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Modeling and Implementation of Direct Wheel Speed Control for Trajectory Tracking Control System of Sampling Robot at Customs Ports

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Abstract

As a vital gateway for national economic exchange, customs ports undertake the critical task of supervising the quality, quantity, and compliance of imported and exported goods. Concentrates, as essential industrial raw materials, play a crucial role in international trade. However, traditional manual sampling methods applied in customs port concentrate inspection suffer from numerous drawbacks, such as low efficiency, insufficient accuracy, and high labor intensity. These issues severely constrain the efficiency and accuracy of customs inspection work, increasing the risks of human error and operational hazards. To address these challenges and enhance the intelligent level of customs port concentrate inspection, this paper proposes an intelligent sampling robot design. This design aims to overcome the limitations of traditional manual sampling through technological innovation, achieving automation and intelligence in the sampling process, thereby improving sampling efficiency, ensuring detection accuracy, reducing labor intensity, and promoting the intelligent construction of customs ports. The sampling robot operates in the complex environment of concentrate ports, where the system is often subject to external disturbances, and its modeling process involves uncertainties. Therefore, ensuring high-performance control under modeling errors and external disturbances holds particular significance for the robustness of the sampling robot system. This paper details the mechanical structure, motor parameter calculation, Permanent Magnet Synchronous Motor (PMSM) modeling, coordinate transformation, Space Vector Pulse Width Modulation (SVPWM), and a wheel-speed-driven kinematic modeling approach for trajectory tracking control. System simulation results based on Matlab/Simulink demonstrate the effectiveness of the designed control system and direct wheel speed control model.

KEYWORDS

sampling robot, trajectory tracking, permanent magnet synchronous motor (PMSM), space vector pulse width modulation (SVPWM), direct wheel speed control, customs

1 | INTRODUCTION

With the rapid growth of global trade, cargo inspection and sampling operations at customs ports are facing unprecedented pressure on efficiency and safety. The traditional manual sampling method is plagued by problems such as high labor intensity, complex operating environments, and high risk of personnel exposure to hazardous materials, creating an urgent need to introduce intelligent and automated robotic technologies to replace manual operations. The centralized customs sampling operation environment is typically characterized by complex conditions including uneven ground, rugged terrain, and narrow spaces, which impose special requirements on the mechanical structure, drive system, and motion control of the robot. In particular, the sampling robot needs to complete precise fixed-point docking, manipulator operation, and sample storage and transfer in a limited space, where the trajectory tracking accuracy of its mobile platform directly determines the success or failure of the entire sampling task[1]. Therefore, developing a sampling robot that can adapt to the complex port environment with high-precision trajectory tracking capability, and establishing its direct wheel speed control model, have important theoretical significance and engineering application value.

Trajectory tracking control of wheeled mobile robots (WMRs) is a classic problem in the field of robotics. Existing research is mainly divided into two categories: kinematic model-based control and dynamic model-based control. The former takes the robot pose (position and heading angle) as the state variable, and designs control laws for linear velocity and angular velocity to make the actual trajectory converge to the desired trajectory[2]. The latter further takes into account dynamic factors such as the interaction force between the wheels and the ground, as well as the drive characteristics of the motors, to achieve higher-precision control[3]. For differential-driven wheeled mobile robots, commonly used control methods include the Pure Pursuit method, Stanley method, Model Predictive Control (MPC), and Backstepping method[4]. However, most of these methods are based on idealized kinematic models, assuming that the wheel speed can respond to control commands without delay or error, while ignoring the dynamic characteristics of the motor drive system (such as the response delay of the current loop and speed loop) and the impact of actual road conditions on wheel speed. In complex ground environments such as customs ports, factors such as wheel slip, uneven ground, and motor load fluctuations will cause significant deviations between the actual wheel speed and the expected value, which in turn leads to increased trajectory tracking error and even system instability[5].

In recent years, some scholars have begun to implement combined control of the motor drive model and the robot kinematic model, for example, by adopting a cascade control structure (with an outer-loop kinematic controller and an inner-loop motor speed controller), or compensating for wheel speed disturbances based on a sliding mode observer[6]. However, existing research mostly focuses on theoretical simulation or simplified experimental platforms, and lacks systematic modeling and verification for the actual working conditions of heavy-load, large-inertia, high-precision sampling robots. Especially in the port sampling scenario, the robot not only needs to carry a sample load of up to 300 kg, but also is required to complete accurate trajectory tracking and fixed-point docking in narrow passages, which imposes extremely high requirements on the real-time performance and robustness of wheel speed control[7]. In addition, there is still a lack of in-depth discussion in existing literature on the kinematic modeling and implementation of direct wheel speed control, that is, controlling the wheel speed directly through motor current/voltage commands without an intermediate speed loop.

To address the above problems, this paper takes the customs port sampling robot as the research object, and designs a four-wheel drive robot system with a three-level modular structure of "manipulator-chassis-storage mechanism". The chassis adopts a configuration with front-wheel drive (directly driven by permanent magnet synchronous motors, PMSMs) and rear-wheel driven (with steering damping bearings), to adapt to slopes and gravel roads commonly found in ports. The manipulator has 3 active degrees of freedom, which can realize 360° horizontal dead-angle-free sampling coverage with a maximum working height of 1.2 m. Its end effector is integrated with a force sensor to ensure the compliance of the sampling operation. The skip storage mechanism at the back adopts an inverted trapezoidal stainless steel box, and the inner wall is sprayed with a polytetrafluoroethylene (PTFE) anti-stick coating, which effectively reduces the adhesion of viscous samples such as mineral concentrate. In terms of the drive system, this paper establishes an electrical model of the PMSM in the dq synchronous rotating

reference frame, including the voltage equation, torque equation, and motion equation, and implements the Clark and Park coordinate transformation as well as the Space Vector Pulse Width Modulation (SVPWM) strategy in detail, which provides a bottom-level drive foundation for precise wheel speed control.

