al. [7] optimized the design logic of adaptive gain to avoid overestimation of control gain. They also verified the practicality of the algorithm through experiments with electric and pneumatic actuators, promoting the engineering implementation of the combined technology of the two. In 2017, Ran Dechao et al. [8] designed an adaptive second-order terminal sliding mode algorithm for spacecraft attitude maneuvering problems, which improved attitude control accuracy through finite time convergence characteristics. In 2018, Amini et al. [9] proposed an adaptive second-order discrete sliding mode control method suitable for uncertain nonlinear systems with multiple inputs and outputs. Based on Lyapunov stability theory and invariance principle, derive adaptive laws to eliminate system uncertainty. In 2019, Zhang Qian et al. [10] applied an adaptive second-order sliding mode observer to sensorless control of permanent magnet synchronous motors, and improved the robustness of motor operation by combining parameter identification technology. In 2025, Samari et al. [11] proposed a recursive data-driven adaptive suboptimal second-order sliding mode control method for single input nonlinear perturbation systems with noisy data. By decomposing the system dynamics characteristics, a virtual controller and data-driven sliding mode variables were designed to ensure semi global asymptotic stability of the system origin in the presence of disturbances, providing a new approach for adaptive second-order sliding mode control in noisy environments.

1.2 | Article arrangement

This article focuses on the research of adaptive second-order sliding mode control, and the specific arrangement is as follows: The first chapter is an introduction, which elaborates on the engineering background of nonlinear system control, sorts out the research status of adaptive second-order sliding mode control, and clarifies the research significance and core objectives of this paper.

Chapter 2 is about sliding mode control design, taking a single joint hydraulic robotic arm as the controlled object, establishing a system model with nonlinearity, parameter uncertainty, and external disturbances, and designing the core structure of an adaptive second-order sliding mode controller, including sliding mode surface construction, adaptive law parameter selection, and control law expression derivation, laying the foundation for subsequent theoretical analysis.

Chapter 3 is a theoretical proof based on Lyapunov stability theory. By constructing appropriate Lyapunov functions, the stability conditions of the system are derived step by step, and the finite time convergence and sliding mode domain reachability of the closed-loop system are strictly proved, verifying the theoretical effectiveness of the proposed control method.

Chapter 4 is a simulation experiment that sets specific parameters and external disturbance conditions for the robotic arm system, designs joint angle tracking control tasks, and verifies the superiority of the method.

Chapter 5 is a summary, summarizing this article and objectively analyzing the advantages and limitations of the proposed control method. Combined with the research trend of adaptive second-order sliding mode control, future research directions that can be further expanded are proposed.

2 | SLIDING MODE CONTROL

2.1 | Sliding mode control

Taking a single joint hydraulic robotic arm as the control object, considering system nonlinearity, parameter uncertainty, and external load disturbances, the model expression is as follows.

$$J\ddot{\theta} + P\dot{\theta} + K \sin \theta = T_L + u \quad (1)$$

among them, $\theta, \dot{\theta}, \ddot{\theta}$ are the joint angle, angular velocity, and angular acceleration of the robotic arm; J is the moment of inertia; P is the viscous damping coefficient; K is the spring stiffness coefficient; T_L is a time-varying load disturbance; u is the output torque for the controller.

2.2 | System Design

Design the controller as

$$u = -\beta_2(t) \times \text{sign}(m) \quad (2)$$

among them, $\beta_2(t)$ is the adaptive law, designed as

$$\beta_2(t) = \begin{cases} \frac{\beta_1}{2} \left| \beta_2(t) - \beta_2^{\max \frac{1}{2}} \right|, & \text{if } W_1(s, s) > a \text{ and } \beta_2^{\min} \leq \beta_2(t) \leq \beta_2^{\max} \\ -\frac{\beta_1}{2} \left| \beta_2(t) - \beta_2^{\max \frac{1}{2}} \right| - \beta_3, & \text{if } W_1(s, s) \leq a \text{ and } \beta_2^{\min} \leq \beta_2(t) \leq \beta_2^{\max} \\ z, & \text{if } \beta_2(t) < \beta_2^{\min} \\ 0, & \text{if } \beta_2(t) < \beta_2^{\max} \end{cases} \quad (3)$$

among them, $\beta_1, a, z > 0$, β_3 is a very small positive number.

There are initial values and boundaries for β_2 , $0 < \beta_2^{\min} \leq \beta_2(0) \leq \beta_2^{\max}$ and $\beta_2^{\max} \geq \frac{\frac{27}{\beta_1^3} + 2^{\frac{3}{2}} \beta_1^2 + 3\beta_1^{\frac{11}{3}}}{C_m}$, $W_1(s, s)$ meets the conditions $W_1(s, s) = \frac{2}{5}|s|^{\frac{5}{2}} + \frac{1}{20}|s + \beta_1[s]^{\frac{1}{2}}|^5$.

Sliding mode design

$$\ddot{s} = A(t, x) + B(t, x)u \quad (4)$$

among them, $|A(t, x)| \leq A_0$, $0 < C_m \leq B(t, x) \leq C_M$, A_0, C_m, C_M all are positive numbers.

The sliding surface range is

$$\Omega = \left\{ (s, \dot{s}) : |s| \leq \left(\frac{5}{2} \left(\frac{\Delta}{c} \right)^{\frac{1}{\alpha}} \right)^{\frac{2}{5}}, |\dot{s}| \leq \left(\frac{1}{4} \left(\frac{\Delta}{c} \right)^{\frac{1}{\alpha}} + \frac{5}{2} \beta_1^5 \left(\frac{\Delta}{c} \right)^{\frac{1}{\alpha}} \right)^{\frac{1}{5}} \right\} \quad (5)$$

among them, a, c are parameters that satisfy the Lyapunov function V

$$V \leq -cV^\alpha + \Delta \quad (6)$$

3 | THEORETICAL PROOF

For variables $x, c \in \mathbb{R}$ the following expression exists

$$\lfloor x \rfloor^c = \text{sign}(x)|x|^c \quad (7)$$

Design Lyapunov function V_1

$$V_1(s) = \frac{2}{5} |s|^{\frac{5}{2}} \quad (8)$$

By taking its derivative, we obtain

$$\dot{V}_1(s) = \lfloor s \rfloor^{\frac{3}{2}} \dot{s} = \lfloor s \rfloor^{\frac{3}{2}} (\dot{s} - s_1) + \lfloor s \rfloor^{\frac{3}{2}} s_1 \quad (9)$$

among them, s_1 is the virtual control law, designed as

$$s_1 = -\beta_1 \lfloor s \rfloor^{\frac{1}{2}} \quad (10)$$

thus obtaining

$$\dot{V}_1(s) \leq \lfloor s \rfloor^{\frac{3}{2}} (\dot{s} - s_1) + \beta_1 s^2 \quad (11)$$

Design Lyapunov function V_2

$$V_2 = V_1 + \int_{s_1}^s \left[\lfloor \mu \rfloor^2 - \lfloor s_1 \rfloor^2 \right]^2 d\mu + \frac{2}{5} |\beta_2(t) - \beta_2^{\max}|^{\frac{5}{2}} \quad (12)$$

among them, $\int_{s_1}^s \left[\lfloor \mu \rfloor^2 - \lfloor s_1 \rfloor^2 \right]^2 d\mu = W_0$.

Derive from V_2 to obtain

$$\dot{V}_2 = \dot{V}_1 + \frac{\partial W_0}{\partial s} \dot{s} + \frac{\partial W_0}{\partial \dot{s}} \ddot{s} + 0 \quad (13)$$

Bring $\dot{V}_1(s) \leq \lfloor s \rfloor^{\frac{3}{2}} (\dot{s} - s_1) + \beta_1 s^2$ in to obtain

$$\dot{V}_2 \leq \lfloor s \rfloor^{\frac{3}{2}} (\dot{s} - s_1) - \beta_1 s^2 + \frac{\partial W_0}{\partial s} \dot{s} + \frac{\partial W_0}{\partial \dot{s}} \ddot{s} + \frac{2}{5} |\beta_2(t) - \beta_2^{\max}|^{\frac{5}{2}} \quad (14)$$

The first item according to reference [12]

$$\left| \lfloor x_1 \rfloor^{b_1 b_2} - \lfloor x_2 \rfloor^{b_1 b_2} \right| \leq 2^{1-b_2} \left| \lfloor x_1 \rfloor^{b_1} - \lfloor x_2 \rfloor^{b_1} \right|^{b_2} \quad (15)$$