At the level of trajectory tracking control, this paper proposes a kinematic modeling and control method based on direct wheel speed drive. Different from traditional methods, this paper does not directly define the control variables as linear velocity and angular velocity. Instead, it directly drives the motor speed through the inverse solution of the desired linear velocity of the left and right wheels, and then updates the actual pose of the robot through the forward kinematic model (based on actual wheel speed feedback), thus forming closed-loop feedback[8]. Specifically, this paper first derives the geometric relationship between the center of mass (CoM) velocity of the differential-driven robot and the linear velocity of the left and right wheels, and presents the kinematic equation in the form of a pose derivative matrix. Then, a mode switching logic based on distance error and heading error is defined: when the position error is large, the system is in the position tracking mode, giving priority to reducing the position deviation; when the position error is less than the threshold while the heading error is still large, the system switches to the heading control mode, giving priority to correcting the heading[9]. This mode switching mechanism effectively avoids the oscillation phenomenon that may occur with the traditional single control law when approaching the target point. For the control laws, proportional control is adopted for the angular velocity, where the target heading is jointly determined by the desired heading and the steering desired heading; on-off logic control is adopted for the linear velocity, which is dominated by the basic linear velocity, with deceleration adjustment only performed when the distance error is small[10].

To realize the closed-loop connection between the wheel speed command and the motor drive, this paper establishes a wheel speed inverse solution and motor speed conversion module: the desired linear velocity of the left and right wheels is inversely solved according to the control variables (linear velocity and angular velocity), and then the desired motor speed is calculated through the wheel radius; meanwhile, the actual wheel linear velocity is inversely solved from the actual motor speed (fed back by the PMSM model), and substituted into the discretized pose recurrence equation (Euler method, sampling time 0.01 s) to update the robot pose[11]. Periodic normalization processing is applied to the heading angle to avoid numerical overflow. This "direct wheel speed control" modeling method enables the control law to directly act on the final execution variable (wheel speed), avoiding the degradation of tracking performance caused by the delay or error of the intermediate speed loop in traditional methods.

To verify the effectiveness of the proposed control system, an overall simulation model integrating PMSM drive control, coordinate transformation, SVPWM modulation, wheel speed kinematic modeling, and trajectory tracking closed-loop control is built based on the Matlab/Simulink simulation platform. The simulation parameters are strictly set in accordance with the actual design specifications and calculated parameters of the robot: total robot weight of 440 kg (including 300 kg sample load), wheel radius of 0.15 m, wheelbase of 450 mm, track width of 380 mm, basic linear velocity of 0.5 m/s, number of pole pairs of the PMSM of 4, rated torque of a single motor of no less than 11.65 N·m, sampling time of 0.01 s, distance threshold of 0.05 m, and heading proportional coefficient of 5. Aiming at the typical working conditions in port sampling operations, namely straight-line trajectory (driving between cargo stacks) and circular arc trajectory (turning in narrow spaces, sampling around cargo stacks), two predefined trajectories are set for simulation verification. Trajectory 1 is a segmented motion trajectory including two straight sections and two circular arc turning sections; Trajectory 2 is an ultra-smooth segmented motion trajectory based on a fifth-order Bezier curve, which has been optimized for smoothness[12]. The simulation results show that: in the straight section, the actual trajectory of the robot has an extremely high degree of coincidence with the reference trajectory, the distance error rapidly converges to within 0.01 m within 0.3 s, the heading error is always controlled within the range of $\pm 0.5^\circ$, and there is no steady-state drift; in the circular arc turning section, the maximum distance error does not exceed 0.03 m, the heading error is controlled within $\pm 1^\circ$, and it rapidly converges to the steady-state range of 0.01 m and $\pm 0.5^\circ$ after the turning is completed. The mode switching logic is correct throughout the whole process, and the linear velocity is stably output at 0.5 m/s without frequent fluctuations. These results fully verify the high-precision tracking capability and robustness of the proposed direct wheel speed control modeling method under complex trajectories.

The main contributions of this paper are as follows: 1) A complete set of mechanical structure and drive system parameters of the sampling robot is designed for the complex operating environment of customs ports; 2) A complete control link from

the bottom-level drive of the PMSM (SVPWM) to the wheel speed kinematic model is established, realizing direct control of the wheel speed; 3) A trajectory tracking control law based on mode switching is proposed, which takes into account both the position convergence speed and the heading correction accuracy; 4) The excellent performance of the system under straight-line and circular arc trajectories is verified through simulation, which provides a theoretical basis for the development of the actual prototype.

The remaining sections of this paper are arranged as follows: Section 2 details the mechanical structure design of the robot, including the chassis, manipulator, and skip storage mechanism; Section 3 presents the calculation and selection basis of the motor parameters; Section 4 establishes the dq-axis model of the PMSM, and describes the implementation of coordinate transformation and SVPWM modulation; Section 5 elaborates the trajectory tracking control method of the mobile robot based on wheel speed drive, including kinematic modeling, mode switching logic, control law design, and discretized pose update; Section 6 presents the simulation results and analysis.

2 | ROBOT MECHANICAL STRUCTURE

The customs concentrate sampling operation environment typically includes complex conditions such as uneven ground, rugged terrain, and confined spaces, which impose special requirements on the robot's mechanical structure and control system[13]. Based on this demand, we designed a four-wheel-drive sampling robot system as shown in Figure 1. The robot adopts a three-level modular design of "manipulator-chassis-storage mechanism." The chassis is a four-wheel structure, with the front two wheels being drive wheels, configured with Permanent Magnet Synchronous Motors (PMSM) for direct drive, and the rear two wheels being idler wheels with steering damping bearings. The chassis four-wheel structure is shown in Figure 2. Its wheelbase is 450mm, track width is 380mm, wheel radius is 0.15m, and minimum ground clearance is 80mm, enabling it to adapt to common slopes and gravel pavement at ports. A three-degree-of-freedom grabbing manipulator is integrated on top of the body, and a scoop storage mechanism is installed on the back.