Thus obtaining

$$\lfloor s \rfloor^{\frac{3}{2}} (\dot{s} - s_1) \leq |\dot{s}|^{\frac{3}{2}} 2^{\frac{1}{2}} |m|^{\frac{1}{2}} \quad (16)$$

among them, $m = \lfloor \dot{s} \rfloor^2 - \lfloor s_1 \rfloor^2$

According to the inequality

$$|x|^{c_1} |y|^{c_2} \leq \frac{c_1}{c_1 + c_2} v(x, y) |x|^{c_1 + c_2} + \frac{c_2}{c_1 + c_2} (v(x, y))^{\frac{c_1}{c_2}} |y|^{c_1 + c_2} \quad (17)$$

$$\lfloor s \rfloor^{\frac{3}{2}} (\dot{s} - s_1) \leq \frac{\beta_1}{4} s^2 + v m^2 \quad (18)$$

among them, $v = \frac{27}{\beta_1^3}$

The third item, expand the partial derivative of s to obtain

$$\frac{\partial W_0}{\partial s} = \left[\lfloor \dot{s} \rfloor^2 - \lfloor s_1 \rfloor^2 \right]^2 \frac{\partial \dot{s}}{\partial s} - \left[\lfloor s_1 \rfloor^2 - \lfloor s_1 \rfloor^2 \right]^2 \frac{\partial s_1}{\partial s} + \int_{s_1}^s 2 \times \left[\lfloor \mu \rfloor^2 - \lfloor s_1 \rfloor^2 \right] \times \frac{\partial}{\partial s} \left(\lfloor \mu \rfloor^2 - \lfloor s_1 \rfloor^2 \right) d\mu \quad (19)$$

among them, the sliding variable s and the first-order derivative of the sliding variable are both state variables of the system, and is the rate of change at s, not directly determined by its value. The two are independent state variables, thus

$$\frac{\partial W_0}{\partial s} = \int_{s_1}^s 2 \times \left[\lfloor \mu \rfloor^2 - \lfloor s_1 \rfloor^2 \right] \times \frac{\partial}{\partial s} \left(\lfloor \mu \rfloor^2 - \lfloor s_1 \rfloor^2 \right) d\mu \quad (20)$$

among them, $\frac{\partial}{\partial s} \lfloor \mu \rfloor^2 = 0$, only taking the partial derivative of $\lfloor s_1 \rfloor^2$ yields

$$\frac{\partial}{\partial s} \left[\lfloor \mu \rfloor^2 - \lfloor s_1 \rfloor^2 \right] = -2 \lfloor s_1 \rfloor \frac{\partial \lfloor s_1 \rfloor}{\partial s} \quad (21)$$

At this moment, obtain

$$\frac{\partial W_0}{\partial s} = -2 \int_{s_1}^s \left[\lfloor \mu \rfloor^2 - \lfloor s_1 \rfloor^2 \right] \times \lfloor s_1 \rfloor \frac{\partial \lfloor s_1 \rfloor}{\partial s} d\mu \quad (22)$$

among them, $\lfloor s_1 \rfloor \frac{\partial \lfloor s_1 \rfloor}{\partial s} = \frac{1}{2} \frac{\partial \lfloor s_1 \rfloor^2}{\partial s}$, thus

$$\left| \frac{\partial W_0}{\partial s} \right| \leq 2 \int_{s_1}^s \left[\lfloor \mu \rfloor^2 - \lfloor s_1 \rfloor^2 \right] d\mu \times \frac{\partial \lfloor s_1 \rfloor^2}{\partial s} \quad (23)$$

$$\int_{s_1}^s \left[\lfloor \mu \rfloor^2 - \lfloor s_1 \rfloor^2 \right] d\mu \leq |\dot{s} - s_1| \left[\lfloor \dot{s} \rfloor^2 - \lfloor s_1 \rfloor^2 \right] \quad (24)$$