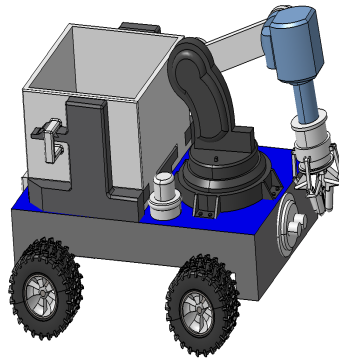


FIGURE 1 Schematic diagram of the overall robot structure

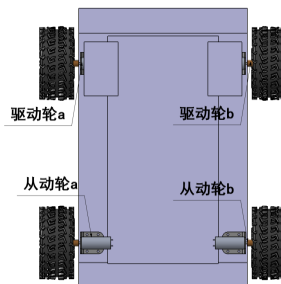


FIGURE 2 Schematic diagram of the chassis structure

The manipulator adopts a "rotating base + two-link" configuration, possessing 3 active degrees of freedom[14]. The base rotation (J1) achieves 360° rotation, driven by a hollow shaft servo motor coupled with a slewing bearing, allowing 360° omnidirectional sampling coverage in the horizontal plane. The boom pitch (J2) can swing from 0° to 120°, using a crank-slider force amplification mechanism, with a maximum grabbing weight of 5kg. The forearm pitch (J3) swings from -30° to 90°, forming a linkage with the boom to expand the vertical sampling range, with a maximum working height of 1.2m and a minimum sampling height of 0.3m. Each joint motor is a 200W DC servo motor with a rated speed of 3000 r/min, torque of 0.63 N·m, paired with a 100:1 harmonic reducer (backlash ≤ 10 arc minutes). The end-effector integrates a force sensor with a range of 50N and an accuracy of 0.1N[15].

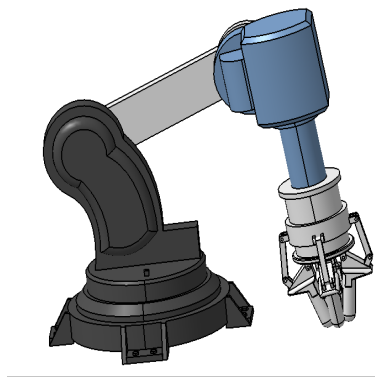


FIGURE 3 Schematic diagram of the manipulator structure

The scoop is an inverted trapezoidal stainless steel container with a volume of 5L and dimensions of 400mm x 300mm x 200mm[16]. The inner wall is coated with a polytetrafluoroethylene (PTFE) anti-adhesion coating with a surface roughness $R_a \leq 0.2\mu\text{m}$ to reduce concentrate adhesion. Its top opening measures 200mm x 150mm, aligning with the end-effector motion trajectory. The opening is equipped with an electric gate driven by a micro servo motor, with an opening time ≤ 1 s.

3 | MOTOR PARAMETER CALCULATION

The total weight of the robot is 440kg (including 300kg sample, 20kg manipulator, 100kg chassis, and 20kg for other components). The rolling friction force f during travel is calculated using a rolling friction coefficient $\mu = 0.02$ (estimated for common pavement): $f = \mu mg = 0.02 \times 440 \times 9.8 \approx 86.24$ N ($g = 9.8$ N/kg).

Considering complex road conditions such as slopes, the total resistance F is taken as 1.5 times the rolling friction: $F = 1.5 \times 86.24 \approx 129.36$ N. Assuming a travel speed $v = 0.5$ m/s (conventional speed for sampling operation), the required total power is $P = Fv = 129.36 \times 0.5 \approx 64.68$ W.

The power is shared by two Permanent Magnet Synchronous Motors, so the power per motor is approximately $64.68 \div 2 = 32.34$ W. Based on the power-torque relationship $P = T\omega$ (where ω is angular velocity), the wheel angular velocity $\omega = v \div r = 0.5 \div 0.15 \approx 3.33$ rad/s ($r = 0.15$ m). The torque per motor $T = P / \omega = 32.34 \div 3.33 \approx 9.71$ N·m. Considering overload capacity, the motor rated torque should be no less than 1.2 times the calculated value ($9.71 \times 1.2 \approx 11.65$ N·m).

Motor speed n is calculated from $\omega = 2\pi n / 60$: $n = \omega \times 60 / (2\pi) = 3.33 \times 60 / (2 \times 3.14) \approx 31.85$ r/min. The motor must meet this speed to ensure the travel velocity.

4 | PERMANENT MAGNET SYNCHRONOUS MOTOR MODEL

The electrical model of the Permanent Magnet Synchronous Motor (PMSM) is established based on the dq-coordinate system, with core equations as follows:

Voltage equations:

$$\begin{cases} u_d = R_s i_d + \omega_e L_q i_q + L_d \frac{di_d}{dt} \\ u_q = R_s i_q + \omega_e L_d i_d + L_q \frac{di_q}{dt} + \omega_e \psi_f \end{cases} \quad (1)$$

where: u_d, u_q are d-axis and q-axis voltages, i_d, i_q are currents, L_d, L_q are inductances, ω_e is rotor electrical angular velocity, ψ_f is permanent magnet flux linkage[17]. The current differential form is simplified as:

The current differential form is simplified as:

$$\begin{cases} \frac{di_d}{dt} = \frac{1}{L_d} u_d - \frac{R_s}{L_d} i_d + \frac{L_q}{L_d} \omega_e i_q \\ \frac{di_q}{dt} = \frac{1}{L_d} u_q - \frac{R_s}{L_d} i_q - \frac{L_d}{L_d} \omega_e i_d - \frac{\omega_e \psi_f}{L_d} \end{cases} \quad (2)$$

Torque equation: Electromagnetic torque includes magnet torque and reluctance torque:

$$T_e = \frac{3}{2} p_n i_q [\psi_f + (L_d - L_q) i_d] \quad (3)$$

Where p_n : is the number of pole pairs[18].

For surface-mounted PMSM ($L_d = L_q$), it simplifies to:

$$T_e = \frac{3}{2} p_n \psi_f i_q$$

Motion equation:

$$T_e - T_L - B \omega_m = J \frac{d\omega_m}{dt} \quad (4)$$

where: Ω_m is mechanical angular velocity (rad/s), J is moment of inertia, B is damping coefficient, T_L is load torque. Key relationships:

$$\omega_m = \omega_e / p_n \quad (5)$$

$$N_r = 30 \omega_m / \pi \quad (6)$$

where: N_r is rotational speed (r/min), θ_e is rotor position angle[19].