Derive that

$$\left[\lfloor \dot{s} \rfloor^2 - \lfloor s_1 \rfloor^2 \right] \leq 2^{1-\frac{1}{2}} |\dot{s} - \lfloor s_1 \rfloor|^2 = 2^{\frac{1}{2}} |\dot{s} - s_1|^2 \quad (25)$$

so

$$\int_{s_1}^s \left[\lfloor \mu \rfloor^2 - \lfloor s_1 \rfloor^2 \right] d\mu \leq 2^{\frac{1}{2}} |\dot{s} - s_1|^3 \quad (26)$$

$$|\dot{s} - \lfloor s_1 \rfloor| = \left| \lfloor \dot{s} \rfloor^{2 \times \frac{1}{2}} - \lfloor s_1 \rfloor^{2 \times \frac{1}{2}} \right| \leq 2^{1-\frac{1}{2}} \left| \lfloor \dot{s} \rfloor^2 - \lfloor s_1 \rfloor^2 \right|^{\frac{1}{2}} \leq 2^{\frac{1}{2}} |m|^{\frac{1}{2}} \quad (27)$$

among them, $|\dot{s} - s_1| = |\dot{s} - \lfloor s_1 \rfloor|$ so

$$|\dot{s} - s_1|^3 \leq 2^{\frac{3}{2}} |m|^{\frac{3}{2}} \quad (28)$$

Finally we can get

$$\left| \frac{\partial W_0}{\partial s} \dot{s} \right| \leq 2^{\frac{3}{2}} |m|^{\frac{3}{2}} \left| \frac{\partial [s_1]^2}{\partial s} \right| \dot{s} \quad (29)$$

The fourth item

$$\frac{\partial W_0}{\partial s} \ddot{s} = [\lfloor \mu \rfloor^2 - \lfloor \dot{s}_1 \rfloor^2]^2 \ddot{s} \quad (30)$$

Virtual control law is designed as $s_1 = -\beta_1 [s]^{\frac{1}{2}}$, so $\left| \frac{\partial [s_1]^2}{\partial s} \right| = \beta_1^2$

$$|\dot{s}| = |\dot{s} - s_1 + s_1| \leq |\dot{s} - s_1| + |s_1| \leq 2^{\frac{1}{2}} |m|^{\frac{1}{2}} + \beta_1 |s|^{\frac{1}{2}} \leq |m|^{\frac{1}{2}} + \beta_1 |s|^{\frac{1}{2}} \quad (31)$$

$$\left| \frac{\partial [s_1]^2}{\partial s} \right| \dot{s} \leq \beta_1^2 |m|^{\frac{1}{2}} + \beta_1^3 |s|^{\frac{1}{2}} \quad (32)$$

Bring it into the third item to obtain

$$\left| \frac{\partial W_0}{\partial s} \dot{s} \right| \leq 2^{\frac{3}{2}} |m|^{\frac{3}{2}} \left(\beta_1^2 |m|^{\frac{1}{2}} + \beta_1^3 |s|^{\frac{1}{2}} \right) = 2^{\frac{3}{2}} \beta_1^2 |m|^2 + 2^{\frac{3}{2}} |m|^{\frac{3}{2}} \beta_1^3 |s|^{\frac{1}{2}} \quad (33)$$

According to the previous inequality, there exists

$$2^{\frac{3}{2}} |m|^{\frac{3}{2}} \beta_1^3 |s|^{\frac{1}{2}} \leq 2^{\frac{3}{2}} \beta_1^3 \left(\frac{1}{4} v |s|^2 + \frac{3}{4} (v)^{-\frac{1}{3}} |m|^2 \right) \leq 2^{-\frac{1}{2}} \beta_1^3 v |s|^2 + 3 \times 2^{-\frac{1}{2}} \beta_1^3 (v)^{-\frac{1}{3}} |m|^2 \quad (34)$$

We make $2^{-\frac{1}{2}} \beta_1^3 v = \frac{1}{4} \beta_1$, and can obtain $v = \frac{1}{2^{\frac{3}{2}} \beta_1^2}$

Bring it back and get

$$2^{\frac{3}{2}} |m|^{\frac{3}{2}} \beta_1^3 |s|^{\frac{1}{2}} \leq \frac{1}{4} \beta_1 |s|^2 + 3 \beta_1^{\frac{11}{3}} |m|^2 \quad (35)$$

so

$$\left| \frac{\partial W_0}{\partial s} \dot{s} \right| \leq 2^{\frac{3}{2}} \beta_1^2 |m|^2 + \frac{1}{4} \beta_1 |s|^2 + 3 \beta_1^{\frac{11}{3}} |m|^2 \quad (36)$$

At this point, for the Lyapunov function V_2

$$\dot{V}_2 \leq -\frac{\beta_1}{2} s^2 + \left(\frac{27}{\beta_1^3} + 2^{\frac{3}{2}} \beta_1^2 + 3 \beta_1^{\frac{11}{3}} \right) m^2 + [m]^2 \dot{s} + \dot{\beta}_2(t) |\beta_2(t) - \beta_2^{\max}|^{\frac{3}{2}} \quad (37)$$

Bringing controller u into this Lyapunov function

$$\dot{V}_2 \leq -\frac{\beta_1}{2} s^2 + \left(\frac{27}{\beta_1^3} + 2^{\frac{3}{2}} \beta_1^2 + 3 \beta_1^{\frac{11}{3}} \right) m^2 + [m]^2 (A(t, x) + B(t, x)u) + \dot{\beta}_2(t) |\beta_2(t) - \beta_2^{\max}|^{\frac{3}{2}} \quad (38)$$

among them, $u = -\beta_2(t) \times \text{sign}(m)$, $\text{sign}(m) = \frac{[m]}{|m|}$ and $[m]^2 = |m|^2$ that is to say

$$[m]^2 B(t, x)u = -B(t, x)\beta_2(t) |m|^2 \times \frac{[m]}{|m|} = -B(t, x)\beta_2(t) |m| [m] \quad (39)$$

$$[m]^2 A(t, x) \leq A_0 |m|^2 \quad (40)$$

$$\dot{V}_2 \leq -\frac{\beta_1}{2} s^2 + r m^2 + A_0 |m|^2 - B(t, x)\beta_2(t) |m| [m] + \dot{\beta}_2(t) |\beta_2(t) - \beta_2^{\max}|^{\frac{3}{2}} \quad (41)$$

among them, $r = \frac{27}{\beta_1^3} + 2^{\frac{3}{2}} \beta_1^2 + 3 \beta_1^{\frac{11}{3}}$, $[m] |m| = [m]^2 = |m|^2$, and $B(t, x) \geq C_m$, so

$$-B(t, x)\beta_2(t) |m| [m] \leq -C_m \beta_2(t) |m|^2 \quad (42)$$

$$\dot{V}_2 \leq -\frac{\beta_1}{2}s^2 + (r + A_0 - C_m\beta_2(t))|m|^2 + \dot{\beta}_2(t)|\beta_2(t) - \beta_2^{\max}|^{\frac{3}{2}} \quad (43)$$

If the conditions for system stability are met, there exist

$$r + A_0 - C_m\beta_2(t) \leq -\frac{\beta_1}{2} \quad (44)$$

$$\beta_2(t) \geq \frac{r + A_0 + \frac{\beta_1}{2}}{C_m} \quad (45)$$

$$\begin{aligned} \dot{V}_2 &\leq -\frac{\beta_1}{2}(s^2 + m^2) + \dot{\beta}_2(t)|\beta_2(t) - \beta_2^{\max}|^{\frac{3}{2}} \\ &= -\frac{\beta_1}{2}\left(s^2 + m^2 + (\beta_2(t) - \beta_2^{\max})^2\right) + \dot{\beta}_2(t)|\beta_2(t) - \beta_2^{\max}|^{\frac{3}{2}} + \frac{\beta_1}{2}(\beta_2(t) - \beta_2^{\max})^2 \end{aligned} \quad (46)$$

There is a Lyapunov function ahead

$$V_2 = \frac{2}{5}|s|^{\frac{5}{2}} + \int_{s_1}^s \left[|\mu|^2 - |s_1|^2\right]^2 d\mu + \frac{2}{5}|\beta_2(t) - \beta_2^{\max}|^{\frac{5}{2}} \quad (47)$$

satisfy the conditions of $\dot{V}_2 \leq -cV_2^\alpha + \varepsilon$, including $c = \frac{\beta_1}{2^{\frac{5}{2}}}$, $\alpha = \frac{4}{5}$, $\varepsilon = \dot{\beta}_2(t)|\beta_2(t) - \beta_2^{\max}|^{\frac{3}{2}} + \frac{\beta_1}{2}(\beta_2(t) - \beta_2^{\max})^2$, according to

$$\dot{V}_2 \leq -cV_2^\alpha + \Delta \quad (48)$$

so $V_2 \leq \left(\frac{\Delta}{c}\right)^{\frac{1}{\alpha}}$. Due to the non negativity of V_2 , it satisfies $\frac{2}{5}|s|^{\frac{5}{2}} \leq V_2 \leq \left(\frac{\Delta}{c}\right)^{\frac{1}{\alpha}}$. Obtain the range of s

$$|s| \leq \left(\frac{5}{2}\left(\frac{\Delta}{c}\right)^{\frac{1}{\alpha}}\right)^{\frac{2}{5}} \quad (49)$$