Motor parameters: DC bus voltage 560 V, speed 3000 rad/s, resistance 0.958 Ω , inductance 0.000525 H, flux linkage 0.1119 Wb, pole pairs 4, moment of inertia 0.0016 kg·m², damping coefficient 0.0002024 N·m/rad/s.

4.1 | Coordinate Transformation Model and Simulation

The transformation from the abc three-phase stationary coordinate system to the dq synchronous rotating coordinate system is achieved through Clark and Park transformations, simplifying model decoupling:

Clark Transform: Maps the abc coordinate system to the stationary $\alpha\beta 0$ coordinate system: Clark Transform: Maps the abc coordinate system to the stationary $\alpha\beta 0$ coordinate system:

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (7)$$

Inverse Clark Transform:

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} \quad (8)$$

The Clark transform simulation model is as follows:

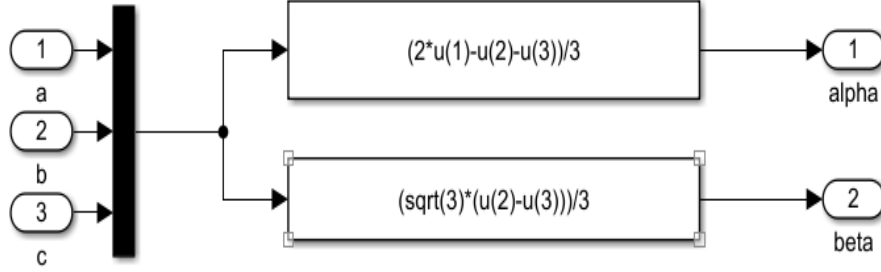


FIGURE 4 MATLAB simulation model of Clark transformation

Park Transform: Transforms the coordinate system to the dq rotating coordinate system:

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} \quad (9)$$

Inverse Park Transform:

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} \quad (10)$$

The Park transform simulation result is as follows:

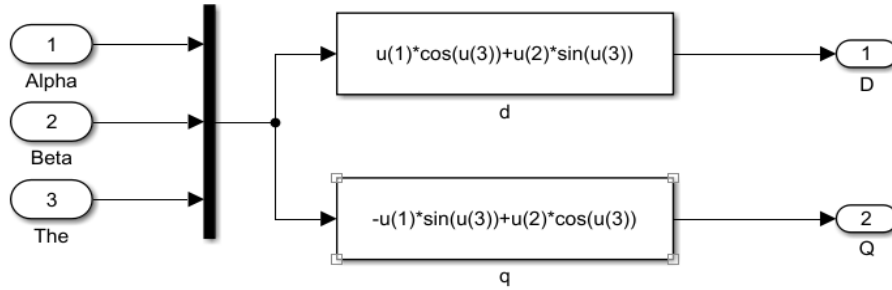


FIGURE 5 Simulation results of Park transformation

In the dq coordinate system, u_d, u_q become decoupled DC quantities, facilitating control[20].

4.2 | Voltage Space Vector PWM (SVPWM)

SVPWM is based on the space vector equivalence principle, using basic voltage vectors to synthesize the target output voltage:

Principle and Sector Selection: The three-phase voltage synthesis vector is:

$$U_s = \frac{3}{2} U_m e^{j(\omega t + \frac{\pi}{2})} \quad (11)$$

When resistive voltage drop is neglected, the voltage and flux linkage relationship is:

$$u_s = \frac{d\psi_s}{dt} \quad (12)$$

The voltage space vector diagram divides the complex plane into six sectors:

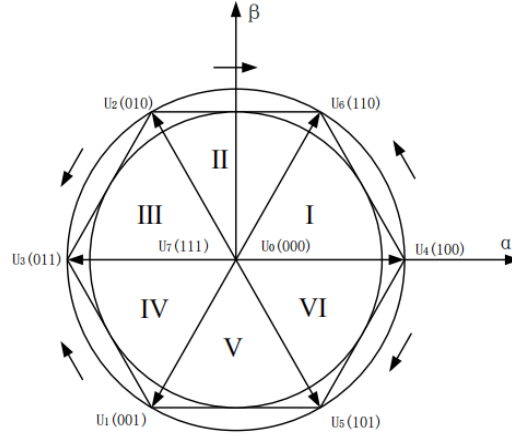


FIGURE 6 Voltage space vector diagram

Sector judgment is based on u_α, u_β components: $U_{ref1} = u_\beta, U_{ref2} = 32u_\alpha - 12u_\beta, U_{ref3} = -32u_\alpha - 12u_\beta$ [6]

Calculate (A/B/C as sign flags), sector correspondence is shown in Table 1:

TABLE 1 Sector control table						
N	3	1	5	4	6	2
Sector	I	II	III	IV	V	VI

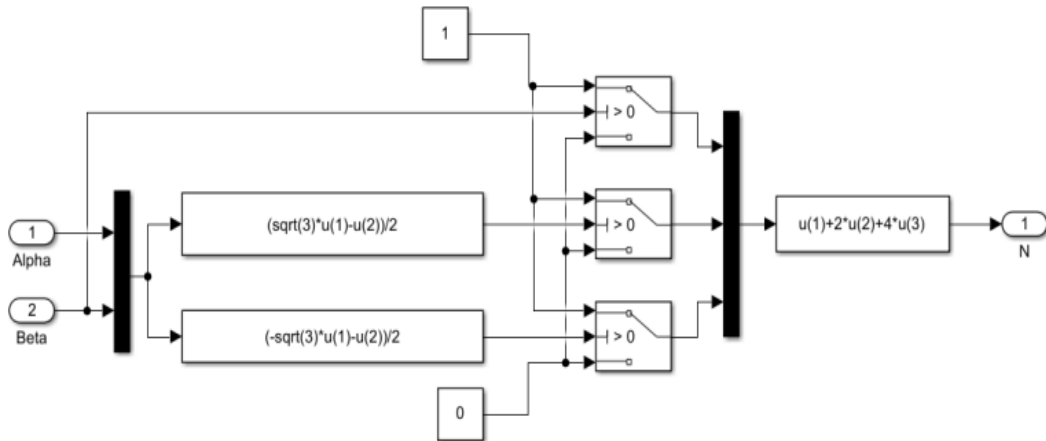


FIGURE 7 Sector selection

Vector Action Time Calculation: In Sector I, the vector synthesis formula is:

$$T_s U_{\text{out}} = T_4 U_4 + T_6 U_6 + T_0 U_0 \quad (13)$$

Action times:

$$T_4 = \sqrt{3} \frac{U_{\text{out}}}{U_{\text{dc}}} T_s \sin\left(\frac{\pi}{3} - \theta\right) \quad (14)$$

$$T_6 = \sqrt{3} \frac{U_{\text{out}}}{U_{\text{dc}}} T_s \sin \theta \quad (15)$$

$$T_0 = T_7 = \frac{1}{2} (T_s - T_4 - T_6) \quad (16)$$

If $T_4 + T_6 > T_s$, modulation is required:

$$T_4 = \frac{T_4}{T_4 + T_6} T_s \quad (17)$$

$$T_6 = \frac{T_6}{T_4 + T_6} T_s \quad (18)$$

The action time simulation model is as follows:

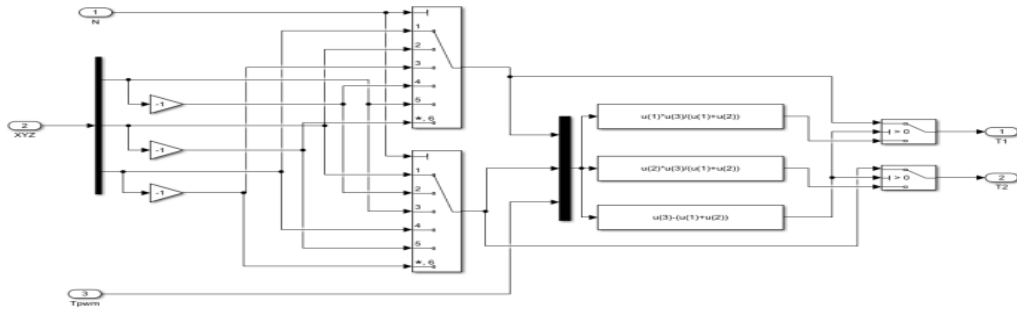


FIGURE 8 Action time of adjacent sector vectors

Define switching points:

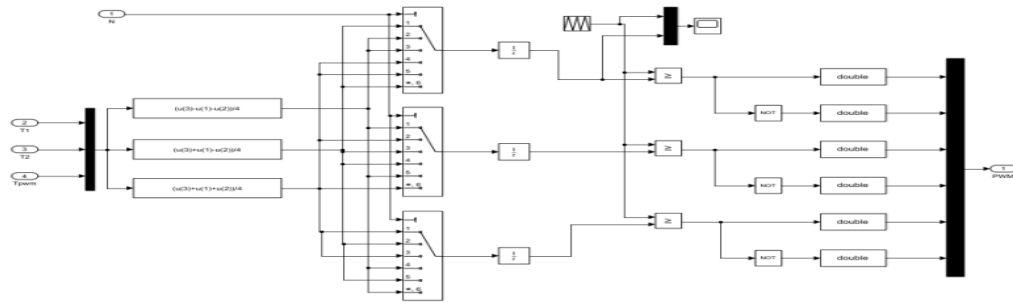
$$T_a = \frac{T_s - T_4 - T_6}{4}, \quad T_b = T_a + \frac{T_4}{2}, \quad T_c = T_b + \frac{T_6}{2} \quad (19)$$

The three-phase switching points $T_{cm1}, T_{cm2}, T_{cm3}$ are determined according to the sector.

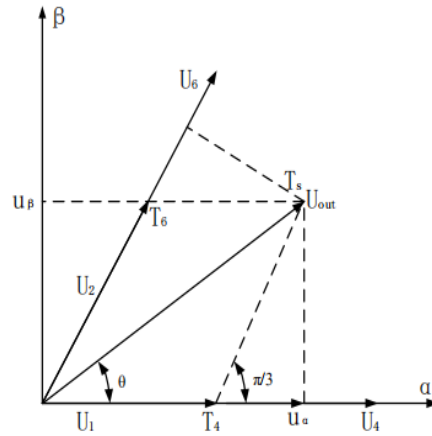
TABLE 2 Sector switching points mapping

Sector	T_{cm1}	T_{cm2}	T_{cm3}
I	T_a	T_b	T_c
II	T_b	T_a	T_c
III	T_c	T_a	T_b
IV	T_c	T_b	T_a
V	T_b	T_c	T_a
VI	T_a	T_c	T_b

The PWM signal generation simulation model is as follows:

**FIGURE 9** Determination of sector vector switching points and PWM signal generation module

A schematic diagram of voltage space vector synthesis is as follows:

**FIGURE 10** Schematic diagram of voltage space vector synthesis

5 | MOBILE ROBOT TRAJECTORY TRACKING CONTROL

5.1 | Overall Structure of the Tracking System

The system consists of four core modules:

1. Trajectory Generator: Generates a continuous smooth reference trajectory $\xi_d(t) = [x_d(t), y_d(t), \theta_d(t)]^\top$ based on time parameterization[21].
2. Controller: Computes the control input $\mathbf{u} = [v, \omega]^\top$ to make the actual pose $\xi(t)$ converge to the reference trajectory[22].
3. Actuator: Converts the control input to left and right wheel linear speeds v_l, v_r , driving the motors.
4. Pose Update: Updates the actual pose $\xi(t)$ based on wheel speed feedback via the forward kinematics model[23].