Actually $W_0 = \int_{s_1}^s \left[|\mu|^2 - |s_1|^2\right]^2 d\mu \geq \frac{1}{20}|s - s_1|^5$ satisfies the lower bound of W_2 , thus obtaining

$$|s| \leq \left(\frac{1}{4}\left(\frac{\Delta}{c}\right)^{\frac{1}{\alpha}} + \frac{5}{2}\beta_1^5\left(\frac{\Delta}{c}\right)^{\frac{1}{\alpha}}\right)^{\frac{1}{5}} \quad (50)$$

thus obtaining the bounded domain of the sliding variable $(s, \dot{s})$.

It should be noted that the system is divided into four situations to analyze the effectiveness of the adaptive law.

Scenario 1, $W_1(s, \dot{s}) \leq a$ and $\beta_2^{\min} \leq \beta_2(t) \leq \beta_2^{\max}$ at this moment, $\dot{\beta}_2(t) = \frac{\beta_1}{2}|\beta_2(t) - \beta_2^{\max}|^{\frac{1}{2}}$, $\dot{V}_2 \leq -cV_2^\alpha$, among them $c = \frac{\beta_1}{2^{\frac{5}{2}}}$, $\alpha = \frac{4}{5}$, the system state converges to the domain $\Omega = \{(s, \dot{s}) : W_1(s, \dot{s}) \leq a\}$ in a finite time;

Scenario 2, $W_1(s, \dot{s}) \leq a$ and $\beta_2^{\min} \leq \beta_2(t) \leq \beta_2^{\max}$, at this point, $\beta_2(t)$ is negative and the gain decreases; If the state leaves domain Ω , switch to situation 1;

Scenario 3, $\beta_2(t) < \beta_2^{\min}$, adaptive law $\beta_2(t) = z > 0$, gain increases and quickly returns to $[\beta_2^{\min}, \beta_2^{\max}]$

Scenario 4, $\beta_2(t) < \beta_2^{\max}$, $\beta_2(t) = 0$, the gain remains at its maximum value, and the state quickly converges to domain Ω and switch to situation 2.

In summary, there are normal numbers $n_1 = \left(\frac{5}{2}\left(\frac{\Delta}{c}\right)^{\frac{1}{\alpha}}\right)^{\frac{2}{5}}$, $n_2 = \left(\frac{1}{4}\left(\frac{\Delta}{c}\right)^{\frac{1}{\alpha}} + \frac{5}{2}\beta_1^5\left(\frac{\Delta}{c}\right)^{\frac{1}{\alpha}}\right)^{\frac{1}{5}}$, and therefore $\|s\| \leq n_1$, $|\dot{s}| \leq n_2$, the system realizes a real second-order sliding mode.

4 | SIMULATION EXPERIMENT

According to the system given earlier, the moment of inertia $J = 1.2 \text{ kg/m}^2$; Viscous damping coefficient $P = 0.8 \text{ N} \cdot \text{m} \cdot \text{s/rad}$; Spring stiffness coefficient $K = 15 \text{ N} \cdot \text{m/rad}$; Time varying load disturbance $T_L = 3 \sin(0.5t) \text{ N} \cdot \text{m}$.

The control objective is to make the joint angle θ track the reference trajectory $\theta_d = 0.5 \sin(\pi t) + 0.3 \cos(2t)$. Design Controller $u = -\beta^2(t) \cdot \text{sign}\left(|\dot{s}|^2 + \beta_1^2 s\right)$, take the tracking error as $e = \theta - \theta_d$, sliding mode variable design is $s = \dot{e} + \beta_1 |e|^{\frac{1}{2}}$ and

adaptive gain $\beta_2(t)$

$$\beta_2(t) = \begin{cases} \frac{\beta_1}{2} |\beta_2(t) - \beta_2^{\max} \frac{1}{2}|, & \text{if } W_1(s, s) > a \text{ and } \beta_2^{\min} \leq \beta_2(t) \leq \beta_2^{\max} \\ -\frac{\beta_1}{2} |\beta_2(t) - \beta_2^{\max} \frac{1}{2}| - \beta_3, & \text{if } W_1(s, s) \leq a \text{ and } \beta_2^{\min} \leq \beta_2(t) \leq \beta_2^{\max} \\ 0, & \text{if } \beta_2(t) < \beta_2^{\max} \text{ or } \beta_2(t) < \beta_2^{\min} \end{cases} \quad (51)$$

among them, $W_1(s, \dot{s}) = \frac{2}{5} |s|^{\frac{5}{2}} + \frac{1}{20} |s + \beta_1 \lfloor s \rfloor^{\frac{1}{2}}|^5$, $a = 3.5 \times 10^{-8}$, $\beta_2^{\min} = 8$, $\beta_2^{\max} = 15$, $\beta_3 = 0.1$.

The simulation results are shown below

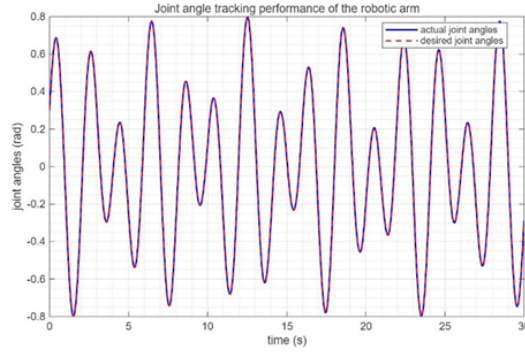


FIGURE 1 Joint angle tracking performance of the robotic arm

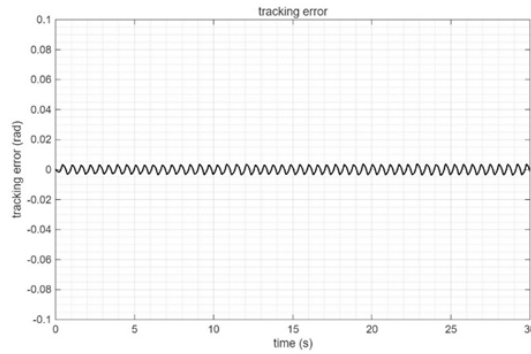


FIGURE 2 Tracking error

As shown in Figure 1, the actual joint angle has a high degree of fit with the expected joint angle, effectively verifying the finite time stability of the ASOSM controller and ensuring tracking accuracy; As shown in Figure 2, it effectively proves that the system can still maintain high-precision tracking under uncertainty and disturbance.

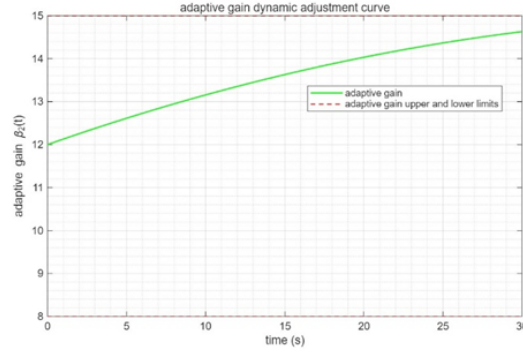


FIGURE 3 Adaptive gain dynamic adjustment curve

As shown in Figure 3, strictly matching the four types of switching logic of the article adaptive law, the gain increases when the system deviates from the domain and decreases when it enters the domain, avoiding gain overestimation; As shown in Figure 4, it reflects the advantage of ASOSM controller in avoiding overestimation of dynamic gain, significantly reducing chattering, which is in line with the design goal of the article.

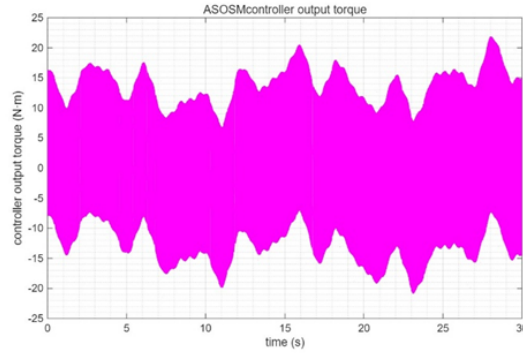


FIGURE 4 ASOSMcontroller output torque

5 | SUMMARY AND OUTLOOK

This article focuses on the problems of gain overestimation and increased chattering in traditional second-order sliding mode control for nonlinear systems with uncertainty. Theoretical research and simulation verification are conducted, and an adaptive second-order sliding mode control strategy based on Lyapunov method is used. The strategy only requires bounded system uncertainty and does not rely on the strict assumption of bounded derivatives. By designing a dynamic adaptive law, the control gain can be adjusted in real time with uncertainty, which fundamentally avoids the problems of insufficient robustness or gain overestimation that are prone to occur in fixed gain controllers.