5.2 | Kinematic Modeling Based on Wheel Speed Drive

5.2.1 | Geometric Relationship of Pose Derivative from Wheel Speed

The linear speeds of the left and right wheels of a differential drive robot directly determine the pose change: Position derivative: The centroid is at the midpoint of the two wheels, and its velocity is the vector sum of the wheel speeds. Because the wheel's forward direction aligns with half the robot's heading angle θ (the tangent direction of the wheel trajectory and the heading is half-angle), the inertial frame projection of the centroid velocity is given by:

$$\dot{x} = \cos \frac{\theta}{2} \cdot v_l + \cos \frac{\theta}{2} \cdot v_r, \quad \dot{y} = \sin \frac{\theta}{2} \cdot v_l + \sin \frac{\theta}{2} \cdot v_r [24]$$

Heading derivative: The speed difference between the left and right wheels generates a turning motion. Define the track width L as the distance between the two wheels. The angular velocity $\dot{\theta}$ of the robot is determined by the wheel speed difference, expressed as:

$$\dot{\theta} = \frac{v_l - v_r}{L} [25]$$

In summary, the matrix form of the pose derivative is:

$$\dot{\xi}(t) = \begin{pmatrix} \cos \frac{\theta}{2} & \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} & \sin \frac{\theta}{2} \\ \frac{1}{L} & -\frac{1}{L} \end{pmatrix} \begin{pmatrix} v_l \\ v_r \end{pmatrix} \quad (20)$$

5.2.2 | Desired Trajectory and Control Input Definition

The desired trajectory is still defined by a time-parameterized function:

$$\xi_d(t) = \begin{pmatrix} x_d(t) \\ y_d(t) \\ \theta_d(t) \end{pmatrix} \quad (21)$$

Pose Error Definition: The error between the actual pose and the reference pose ξ_d :

$$\begin{aligned} e_p &= \sqrt{(x_d - x)^2 + (y_d - y)^2}, \\ e_\theta &= \theta_d - \theta. \end{aligned} \quad (22)$$

where, e_p is the distance error, e_θ is the heading mode identifier.

When $t < 20$ s or $e_p \geq \lambda$, mode = 0 (position tracking mode); When $t \geq 20$ s and $e_p < \lambda$, mode = 1 (heading control mode). (λ is the distance threshold, e.g., $\lambda = 0.01$ m.)

Control Law Derivation: Angular velocity control (proportional control):

$$\omega = k_p [\text{mode} \cdot (\theta_{\text{des}} - \theta) + (1 - \text{mode}) \cdot (\theta_g - \theta)] \quad (23)$$

where, $\theta_g = \arctan 2(y_d - y, x_d - x)$ is the target heading, $\theta_{\text{des}} = \pi/2$ is the steering desired heading, k_p is the proportional

coefficient[27]. Linear velocity control (switch logic):

$$v = \begin{cases} v_0, & \text{mode} = 0 \text{ and } e_p \geq \lambda, \\ 0, & \text{otherwise.} \end{cases} \quad (24)$$

(v_0 is the base linear velocity, e.g., $v_0 = 0.05 \text{ m/s}$)

5.3 | Wheel Speed Drive and Pose Update

5.3.1 | Wheel Speed Inverse Solution and Motor Drive

Desired Wheel Speed Calculation: The control input v, ω is converted to left and right wheel linear speeds:

$$\begin{cases} v_{l,\text{ref}} = v + \frac{\omega L}{2}, \\ v_{r,\text{ref}} = v - \frac{\omega L}{2}. \end{cases} \quad (25)$$

Desired Motor Speed Calculation: The relationship between wheel linear speed and motor speed is determined by the wheel radius:

$$\begin{cases} n_{l,\text{ref}} = \frac{60 \cdot v_{l,\text{ref}}}{2\pi r}, \\ n_{r,\text{ref}} = \frac{60 \cdot v_{r,\text{ref}}}{2\pi r}. \end{cases} \quad (26)$$

Actual Motor Speed to Actual Linear Speed:

$$\begin{cases} v_{l,\text{real}} = \frac{2\pi r \cdot n_{l,\text{real}}}{60}, \\ v_{r,\text{real}} = \frac{2\pi r \cdot n_{r,\text{real}}}{60}. \end{cases} \quad (27)$$

5.3.2 | Pose Update: Discretization and Normalization

Discretized Pose Recursion Using Euler discretization (sampling time T), assuming constant wheel speed and heading angle within a sampling period, the pose update is:

$$\xi(t+T) = \xi(t) + T \cdot \begin{bmatrix} \cos \frac{\theta(t)}{2} & \cos \frac{\theta(t)}{2} \\ \sin \frac{\theta(t)}{2} & \sin \frac{\theta(t)}{2} \\ \frac{1}{L} & -\frac{1}{L} \end{bmatrix} \begin{bmatrix} v_l(t) \\ v_r(t) \end{bmatrix} \quad (28)$$

$$\begin{cases} x(t+T) = x(t) + T \cdot \cos \frac{\theta(t)}{2} \cdot (v_l(t) + v_r(t)), \\ y(t+T) = y(t) + T \cdot \sin \frac{\theta(t)}{2} \cdot (v_l(t) + v_r(t)), \\ \theta(t+T) = \theta(t) + T \cdot \frac{v_l(t) - v_r(t)}{L}. \end{cases} \quad (29)$$

This formula directly describes the physical process of wheel linear speed $\rightarrow$ pose change and is the core of the closed-loop feedback. In discrete-time notation (k):

$$\begin{cases} x(k+1) = x(k) + T \cdot \cos \frac{\theta(k)}{2} \cdot (v_{l,\text{real}}(k) + v_{r,\text{real}}(k)), \\ y(k+1) = y(k) + T \cdot \sin \frac{\theta(k)}{2} \cdot (v_{l,\text{real}}(k) + v_{r,\text{real}}(k)), \\ \theta(k+1) = \theta(k) + T \cdot \frac{v_{l,\text{real}}(k) - v_{r,\text{real}}(k)}{L}. \end{cases} \quad (30)$$

where, $v_{l,\text{real}}, v_{r,\text{real}}$ are the actual wheel linear speeds from the motor drive system[28].