Based on the Lyapunov stability theory, the finite time stability of the closed-loop system was rigorously proved, and the conditions for the system state to converge to the true sliding mode domain were clarified. Through the analysis of the adjustment logic of the adaptive law under four typical operating conditions, the rationality and reliability of the control strategy were verified. Simulation experiments were conducted on a single joint hydraulic robotic arm as the control object, and the results showed that the proposed controller can achieve high-precision trajectory tracking of joint angles, with tracking errors maintained within a very small range. The adaptive gain can be dynamically adjusted according to the system state, effectively reducing the inherent chattering phenomenon of sliding mode control. The smoothness of the controller output torque was significantly improved, verifying the engineering practicality of the method.

However, the method still has certain limitations: its application in multi input multi output strongly coupled nonlinear

systems has not been fully validated, and its ability to suppress measurement noise needs to be further improved. Future research directions that can be expanded include: extending the proposed control strategy to multi input multi output complex systems, and combining data-driven methods to optimize adaptive law design to enhance anti-interference capabilities; Explore the integration path with intelligent algorithms to further improve control accuracy and dynamic response speed, providing a more comprehensive solution for high-performance control of complex and uncertain systems.

REFERENCES

- [1] Qian Xuesen, Song Jian Engineering Control Theory [J]. Revised Edition Science Press, Beijing (1983), 1957.
- [2] LEVANT A. Sliding order and sliding accuracy in sliding mode control[J]. International Journal of Control, 1993, 58(6): 1247-1263.
- [3] SIRA-RAMIREZ H, LLANES-SANTIAGO O. Adaptive dynamical sliding mode control via backstepping[C]//Proceedings of 32nd IEEE conference on decision and control. IEEE, 1993: 1422-1427.
- [4] RIOS-BOLIVAR M, ZINOBER A, SIRA-RAMIREZ H. Dynamical adaptive sliding mode output tracking control of a class of nonlinear systems[J]. International Journal of Robust and Nonlinear Control: IFAC-Affiliated Journal, 1997, 7(4): 387-405.
- [5] WHEELER G, SU C Y, STEPANENKO Y. A sliding mode controller with improved adaptation laws for the upper bounds on the norm of uncertainties[C]//Proceedings of the 1997 american control conference (cat. No. 97CH36041): Vol. 3. IEEE, 1997: 2133-2137.
- [6] SHTESSEL Y B, MORENO J A, PLESTAN F, et al. Super-twisting adaptive sliding mode control: a lyapunov design[C]49th IEEE conference on decision and control (CDC). IEEE, 2010: 5109-5113.
- [7] SHTESSEL Y, TALEB M, PLESTAN F. A novel adaptive-gain supertwisting sliding mode controller: methodology and application[J]. Automatica, 2012, 48(5): 759-769.
- [8] Ran Dechao, Ni Qing, Sheng Tao, etc Finite time attitude maneuver algorithm for spacecraft based on adaptive second-order terminal sliding mode [J]. Journal of National University of Defense Technology, 2017, 39 (1): 6-10.
- [9] AMINI M R, SHAHBAKHTI M, PAN S. Adaptive discrete second-order sliding mode control with application to nonlinear automotive systems[J]. Journal of Dynamic Systems Measurement Control, 2018, 140(12): 121010.
- [10] Zhang Qian, Li Dong Adaptive second-order sliding mode observer pmsm sensorless vector control with parameter identification [J]. Control and Decision, 2019, 34 (7): 1385-1393.
- [11] SAMARI B, INCREMONA G P, FERRARA A, et al. Data-driven adaptive second-order sliding mode control with noisy data[J]. arXiv Preprint arXiv:2508.02357, 2025.
- [12] DING S, MEI K, YU X. Adaptive Second-Order Sliding Mode Control: A Lyapunov Approach[J]. IEEE Transactions on Automatic Control, 2022, 67(10): 5392-5399.

REFERENCES



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