Heading Angle Periodic Normalization

$$\theta(k+1) = \text{mod}(\theta(k+1), 2\pi) \quad (31)$$

6 | SYSTEM SIMULATION RESULTS AND ANALYSIS

To verify the effectiveness, accuracy, and robustness of the designed trajectory tracking control system and direct wheel speed control model for the customs port sampling robot, a comprehensive simulation model integrating Permanent Magnet Synchronous Motor (PMSM) drive control, Clark/Park coordinate transformation, SVPWM modulation, wheel speed kinematic modeling, and trajectory tracking closed-loop control was built based on the Matlab/Simulink platform. The model replicates the actual working conditions of port sampling operations, conducting comprehensive tests on motor drive performance, trajectory tracking accuracy, mode switching logic, and system disturbance rejection[29]. Simulation parameters strictly follow the robot's actual design specifications and calculated parameters to ensure the engineering reference value of the simulation results[13]. The core simulation parameters are shown in Table 3.

TABLE 3 Core simulation parameters

Parameter Category	Parameter Name	Value
Robot Body	Wheelbase / Track Width	450mm / 380mm
	Wheel Radius	0.15m
	Total Weight	440kg
	Base Linear Velocity	0.5m/s
Motor Parameters	PMSM Pole Pairs	4
	Wheel Angular Velocity	3.33rad/s
	Single Motor Rated Torque	$\geq 11.65\text{N}\cdot\text{m}$
Trajectory Control	Sampling Time (Euler)	0.01s
	Distance Threshold (λ)	0.05m
	Heading Proportional Coefficient (K_p)	5

6.1 | Trajectory Tracking Core Module Simulation Result Analysis

During the operation of the customs port sampling robot, straight-line trajectories (straight-line travel between cargo stacks) and circular arc trajectories (turning in confined spaces, sampling around cargo piles) are predominant. This simulation verifies the effectiveness of the trajectory tracking control law, mode switching logic, and pose update module for these two typical reference trajectories, while analyzing the convergence characteristics of distance and heading errors[30].

6.1.1 | Preset Trajectory 1

This trajectory is a segmented motion path of the robot from 0 to 40 seconds, consisting of two straight segments and two circular arc turning segments. The straight-line speed is a constant 5 m/s, and the turning radius is approximately 47.75 m.

0–10 s: The robot moves straight at a uniform speed along the positive X-axis, traveling from the origin (0,0) to (50,0) with a heading angle maintained at 0 rad.

10–20 s: It performs a left-turn circular arc motion, rotating 60° counterclockwise, and the heading angle synchronously increases to $\pi/3$ rad.

20–30 s: It switches to a right-turn circular arc motion, rotating 60° clockwise, and the heading angle gradually returns to zero.

30–40 s: It moves straight along the positive X-axis for the second time, advancing at a uniform speed with a zero heading

angle.

The overall trajectory starts with linear movement, followed by left and right circular arc turns to reset the heading, and ends with a straight-line segment, featuring smooth motion and coherent transitions. The simulation results show that:

The actual motion trajectory of the robot highly coincides with the reference linear trajectory. The distance error converges rapidly to within 0.01 m within 0.3 s with no steady-state drift, which is far below the 0.05 m distance threshold.

The heading error is always controlled within $\pm 0.5^\circ$ without cumulative deviation, indicating that the kinematic modeling of wheel speed drive is accurate, the linear velocities of the left and right wheels are evenly distributed, and the yaw deviation during straight travel is effectively avoided.

The position tracking mode is maintained throughout the process, with a stable linear velocity output of 0.5 m/s and no frequent fluctuations, meeting the constant-speed motion requirements of sampling operations.

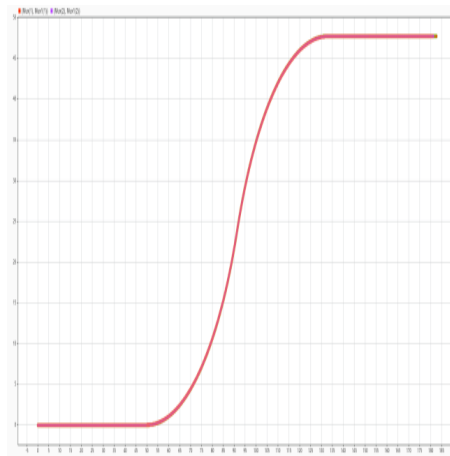


FIGURE 11 Comparison of Preset Trajectory and Actual Trajectory

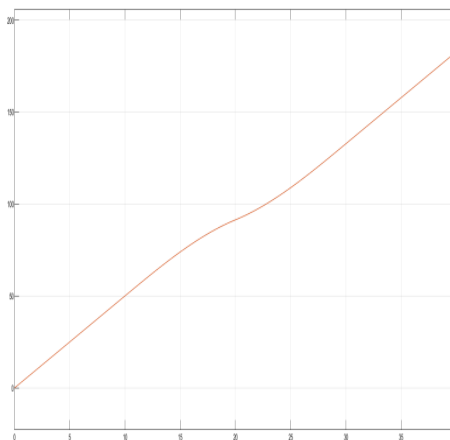


FIGURE 12 Comparison of X-axis Tracking

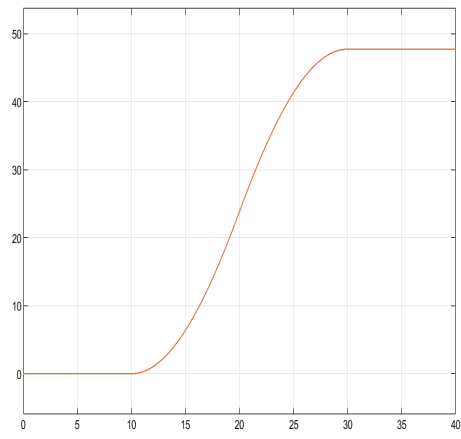


FIGURE 13 Comparison of Y-axis Tracking

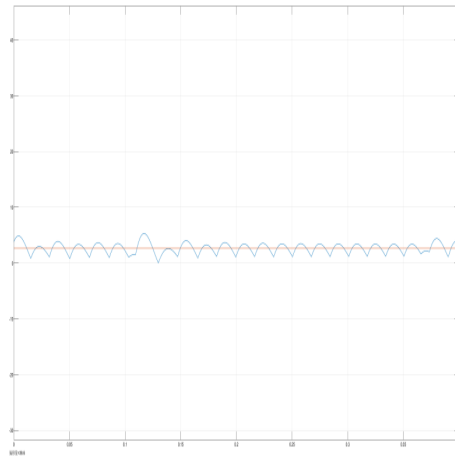


FIGURE 14 Tracking curve of the left wheel motor

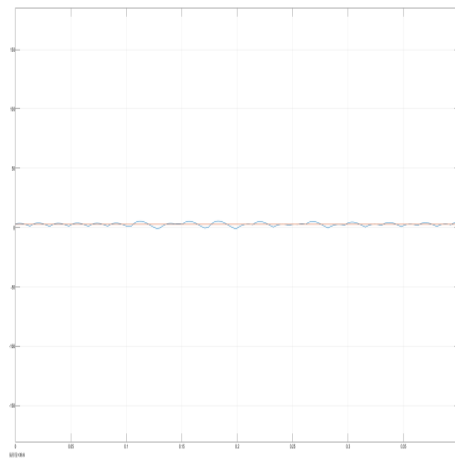


FIGURE 15 Tracking curve of the right wheel motor

6.1.2 | Preset Trajectory 2

This trajectory is an ultra-smooth segmented motion trajectory for the robot within 40 seconds, with no abrupt changes in position or angle throughout the process.

0–16 s: The robot moves horizontally straight along the positive X-axis at a constant speed from the origin (0,0) to (30,0), with the heading angle maintained at 0 rad.

16–28 s: A fifth-order Bézier curve is adopted for the robot's path, starting at (30,0) and ending at (80,0), ensuring a direction change of 20° , i.e., the heading angle changes from 0 rad to $\pi/9$ rad.

28–40 s: It moves straight along the 20° inclined direction to (80,30), with the heading angle stabilized at 20° , and stops at the end point if overtime occurs.

The system additionally performs multiple rounds of smoothing optimization on position coordinates and heading angles through a 5-point weighted moving window average and three rounds of neighborhood smoothing iteration. The curve end point is automatically calibrated to ensure that the direction error of 20° is less than 0.01 rad, resulting in extremely smooth overall trajectory transitions.

The simulation results show that: During the robot's turning process, the maximum distance error is no more than 0.03 m, the heading error is controlled within $\pm 1^\circ$, and both converge rapidly to the steady-state ranges of 0.01 m and $\pm 0.5^\circ$ respectively after the turn is completed.

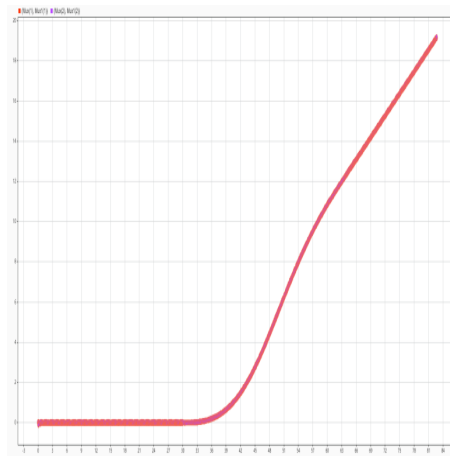


FIGURE 16 Comparison of Preset Trajectory and Actual Trajectory

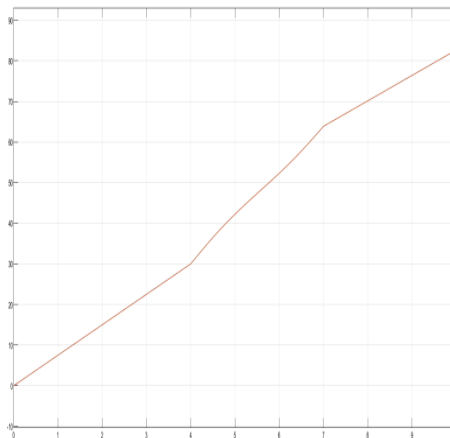


FIGURE 17 Comparison of X-axis Tracking

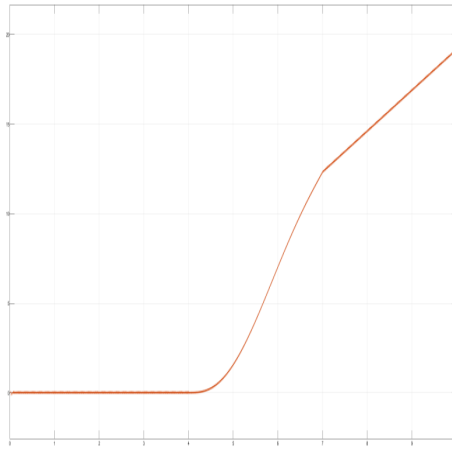


FIGURE 18 Comparison of Y-axis Tracking

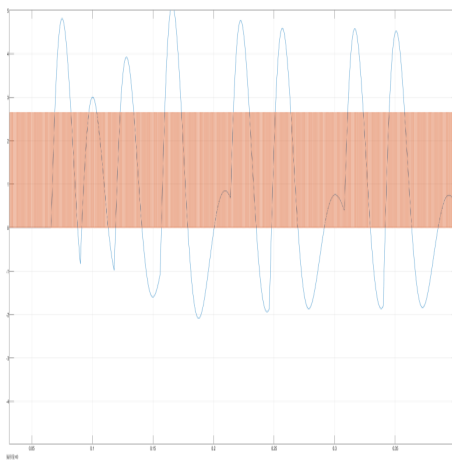


FIGURE 19 Tracking curve of the left wheel motor

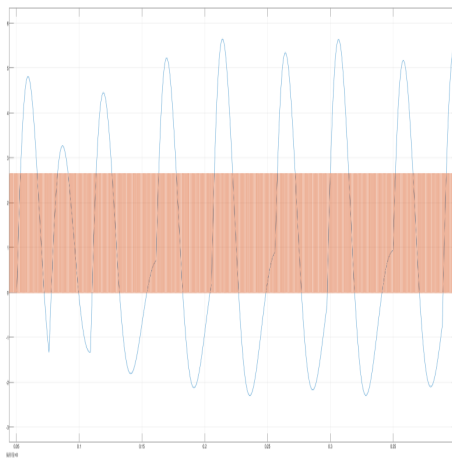


FIGURE 20 Tracking curve of the right wheel motor

7 | ACKNOWLEDGEMENT

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